James Clerk Maxwell on Theory Constitution and Conceptual Chains

Igor Hanzel

Comenius University in Bratislava

Abstract: The aim of this paper is to analyze J. C. Maxwell's *Treatise on Electricity and Magnetism* at the levels of scientific theory and methodology in order to show that it displays certain highly specific epistemic features. We shall start with a reconstruction of the method by means of which is it constituted by briefly delineating the main partial theories utilized up by Maxwell. Next, we shall show that the *Treatise* also involves potentially also an additional method of theory constitution that was not, however, employed by Maxwell. This method, as we will show next, enables one to employ the *Treatise* as a 'structuro-anatomic key' for the reconstruction of those theories in which the *Treatise* initially originated. Then will provide a critical reflection on Maxwell's views on the swing from reflections on vortices and "idle-wheel" particles, which he introduced in the article "On Physical Lines of Force" to the examination of the employed Lagrange's method of analytic mechanic. Finally, we shall employ Maxwell's *Treatise* as a "key" for the analysis of the more recent discussion of the so-called 'Inconsistency of Classical Electrodynamics.'

Keywords: J. C. Maxwell, *Treatise on Electricity and Magnetism*, *On Physical Lines of Force*, methodology, scientific theory, method of theory construction

1 Introduction

The aim of this paper is to analyze J. C. Maxwell's *Treatise on Electricity and Magnetism* (hereafter, *Treatise*) at the levels of *scientific theory* and *methodology* in order to show that it displays certain highly specific epistemic features. We shall start with a reconstruction of the method by means of which the *Treatise* is constituted by briefly delineating the main partial theories utilized up by Maxwell as the starting points for the creation of a theory of electromagnetism.

Next, we shall show that the *Treatise* also involves, at least potentially, also an additional method of theory constitution that was not, however, employed by Maxwell. This method, as we will show next, enables one to employ the *Treatise* as a 'structuro-anatomic key' for the reconstruction of those theories in which the *Treatise* initially originated. We then will provide a critical reflection on Maxwell's views on the swing from reflections on vortices and "idle-wheel" particles, which he introduced in the article "On Physical Lines of Force" (hereafter "Physical"), to the examination of Lagrange's method of analytic mechanic employed in the *Treatise* which seems to be free from these reflections.

Finally, we shall employ Maxwell's *Treatise* as a "key" for the analysis of the more recent discussion of the so-called 'Inconsistency of Classical Electrodynamics.'

2 *Treatise*'s Method of Theory Constitution

Maxwell employs three partial theories as the basis for his conceptual synthesis in the *Treatise*—electrostatics, electrokinematics, and the theory of magnetism.

2.1 Electrostatics

In the framework of electrostatics, Part I of the *Treatise*, understood as a study of the phenomena of electricity in repose, Maxwell takes as the point of departure the description of phenomena produced in certain experiments: the phenomenon of electrification by friction yielding concepts like positive and negative electrification, electrification by induction and conduction based on experiments with a metal hollow vessel subject to electrification under various conditions, yielding, finally, the term "electricity" understood as a physical quantity synonymous with the total electrification of a body (1954, Vol. I, Arts 34–35, 38–39).[1]

[1] In our references to and quotations from Maxwell's *Treatise* we provide the respective article number in the *Treatise* as "Art."

Next, he starts reflecting on the force interaction between two bodies charged by e and e' of units of electricity so that for f, as the force of repulsion between charges e and e' holds $f = ee'r^{-2}$ (1954, Vol. I, Art. 41, 45–46). This enables him to pass to the term "electric field" understood as "the portion of space in the neighbourhood of electrified bodies, considered with reference to electric phenomena" (1954, Vol. I, Art. 44, 47), and where the investigation of this field he regards as the essential feature of his own investigation into the nature of electricity. If e stands for the charge of a body, F for the force acting on this body, then it holds that $F = Re$, where R is a function of the distribution of electricity in other bodies in the field and it is labeled by Maxwell as the "resultant electromotive intensity at the given point of the field" (1954, Vol. I, Art. 44, 48). Its vector notation is given by Maxwell via the German letter $\mathfrak{E}$, we use for it the bold letter **E**.[2] Based on the concept of electric intensity he considers, as a thought experiment, that if charge e is shifted from point A to point B, then **E** is the total electromotive force along the path AB and e**E** is the work done by this force on that body. Once he chooses B as a reference point for all other points in the field, then **E** along the path from A to B is labeled as "potential of point A" (1954, Vol. I, Art. 45, 48), to which, as a physical quantity, he later assigns the sign Ψ, while B is placed, at least in mind, at an infinite distance from other points.

Based on the concept of potential he then labels the difference of potentials between two points of a conductor as the "electromotive force between two points" (1954, Vol. I, Art. 49, 52). He also introduces also the concept of *line of force* as "the line described by a point moving always in the direction of the resultant intensity" (1954, Vol. I, Art. 47, 51), the concept of *electric tension* viewed as a force acting on a (unit) area of the surface of a conductor (1954, Vol. I, Arts. 47–48, 51) and, later on, also the concept of the *capacity of a conductor* as given by the ratio of the electric charge of the conductor and of its potential. As a conclusion of Chapter I, Maxwell introduces the term "specific inductive capacity" or "dielectric constant of the substance," symbo-

2 In this paper we replace Maxwell's German letters standing for vector quantities by bold Latin letters. We hold to Maxwell's notation that uses the same notation for a partial as for a total differential.

lized later as K, and understood as the capacity of an accumulator with a dielectric as compared to that with air as substance (1954, Vol. I, Art. 52, 55), as well as the term "electric polarization" understood as a state in which each particle under the impact of electric intensity becomes positive on one side and negative on the other, thus electric polarization "consists of electric displacement" (1954, Vol. I, Art. 60, 65).

The chain of introduction of new terms for physical quantities goes on in Chapter II. Here Maxwell brings in the terms "electric volume-density," ρ, "electric surface-density," σ, "electric line-density," λ, and then considers again the resultant electric vector intensity **E** as causing a displacement of electricity in the body, and thus causing, if the body is a dielectric, a displacement of electricity in it. If the vector **D** stands for displacement and K for the specific inductive capacity of the dielectric, then $\mathbf{D} = \frac{1}{4\pi} K\mathbf{E}$ holds (1954, Vol. I, Art. 68, 75–76). According to Maxwell, as long as **E** is continuously acting on the conductor, then what is produced is the so-called *current of conduction*.

Next, Maxwell defines the term *potential at a point* as "the work to be done by an external agent in order to bring the unit of positive electricity from an infinite distance to the given point" (1954, Vol. I, Art. 70, 78) and for which he states the *Poisson equation* $\nabla^2\Psi = 4\pi\rho$,[3] that is, electrical density when multiplied with 4π, yields the concentration of the potential.

In Chapter IV, Part I of the *Treatise*, Maxwell shows that the quantity **E** equals the space variation of the potential Ψ, and it holds $\mathbf{E} = -\mathbf{grad}\ \Psi$, and mutually relates the quantities **D** and **E** by means of the equation $\mathbf{E} = 4\pi\mathbf{D}$ for standard media where **D** has the components f, g, h in the direction of the axes x, y, z respectively, while for isotropic media holds $\mathbf{E} = 4\pi K\mathbf{D}$ (1954, Vol. I, Art 101e, 145).

2.2 Electrokinematics

In Part II of the *Treatise* Maxwell deals with quantities and their relations in framework of *electrokinematics* as the theory dealing with the

3 Here ∇^2 stands for the square of the nabla operator ∇.

motion of electric charges in its most abstract aspect, that is, without paying attention to the causes of this motion (Maxwell 1952, 26).

Based on the concept of electric potential and considering at the same time the potentials of two conductors *A* and *B*, so that for their respective potentials it holds that $\Psi_A > \Psi_B$, one obtains by connecting *A* and *B* by means of a conductor *C*, *electric current*, that he then differentiates into a transient and a steady current; the latter being produced, for example, by a voltaic battery. Once the latter becomes a part of a circuit, Maxwell brings in *Ohm's law*.

2.3 Theory of magnetism

In Part III of the *Treatise* Maxwell presents the theory of magnetism by introducing terms like "axes of a magnet," "direction of magnetic force," and "strength of the pole of a magnet," the last of which he views as synonymous with "quantity of 'magnetism'", where in each magnet it holds that this quantity as a total is zero (1954, Vol. II, Art 377, 4). He then introduces the term "magnetic moment" understood as the product of the length of a uniformly and longitudinally magnetized bar magnet with the strength of its positive pole, as well as the term "intensity of magnetization" of a magnetic particle. This quantity is initially represented as *I* and then as vector **J**, understood as the ratio of the magnetic moment of that particle to its volume; its direction being defined by the direction cosines λ, μ, and ν.[4] Based on quantity *I* and λ, μ, ν, one can then describe the *magnetization of a magnet* by means of the three components *A*, *B*, and *C* of **J**, so that $A = I\lambda$, $B = I\mu$, $C = I\nu$.

Next, Maxwell introduces α, β, and γ as the components of an *external magnetic force*, in vector notation symbolized as **H**. Based on **H** and **J**, Maxwell then brings in the description of the actual force on a unit pole by means of the vector **B** (and its components *a*, *b*, *c*), so that it holds $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{J}$ (1954, Vol. II, Art. 399, 25). Here **B** stands for the vector of *magnetic induction*, **H** for the vector of *magnetic force*, and **J** for the vector of *magnetization*.

Maxwell then, with respect to **B**, introduces yet another quantity, namely, **A**, labeled initially as the *vector potential of magnetic induction*,

4 For an explication of this concept see Simpson (1998, 183–184)

with *F*, *G*, and *H* as its components, so that it holds that this magnetic induction is the **curl** of its vector potential; in symbols: **B** = **curl A** (1954, Vol. II, Art. 406, 31–32).

In a final step, Maxwell reflects on magnetization from the point of view of its production and change and starting from the supposition that **J** is proportional to **H**, he labels their mutual ratio as the *coefficient of induced magnetization*, κ, so that it holds that **J** = κ**H** (1954, Vol. II, Art. 426, 50). By unifying the last equation with the equation **B** = **H** +4π**J** one obtains the equation **B** = (1 + 4πκ)**H**, so that the ratio of **B** to **H** equals 1 + 4πκ. The expression 1 + 4πκ Maxwell labels as *magnetic inductive capacity of a substance* that, as a quantity, differs from the quantity κ.

3 The Synthesis

Part IV of the *Treatis*e unifies the partial theories outlined already into one system of equations for quantities. As a point of departure Maxwell draws on Örsted's experiments showing that a wire connecting the ends of a voltaic battery affects a nearby magnet. According to Maxwell, behind this impact is a force in the space surrounding the wire that is acting on the magnet. He views this space as a magnetic field because the force depends on both the strength of the current in the wire and the location of the latter. So, he applies the quantities like **B** and **H** for the description of this field and the determination of the force acting on any portion of an electric circuit placed into a magnetic field.

Next, Maxwell deals with the phenomenon of magneto-electric induction in an experimental set up involving two conducting circuits, the so-called primary and secondary, so that the former is connected to a voltaic battery and the latter to a galvanometer, and both have mutually parallel straight parts. He considers the case when current is send through the straight part of the primary circuit and the galvanometer indicates, as an effect, at the instance of sending the current, a current in the secondary circuit that is of a direction opposite to the current in the primary circuit; this he labels as the *induced current* (1954, Vol. II, Art. 530, 178). It also holds that any variation of the primary current generates an electromotive force in the secondary circuit, so that an increase (or decrease) of the primary current generates an electromotive force of an opposite (or the same) direction as compared to the current; and when the primary current is constant, no electromotive force is produced.

Then, Maxwell considers cases of induction caused by the motion of either the primary or the secondary circuit. He then generalizes these cases by into an experimental set up with a primary coil *A* and a secondary coil *B*, and shows (1954, Vol. II, Art. 539, 186) that when simultaneous movements of coil A from A_1 to A_2 and of coil B from B_1 to B_2 take place, while in coil A the current changes from γ_1 to γ_2, the total induction current depends only the initial and final states represented by the triples A_1, B_1, γ_1 and A_2, B_2, γ_2. In Maxwell's notation the total induction current is represented as $F(A_2, B_2, \gamma_2) - F(A_1, B_1, \gamma_1)$, where *F* is a function of positions *A*, *B* and of the current γ. This function is further specified by Maxwell as *M*, so that it then holds $C\{M_1\gamma_1 - M_2\gamma_2\}$, where *C* stands for the conductivity of the second circuit and $M_1\gamma_1$ ($M_2\gamma_2$) for the initial (or final) values of *M* and γ. Thus, Maxwell's interpretation is that the total current of induction depends of the change of a quantity represented as *M*γ, where γ stands for the primary current and *M* has its origin in the movements of the circuits, while *M*γ as a whole is interpreted as the *number of lines of force that pass at any instance through the circuit*.

Next, Maxwell brings in the concept of the electromotive force and unifies it with that of the number of lines of forces so that it holds that "[t]he total electromotive force acting around a circuit at any instant is measured by the rate of decrease of the number of lines of magnetic force which pass through it" (1954, Vol. II, Art. 541, 189), and where instead of "number of lines of magnetic force" one can use the term "magnetic induction through the circuit."

Chapter IV of Part IV is important, from the point of view of our paper, because for the first time Maxwell states at the general level—in the framework of the *Treatise*—his ambition not to define the forces at work in electromagnetic phenomena but to *get rid of them* (1954, Vol. II, Art 552, 198):

> We are … led to inquire whether there may not be some motion going on in the space outside the wire, which is not occupied by the electric current, but in which the electromagnetic effects of the current are ma-

> nifested. I shall not at present enter on the reasons for looking in one place rather than another for such motions, or for regarding these motions as of one kind rather than another. What I propose now to do is to examine the consequences of the assumption that the phenomena of the electric current are those of a moving system, the motion being communicated from one part of the system to another by forces, the nature and laws of which we do not even attempt to define, because we can eliminate these forces from the equations of motion by the method given by Lagrange for any connected system. In the next five chapters of this treatise I propose to deduce the main structures of the theory of electricity from a dynamical hypothesis of this kind.

Accordingly, with respect to the phenomenon of electric current this means the following (1954, Vol. II, Arts. 551-552, 197–198):

> We have already shown that it has something very like momentum, that it resists being suddenly stopped, and that it can exert, for a short time, a great electromotive force. But a conducting circuit in which a current has been set up has the power of doing work in virtue of this current, and this power cannot be said to be very like energy, for it is really and truly energy. … It appears, therefore, that the system containing an electric current is a seat of energy of some kind; and since we can form no conception of an electric current except as a kinetic phenomenon, its energy must be kinetic energy. … We have already shewn that the electricity in the wire cannot be considered as the moving body in which we are to find this energy [since] … the presence of other bodies near the current alters its energy. … What I propose now to do is to examine the consequences of the assumption that the phenomena of electric current are those of a moving system.

Maxwell's desire to get rid of the concept of force is then realized by Maxwell step by step in Chapters V through VII. In Chapter V, he applies Lagrange's method of analytic mechanic, to his considerations of a physical system which is (1954, Vol. II, Art. 555, 200)

> connected by means of suitable mechanism with a number of moveable pieces, each capable of motion in straight line, and no other kind of motion. The imaginary mechanism … connects each of these pieces with the system. The use of this mechanism is merely to assist the imagination in ascribing position, velocity, and momentum to what appear in Lagrange's investigation, as pure algebraic quantities.

He assigns the symbols "q_1," "q_2," … to the positions of the different pieces of the system, while "$\dot{q}_1$," "$\dot{q}_2$," … stand for their velocities. Next, Maxwell assigns to the system the momentum p for which holds $p = \int F dt$, where F stands for the system of forces, the whole right side for "Impulse of the force" (1954, Vol. II., Art.. 558, 201), and T for the total kinetic energy of the system. The quantities q, p and T are then unified in the equation $F_x = \frac{dp_r}{dt} + \frac{dT_p}{dq_r}$,[5] and for kinetic energy holds $T_p = \frac{1}{2}(p_1 \frac{dT_p}{dp_1} + p_2 \frac{dT_p}{dp_2} + etc.)$. [6] Then, Maxwell proves that for kinetic energy expressed in terms of momenta and velocities holds $T_{p\dot{q}} = \frac{1}{2}(p_1\dot{q}_1 + p_2\dot{q}_2 + etc.)$. Finally, a third way of expressing the kinetic energy of the system is by employing T involving only velocities, $T_{\dot{q}}$.

Based on the equation for T_p, Maxwell then claims that he can get rid of the term "impulsive force" by doing the following (1954, Vol. II, Art. 560, 204). He considers, first, the total variation of T_p. So as $T_p = T_p(p, q)$, one obtains

$$\delta T_p = \sum(\frac{dT_p}{dp}\delta p) + \sum(\frac{dT_p}{dp}\delta q).$$

If one now considers the case of an instantaneous impulse, that is, $\delta t \to 0$, then $\delta q \to 0$ holds. The last equations then turns into $\delta T_p = \sum(\frac{dT_p}{dp}\delta p)$. And, if at the same time one views in general the increment δT as caused by the action of an infinitesimal impulse with components δp_1, δp_2, etc., then for this increment it holds that $\delta T = \sum(\dot{q}\delta p)$.

---

5 Here "T_p" stands for kinetic energy in terms of the variables q and the momentum p.

6 The subscript "r" indicates that the respective magnitude belongs to the variable q_r.

By comparing the last two equations one obtains $\dot{q} = \frac{dT_p}{dp}$, that is "*the velocity corresponding to the variable q is the differential coefficient of* T_p *with respect to the corresponding momentum*" (1954, Vol. II, Art. 560, 204). From this resulting equation, to which Maxwell assigns the numeral (3), he draws the following conclusion (1954, Vol. II, Art. 560, 204):

> We have arrived at this result by the consideration of impulsive forces. By this method we have avoided the consideration of configuration during the action of the forces. But the instantaneous state of the system is in all respects the same, whether the system was brought from a state of rest to the given state of motion by the transient application of impulsive forces, or whether it arrived at that state in any manner, however gradual. … Hence, the equation (3) is equally valid, whether the state of motion of the system is supposed due to impulsive forces, or to forces acting in any manner whatever. We may now therefore dismiss the consideration of impulsive forces.

In Chapter VI, entitled "Dynamical Theory of Electromagnetism," Maxwell applies the apparatus developed in Chapter V to a system of conductors, which he now views as a dynamical system with kinetic energy, and about which he claims in a similar vein as he did in Chapter IV: "The nature of the connexions of the parts of this system is unknown to us, but as we have dynamical methods of investigation which do not require a knowledge of the mechanism of the system, we apply them to this case" (1954, Vol. II, Art 570, 213).

In Chapter VII, Maxwell applies Lagrange's method to conducting circuits A_1, A_2, etc. so that now x_1, x_2, etc. stand for the form and position of the conductor while $\dot{y}_1$, $\dot{y}_2$, etc. etc. stand for the actual current. For the electrokinetic energy of the system it holds that $T = \frac{1}{2}L_1\dot{y}_1^2 + \frac{1}{2}L_2\dot{y}_2^2 + etc. + M_{12}\dot{y}_1\dot{y}_2 + etc.$ It is worth noticing here how Maxwell interprets the terms L_1, L_2, M_{12}, etc. In the language of dynamics they stand for the electric moment of inertia of A_1, A_2, etc. and the electric product of inertia of two circuits A_1, A_2, etc. On the other hand, with respect to the very electromagnetic process L_1, L_2, etc. stand for the *coefficients of self-induction* of the circuits A_1, A_2, etc., while M_{12}, etc. stand for the *mutual induction* of A_1 and A_2, etc. Such an electromagnetic interpretation of the coefficients and variables appearing in Lagrange's method is then applied to concepts like electromotive force, electromagnetic force, as well as to cases when two circuits interact (1954, Vol. II, Arts. 579–583, 224–227).

In Chapter VIII the application of Lagrange's method leads to the formulation of the equations (A) through (C). Worth noting here is Maxwell's claims that starting from this chapter, he is taking a completely fresh approach in his theory construction, and that he shall "begin from a new foundation, without any assumptions except those of the dynamical theory as stated in Chapter VII" (1954, Vol. II, Arts. 585, 229). By further developing his ideas on the quantity he denoted as "*M*," he is able to arrive at the derivation of a vector whose components are *F*, *G*, *H* in the direction of the axis *x*, *y*, and *z* respectively. He denotes this vector as **A**, and states about it the following (1954, Vol. II, Arts. 590, 232):

> The vector **A** represents in direction and magnitude the time-integral of the electromotive intensity which a particle placed at a point (*x*, *y*, *z*) would experience if the primary circuit were suddenly stopped. We shall therefore call it the Electrokinetic Momentum *at the point* (*x*, *y*, *z*). It is identical with the quantity which we investigated in Art. 405 under the name of the vector-potential of magnetic induction.

Subsequently, he introduces three new quantities *a*, *b*, *c* as components of a vector **B**, so that it holds in the so-called quaternion (or vector) notation that

(A) $\mathbf{B} = \mathbf{curl\ A}$

Then, Maxwell derives the equations for electromotive intensity, that is, a set of three equations for the components *P*, *Q*, *R* of the vector **E**, so that it holds in the quaternion form that

$$\mathbf{E} = \mathrm{V} \cdot \mathbf{GB} - \dot{\mathbf{A}} - \mathbf{grad}\ \Psi,$$

The symbol "V" expresses the idea that from the multiplication of the vectors **G** (vector of velocity of a point) and **B**, only its vector component should be taken into account. In modern notation this can be expressed as "**G** × **B**." We thus have:

(B) $\mathbf{E} = \mathbf{G} \times \mathbf{B} - \dot{\mathbf{A}} - \mathbf{grad}\ \Psi.$

Finally, he derives the equations for the components X, Y, Z of the vector of electromagnetic force **F**, so that it holds in vector notation that

$$\text{(C)} \qquad \mathbf{F} = \mathbf{C} \times \mathbf{B} + e\mathbf{E} - m\,\mathbf{grad}\,\Omega,$$

where **C** stands for the total (true) electric current (with components u, v, w), e for electric volume density, m for volume density of magnetic matter, and Ω for the magnetic potential.

In Chapter IX, Maxwell presents the equations (D) through (L) which he labels as the "general equations of the electromagnetic field." From the point of view of this paper, the most important of these are the following. The equation of electric current

$$\text{(E)} \qquad 4\pi\mathbf{C} = \mathbf{curl}\ \mathbf{H},$$

and the equation of true current

$$\text{(H)} \qquad \mathbf{C} = \mathbf{K} + \frac{d\mathbf{D}}{dt}.$$

4 Treatise and its Method of Constitution

As shown earlier, the point of departure of Maxwell's theory constitution in the *Treatise* are three partial theories: electrostatics with the quantities **E**, **D**, Ψ; electrokinematics with the quantities total electric current **C** and resistance R; and the theory of magnetism with quantities **J**, **A**, **B**, **and H**. So, the road from these partial theories with their respective quantities to the theory of electromagnetism in Part IV of the *Treatise* can be represented by Figure 1, below.

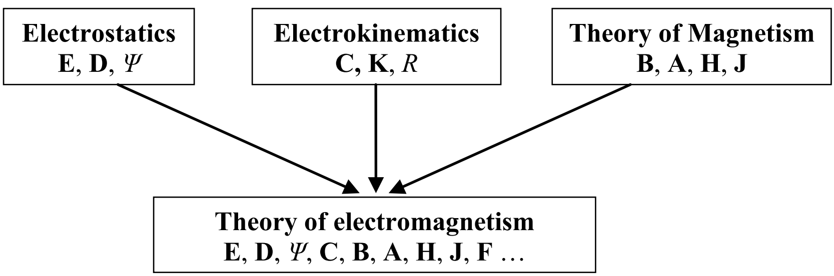

Fig. 1 Maxwell's synthesis of partial theories in the *Treatise*

The *Treatise* contains also—at least potentially—another method of theory constitution which, however, Maxwell did not employed here, to best of my knowledge.[7] What we have here in mind is that *once the equations (A) through (L) are derived, one can start to reflect on conditions under which the concepts and equations initially stated in the framework of the partial theories are still valid, and also to find out why historically these partial theories were the starting point for the derivation of the theory of electromagnetism*.

In this view of theory constitution equations (B) and (E). appear to play a central role. According to the former one obtains an electric field not only due to the contribution of the electric potential Ψ, but also due to changes pertaining to the magnetic field, where **G** stands for the speed of the *movement* of the magnetic field while $\dot{\mathbf{A}}$ stands for the time-change of the electromagnetic momentum. And in equation (E) the rotation of the magnetic field generates the current **C**.

Based on this, one can perform the following *thought operations with physical conditions*. If in (B) both **G** and $\dot{\mathbf{A}}$ are put equal to zero, one ends up with a electric field described by the equation

(B*) $\mathbf{E} = -\,\mathbf{grad}\,\Psi$,

that is, by the equation already given in the *Treatise* in the description of the electrostatic field. And for a field free of rotations of magnetic field, that is,

(E*) $\mathbf{curl}\,\mathbf{H} = 0$,

one ends up with $\mathbf{C} = 0$, thus, with a field free of (true) electric current.

One can proceed even further and suppose that all magnetic magnitudes **A**, **B**, **H**, **J**, and Ω are put equal to zero, that is, that the magnetic field is literally switched off. One then ends up with a "pure" electric field. In the same manner, one can in mind switch off the electric field by supposing that Ψ, E, **D**, *e*, **K**, **D**, and **C** are all equal to zero, thus ending up with a "pure" magnetic field.

---

7 Professor T. Blažek from the Department of Theoretical Physics at Comenius University provided valuable advice in our reconstruction of this method of theory constitution.

Based on these thought operations with physical conditions one can complete Figure 1 and obtain Figure 2.

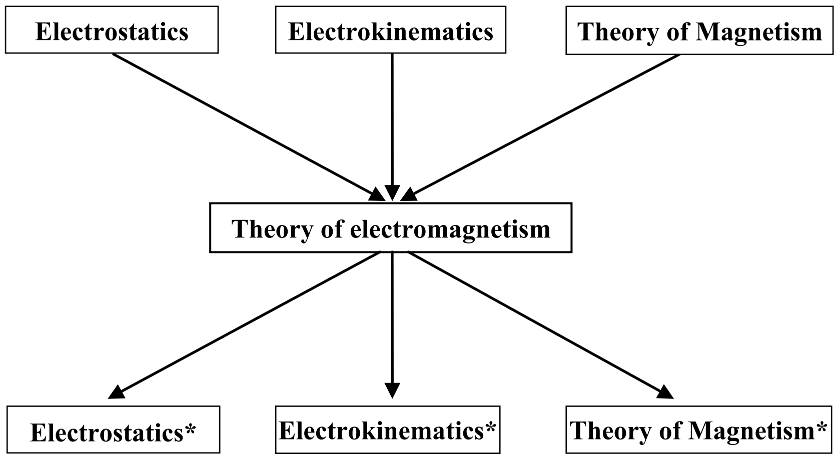

Fig. 2 Method of theory constitution in Maxwell's *Treatise*

Figure 2 shows that Maxwell's theory of electromagnetism in the *Treatise* can serve as a point of departure from which one can derive, as special cases, theories that point back to the historical background and basis of the *Treatise*. To the names of those theories we have assigned a star (i.e., *) in order to indicate that they differ from the theories representing that background and basis. Four our purposes, these "starred" theories stand for the 'anatomic keys' that—once the derivations are accomplished—can serve as a theoretical key and guide for the analysis of the respective information on the history of theories of electricity and magnetism. In this way, we may obtain a preliminary means for answering the question *why physicists initially stuck to the partial (non-starred) theories*. The preliminary answer is that these physicists were in an *unintentional* way theoretically and practico-experimentally involved in situations that involved special conditions like $\mathbf{G} = 0$, $\dot{\mathbf{A}} = 0$, $\mathbf{curl\ H} = 0$, etc.

What has to be emphasized here is that that preliminary answer to that question is at the same time just a superficial one because from the non-starred theories what is reconstructed is only what was preserved inside Maxwell's theory of electromagnetism as given in Part IV of the *Treatise*. The latter can thus serve merely as *a theoretical key and guide for an analysis of the history of electricity and magnetism, but it cannot replace that analysis*, which would involve a substantial submersion into the works of physicists like Faraday, Örsted, and others.

5 The Background of the Treatise: A Conceptual Retrospective

As quoted already above, Maxwell claims in Chapter V, Part IV, about the employment of Lagrange's method the following: "I have applied this method so as to avoid the explicit consideration of the motion of any part of the system except the coordinates or variables, on which the motion of the whole depends" (1954, Vol. II, Art. 554, 200). On the other hand, in Chapter VI one reads that "the nature of the connections of the system is unknown to us, but as we have dynamical methods of investigation which do not require a knowledge of the mechanism of the system [of conductors], we shall apply them to this case" (1954, Vol. II, Art. 570, 213). And then in Chapter VIII he claims that from that point on, he will draw only on the dynamical theory explicated here.

Maxwell even states the following *scientific principle* which is at the basis of this strategy (1965f, 783–784):

> in the study of any complex object, we must fix our attention on those elements of it which we are able to observe and to cause to vary, and ignore those which we can neither observe nor cause to vary. In an ordinary belfry, each bell has one rope which comes down through a hole in the floor to the bellringers' room. But suppose that each rope, instead of acting on one bell, contributes to the motion of many pieces of the machinery … and suppose, further, that all this machinery is silent and utterly unknown to the men at the ropes, who can only see as far as the holes in the floor above them. Supposing all this, what is the scientific duty of the men below? They have full command of the ropes, but of anything else. … These data are sufficient to determine the motion of every one of the ropes[…] This is all that the men at the ropes can ever know. … There is no help for it.

And in another article he states about the application of this principle (1965e, 309) as follows.

> I have applied this method in such a way as to get rid of the explicit consideration of the potion of any part of the system except the coordinates or variables on which the motion of the whole depends. It is important to the student to be able to trace the way in which the motion of each part is determined by that of the variables, but I think it is desirable that the final equation should be obtained independently of this process.

Let us now try to find out, if Maxwell really holds to these claims. *First*, it is noticing that even in Chapter VII, when dealing with interactions between several electric circuits, he brings in concepts like electromotive force and electromagnetic force, thus explicitly drawing on the conceptual resources given prior to Chapter VII, Part IV, of the *Treatise*. Here he introduces the vector **A** with its components *F*, *G*, *H* and declares that it "is identical with quantity which we investigated in Art. 405 [Chapter II, Part III] under the name of the vector-potential of magnetic induction" (1954, Vol. II, Art. 590, 232). He proceeds in a similar manner when, in the process of applying Lagrange's method to conducting circuits, he interprets terms L_1, L_2, etc. as the coefficients of self-induction of these circuits and terms M_{12}, etc. as coefficients of mutual induction of these circuits.

Second, in Chapter VIII, Part IV, Maxwell *claims* to brings in *new* quantities *a*, *b*, *c* with their respective equations which, he declares, are components "of a new vector **B**" (1954, Vol. II, Art. 591, 233). But here again he is drawing on already existing conceptual resources in the *Treatise*, when he states that: "we must regard the vector **B** and its components *a*, *b*, *c* as representing what we are already acquainted with as the magnetic induction and its components" (1954, Vol. II, Art. 592, 234).

Maxwell himself comments on this reliance upon to conceptual resources of the *Treatise* which antedate his dynamico-Lagrangian reflections as follows (1954, Vol. II, Art. 592, 234):

> In the present investigation we propose to deduce the properties of this vector from the dynamical principles stated in the last chapter, with as few appeals to experiment as possible. In identifying this vector, which has appeared as the result of a mathematical investigation, with the magnetic induction, the properties of which we have learned from experiments on magnets, we do not depart from this method, for we introduce no new fact into the theory, we only give a name to a ma-

> thematical quantity, and the propriety of so doing is to be judged by the agreement of the relations of the mathematical quantity with those of the physical quantity indicated by the name.

This means that we have here semantical relations which can be expressed in Figure 3 as follows.

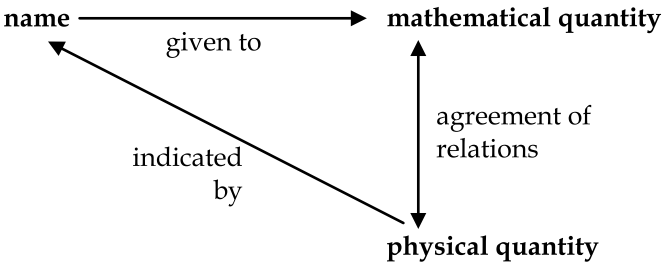

Fig. 3 Maxwell on the relation between mathematical and physical quantities

But this in turn means that what Maxwell labels as "physical quantity" is in fact the *meaning* of a name, where this name is the name of the respective physical quantity, while what he labels as "agreement of relations" stands for the identity of relations into which the respective meanings of the names of mathematical and physical quantities enter. So, the interpretation of quantities *a*, *b*, *c* (and of their unification into vector quantity **B**) as the quantity of magnetic induction is based on bringing in the *meanings* of names derived *before* bringing in Lagrange's method in Chapter VI.

A *third* difficulty with Maxwell's claims that he will draw exclusively on Lagrange's method where the latter is free of any reflections on the mechanism connecting the parts of the system under investigation becomes readily seen in his approach to the action of magnetism on polarized light and light in general. Even though he deals with it *after* he has already dealt with Lagrange's method, nevertheless he claims (1954, Vol. II, Art. 822, 461) that

> The consideration of the action of magnetism on polarized light leads … to the conclusion that in a medium under the action of magnetic force something belonging to the same mathematical class as an angu-

lar velocity, whose axis is in the direction of the magnetic force, forms a part of the phenomenon. This angular velocity cannot be that of any portion of the medium of sensible dimensions rotating as a whole. We must therefore conceive the rotation to be that of very small portions of the medium, each rotating on its own axis. This is the hypothesis of molecular vortices.

Once Maxwell brings the concept of vortex into his description of light and at the same time views light as a type of an electromagnetic wave, then the concept of vortex is spread from the description of light to that of the electromagnetic field. For this reason, at least in our view, Maxwell arrives at the following conclusion (1954, Vol. II, Art. 831, 470)

I think we have good evidence fir the opinion that some phenomenon of rotation is going on in the magnetic field, that this rotation is performed by a great number of very small portions of matter, each rotating on its own axis, this axis being parallel to the direction of the magnetic force, and that the rotations of these different vortices are made to depend on one another by means of some kind of mechanism connecting them.

What he has in mind here is his "vortices-idle-wheels" hypothesis presented in his article of 1861-1862, "On Physical Lines of Force" (hereafter, "Physical"). Let us therefore turn to this article and find out how Maxwell arrived at the concept of vortices, and how it found its way into the *Treatise*.

In part I of "Physical" Maxwell considers "the magnetic influence as existing in the form of some kind of pressure or tension, or, more generally, of *stress* in the medium" (1965c, 453), and he supposes as well (1965c, 455)

that the phenomena of magnetism depend on the existence of a tension in the direction of the lines of force, combined with a hydrostatic pressure; or in other words, a pressure greater in the equatorial that in the axial direction: the next question is, what mechanical explanation can we give of this inequality of pressure in a fluid or mobile medium? The explanation which most readily occurs to the mind is that the excess of pressure in the equatorial direction arises from the centrifugal force of vortices or eddies in the medium having their axes in directions parallel to the lines of force.

This supposition Maxwell then treats mathematically and shows that for the mechanical stress at work it holds, in vector notation,[8] that $\mathbf{T} = \frac{\mu}{4\pi}\mathbf{HH} - p_1$ (1965c, 457–458). He then provides an interpretation of each term of this equation as follows: **H** (with components α, β, γ) stands for the force acting on the end of a unit magnetic bar pointing to the north.; μ stands for the magnetic inductive capacity of the medium referred to air as a standard, and **B** (with components μα, μβ, μγ) represents the magnetic induction through unit of area perpendicular to the axes *x*, *y*, *z* respectively. Based on **T** and the equation **F** = **div T**, one can derive the quantity of magnetic force **F**. Now it holds

$$\mathbf{F} = \mathbf{H}\left(\frac{1}{4\pi}\,\mathbf{div}\,\mu\mathbf{H}\right) + \frac{\mu}{8\pi}\,\mathbf{grad}\,\mathbf{H}^2 + \mu\mathbf{H} \times \frac{1}{4\pi}\,\mathbf{curl}\,\mathbf{H} - \mathbf{grad}\,p_1.$$

The fact that the introduction of the concept of vortices yielded the equation for the quantity **F** enables Maxwell in Part II of "Physical" to "inquire into the physical connexions of … vortices with electric currents" (1965c, 468). At the same time, Maxwell faces a problem, namely, (1965c, 468) that

> in conceiving of the existence of vortices in a medium, side by side, revolving in the same direction about parallel axes. The contiguous portions of consecutive vortices must be moving in opposite directions; and it is difficult to understand how the motion of one part of the medium can coexist with, and even produce, an opposite motion of a part in contact with it.

As a solution to this problem he proposes, by drawing on the construction of mechanical devices, the following (1965c, 468):

> In mechanism, when two wheels are intended to revolve in the same direction, a wheel is placed between them so as to be in gear with both, and this wheel is called an "idle wheel." The hypothesis about the vortices which I have to suggest is that a layer of particles, acting as idle wheels, is interposed between each vortex and the next, so that each vortex has a tendency to make the neighbouring vortices revolve in the same direction with itself.

---

8 Here we draw on (Siegel 1991, 60–61). Maxwell uses in "Physical" the component and not the vector notation.

Maxwell then brings in the quantity of electric current per unit of area, **K** for short (with components *p*, *q*, *r*), and on the basis of kinematic reflections shows that it holds that $\mathbf{K} = \frac{1}{4\pi}\,\mathbf{curl\ H}$. He argues that this equation should express "the relation between the quantity of an electric current and the intensity of the lines of force surrounding it" (1965c, 471) and "[i]t appears therefore that, according to our hypothesis, an electric current is represented by the transference of the moveable particles interposed between the neighbouring vortices" (1965c, 471).

In Part II of "Physical" Maxwell also applies his "vortices-idle-wheels" hypothesis to the phenomenon of magnetic induction and derives the equation $-\,\mathbf{curl\ E} = \mu\frac{d\mathbf{H}}{dt}$ relating the changes in the state of the magnetic field to the electromotive forces generated by that changes. The "vortices-idle-wheels" hypothesis thus enables Maxwell to obtain the following two equations relating electric and magnetic quantities: $\mathbf{E} = \frac{d\mathbf{A}}{dt}$, $\mathbf{E} = \mathbf{G} \times \mathbf{B} + \frac{d\mathbf{A}}{dt} - \mathbf{grad}\ \Psi$, as well as the equation $-\,\mathbf{curl\ A} = \mathbf{B}$. Finally, in Part III of "Physical" he shows that the relation of **E** to **D** is $\mathbf{E} = -\,4\pi E^2\mathbf{D}$ and so as a time variation of **D** is a current, $\mathbf{K} = \mathbf{curl\ H}$ turns now into $\mathbf{K} = \frac{1}{4\pi}(\mathbf{curl\ H} - E^{-2}\frac{d\mathbf{E}}{dt})$.

In our view, these three equations are the result of the employment of the "vortices-idle-wheels" hypothesis. But if this is the case, then a comparison of the employment of quantities in "Physical' and the *Treatise*, on the other hand, and of their relations via equations, on the other hand, shows how the latter as a *whole* (i.e., before Chapter VII (Part IV), in Chapter VII itself, and after Chapter VII) still draws on that hypothesis. Let us make such a comparison by means of the Tables 1 and 2 below (see the next page).

From these two comparison tables is becomes apparent that the *Treatise* as a hierarchically built system of knowledge draws on the knowledge found in the earlier article "Physical." But since "Physical." draws on the "vortices-idle-wheels" hypothesis, so the whole of Maxwell's *Treatise* is based on this hypothesis, even if he believed that he could appropriate the quantities and their relations from "Physical" and at the same time dispose of that hypothesis. Thus, *it does not hold what Maxwell claimed, namely, that the final equations could be obtained independently of the epistemic/cognitive process by means of which one traces the relation of a mechanism to its parts* (1965e, 309).

Symbol of quantity (components of the quantity)	**Its meaning in**	
	On Physical Lines of Force	***Treatise on Electricity and Magnetism***
A (*F*, *G*, *H*)	Faraday's "electrotonic state"	First, vector-potential of magnetic induction, then electromagnetic momentum at a point
B (μα, μβ, μγ) (*a*, *b*, *c*)	Magnetic induction	Magnetic induction
C (*u*, *v*, *w*)	–	Total (true) current
D (*f*, *g*, *h*)	Displacement	Displacement
E (*P*, *Q*, *R*)	Force exerted by the vortices on the idle wheels	Electromotive intensity (force)
F (*X*, *Y*, *Z*)	Force on an element of the medium arising from the variation of the internal stress.	Mechanical force acting on a unit volume of the conductor
H (α, β, γ)	Force acting on the end of a unit magnetic bar pointing North.	Magnetic force
J (*A*, *B*, *C*)	–	Magnetization
K (*p*, *q*, *r*)	First, electric current per unit of area, then total current	Density of current of conduction

Tab. 1 Comparison of quantities employed in "Physical" and the *Treatise*

Equations in "Physical"	Equations in the *Treatise*
$\mathbf{F} = \mathbf{H}(\frac{1}{4\pi}\,\mathbf{div}\,\mu\mathbf{H}) + \frac{\mu}{8\pi}\,\mathbf{grad}\,\mathbf{H}^2 +$	$f = -ee'r^{-2}$
$\mu\mathbf{H} \times \frac{1}{4\pi}\,\mathbf{curl}\,\mathbf{H} - \mathbf{grad}\,p_1$	$\mathbf{D} = \frac{1}{4\pi}\mathrm{K}\mathbf{E}$
$-\,\mathbf{curl}\,\mathbf{A} = \mathbf{B}$	$\mathbf{E} = -\,\mathbf{grad}\,\Psi$
$\mathbf{K} = \frac{1}{4\pi}\,\mathbf{curl}\,\mathbf{H}$	$E = CR$ (Ohm's law)
$-\,\mathbf{curl}\,\mathbf{E} = \mu\frac{d\mathbf{H}}{dt}$	$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{J}$
$\mathbf{K} = \frac{d\mathbf{D}}{dt}$, $\mathbf{E} = -\,4\pi\mathrm{E}^2\mathbf{D}$	$\mathbf{J} = \kappa\mathbf{H}$
$\mathbf{K} = \frac{1}{4\pi}(\mathbf{curl}\,\mathbf{H} - \mathrm{E}^{-2}\frac{d\mathbf{E}}{dt})$.	$\mathbf{B} = \mathbf{curl}\,\mathbf{A}$
$\mathbf{E} = \frac{d\mathbf{A}}{dt}$, $\mathbf{E} = \mathbf{G} \times \mathbf{B} + \frac{d\mathbf{A}}{dt} - \mathbf{grad}\,\Psi$	$\mathbf{B} = \mu\mathbf{H}$
	$\mathbf{F} = \mathbf{C} \times \mathbf{B}$
	Equations (A) through (L)

Tab. 2 A comparison of equations employed in "Physical" and *Treatise*

If such an inherent determination of the knowledge in *Treatise* by the knowledge in "Physical" really took place, then we can arrive at a new view on the employability of what Maxwell labeled in the *Treatise* as "Lagrange's method." Usually it is claimed that while Lagrange's method is of such a general nature that makes it is applicable to disparate physical phenomena that can be conceptually unified, still (Morrison 2000, 64, 83)

> this generality has a drawback: By not providing an account of the way physical processes take place, the unifying power is achieved as the expense of explanatory power. … the unifying power of the Lagrangian approach lay in the fact that it ignored the nature of the system and the details of its motion.

However, an analysis of the employment of Lagrange's method in the *Treatise* as well as in Maxwell's article "A Dynamical Theory of the Electromagnetic Field" (hereafter, "Dynamical"), once it is put into relation with the paper "Physical," enables one to discover yet another important feature of Lagrange's method. In Chapter VII, Part IV, of the *Treatise* Maxwell utilized Lagrange's method to deal with the mutual action of two circuits.

The dynamical illustration given by this model goes back to Maxwell's treatment of the mutual action of two currents as given in the article "Dynamical," where Maxwell declares that (1965d, 536)

> the effect of the connexion between the current and the electromagnetic field surrounding it, is to endow the current with a kind of momentum, just as the connexion between the driving-point of a machine and a fly-wheel endows the driving-point with an additional momentum, which may be called the momentum of the fly-wheel reduced to the driving-point.

The dynamical illustration here involves a system of three elements *A*, *B*, and *C*, so that *C* is the body suspended between *A* and *B*, and where *A* and *B* stand for both bodies and driving points. After providing the details of the dynamical illustration, Maxwell states the following: "This dynamical illustration is to be considered merely as assisting the reader to understand what is meant in mechanics by Reduced Momentum" (1965d, 538).

The concept of reduced momentum and the whole dynamical illustration, however, go back to Maxwell's "vortices-idle-wheels" hypothesis in "Physical," where he states the following (1965c, 478):

> The electromotive force … arises from the action between the vortices and the interposed particles, when the velocity of rotation is altered in any part of the field. It corresponds to the pressure on the axle of a wheel in a machine when the velocity of the driving wheel is increased or diminished.

And with respect to the quantity of electrotonic state (later labeled as vector quantity **A**) he claims that (1965c, 478–479):

> it corresponds to the *impulse* which would act on the axle of a wheel in a machine if the actual velocity were suddenly given to the driving wheel, the machine being previously at rest. If the machine were suddenly stopped by stopping the driving wheel. … This impulse may be calculated for any part of a system of mechanism, and may be called the *reduced momentum* of the machine for that point.

Thus, if Maxwell's articles "Physical" and "Dynamical" along with the *Treatise* are considered *in their conceptual unity*, it is does not hold what Morrison claims about the employment of Lagrange's method, namely, that "one need not take into account of the underlying causes that produce phenomena" (Morrison 2000, 78) are not valid. Our comparison of "Physical," "Dynamical," and *Treatise* leads inescapable to the conclusion that Maxwell can employ Lagrange's method in the theory of electromagnetism as a method seemingly free of any involvement into the conceptual grasping of the mechanisms at work in certain kinds of phenomena only *after* these mechanisms have already been conceptually reflected upon and the respective quantities and their relations have been already derived from such reflections. Only then can he take over the results of such reflections, integrate them into Lagrange's method, all the while pretending to be able to dispose of those reflections. *The order (i.e., first reflections on the mechanisms at work, followed by derivation of quantities and of their relations describing this mechanism, then the introduction of Lagrange's method, finally, integration of those quantities and of their relations into Lagrange's method*) *cannot be reversed*.

6 The So-Called 'Inconsistency of Classical Electrodynamics'

The discussion on the so-called inconsistency of Classical Electrodynamics (hereafter, CED) was triggered by the views presented in (Frisch 2005).[9] According to its author, the inconsistency of CED[10] surfaces when one deals with advanced/retarded solutions of Maxwell-Lorentz equations. He introduces this inconsistency as follows (Frisch 2005, 4):

> Radiation phenomena exhibit a temporal asymmetry … it can be expressed as follows. There are coherent, diverging electromagnetic waves, but not coherent converging waves, and is so despite the fact

---

9 For comments and critique of these views see (Belot 2007), (Muller 2007) and (Vickers 2008); for Frisch's reactions to them, see (Frisch 2008).

10 Another inconsistency is related to the problem of self-interactions in CED.

> that the wave equation that can be derived from the Maxwell equations … allows for both types of waves.

For these equations there exist two types of solution. The so-called *retarded* solutions stand for a field (and/or its changes) diverging from the source/charge and propagating in such a way that the points/states of the field further and further from source/charge are determined (caused) by the propagation of disturbances from the past to the future. The so-called *advanced* solutions stand for a field converging in such a way that the solutions are correlated with the future state of the source/charge. Frisch then states the following (2009, 15):

> [C]oherently diverging disturbances of the field in the future of a radiating charge are explained by the charge's action as common cause of the disturbances. By contrast, coherently converging radiation is extremely implausible, unless it, too, has a common cause explanation in its past. (For example, a coherently converging wave might be produced by a radiating source in the center of a spherical mirror, which reflects the diverging wave due to the source back towards the center.)

In our view, there is one feature of contemporary CED that can be traced back to Maxwell's electrodynamics—and which is still operative in contemporary CED—and which can help in getting to the roots of this inconsistency. If one looks at Maxwell's equations (A) through (L), one finds out that a *charge can generate a field* (for example, a moving electric charge generates a magnetic field and an electric charge at rest generates an electrostatic field), but the opposite does not hold, namely, that *an electric charge is not produced by a field, say, in the point where radiation waves are converging*. This feature of Maxwell's electrodynamics thus enables to explain why in CED the case used by Frisch as an example, namely, a source/charge emitting radiation, is conceptualized as physically meaningful, while the case of a radiation coherently converging on a source/charge (or even generating it) is viewed as physically implausible.

It is noteworthy that this feature of Maxwell's electrodynamics contradicts another feature of Maxwell's electrodynamics, namely, the one that *assigns conceptual priority to fields as compared to charges and currents*. That this is so is readily seen in the equation (E). Here **curl H** stands on the right side as the known quantity while **C**, characterized via equation (H), stands on the left side as a *calculable* quantity.[11] In addition, (E) states that the *rotation of the magnetic field is the cause/source of current as its effect*.[12] In the article "Physical," for example, where equation (E) appears as $\mathbf{K} = \frac{1}{4\pi}\mathbf{curl\ H}$, Maxwell characterizes this equation as an "equation of … electric currents" (1965c, 496).

Finally, it is also worth noting that the conceptual priority of fields as compared to charges and currents disappears in the development of electrodynamics after Maxwell. The equation (E), as restated in *contemporary* CED, has the following form:[13]

$$\mathbf{curl\ H} = 4\pi\mathbf{J} + \frac{d\mathbf{D}}{dt}$$

That is, *magnetic field is* now *conceptualized as an effect that has two causes/sources: free electric current and displacement current*. And it is in these ways that Maxwell's *Treatise* can serve as a helpful "key" in the analysis and explication of the more recent discussions of the so-called "Inconsistency of Classical Electrodynamics.'

Department of Logic and Methodology of Sciences
Philosophy Faculty
Comenius University
Šafárikovo nám. 6
814 99 Bratislava
Slovak Republic
hanzel@fphil.uniba.sk

REFERENCES

BELOT, G. (2007): Classical Electrodynamics. *Canadian Journal of Philosophy* 60, 263–282.

---

11 In the article "On Faraday's Lines of Force" Maxwell states "These equations enable us to deduce the distributions of the currents of electricity whenever we know the values of α, β, γ" (Maxwell 1965b, 195).

12 Here we draw on (Siegel 1986).

13 Here **J** stands for density of free current and **D** for electric displacement field.

FRISCH, M. (2005): *Inconsistency, Anomaly, and Non-Locality*. Oxford: Oxford University Press.

FRISCH, M. (2008): Conceptual Problems in Classical Electrodynamics. *Philosophy of Science* 75, 93–105.

FRISCH, M. (2009): Philosophical Issues in Classical Electrodynamics. (Manuscript at http://www.philosophy.umd.edu/Faculty/mfrisch/papers/problems_in_ced.pdf).

MAXWELL, J. C. (1952): *Matter and Motion*. New York: Dover.

MAXWELL, J. C. (1954): *A Treatise on Electricity and Magnetism*, Vols. 1-2. New York: Dover.

MAXWELL, J. C. (1965a): *The Scientific Papers of James Clerk Maxwell*, Vols. 1-2. New York: Dover.

MAXWELL, J. C. (1965b): On Faraday's Lines of Force. In: Maxwell 1965a, Vol. 1, 155–229.

MAXWELL, J. C. (1965c): On Physical Lines of Force. In: Maxwell 1965a, Vol. 1, 451–513.

MAXWELL, J. C. (1965d): A Dynamical Theory of the Electromagnetic Field. In: Maxwell 1965a, Vol. 1, 526–597.

MAXWELL, J. C. (1965e): On the Proof of the Equations of Motion of a Connected System. In: Maxwell 1965a, Vol. 2, 308–309.

MAXWELL, J. C. (1965f): Thomson and Tait's Natural Philosophy. In: Maxwell 1965a, Vol. 2, 776–785.

MORRISON, M. (2000): *Unifying Scientific Theories*. Cambridge: Cambridge University Press.

MULLER, F. A. (2007): Inconsistency in Classical Electrodynamics? *Philosophy of Science* 74, 253–277.

SIEGEL, D. M. (1986): The Origin of Displacement Current. *Studies in the History and Philosophy of Science* 17, 99–146.

SIEGEL, D. M. (1991): *Innovation in Maxwell's Electromagnetic Theory*. Cambridge: Cambridge University Press.

SIMPSON, T. K. (1998): *Maxwell on the Electromagnetic Field*. New Brunswick (NJ): Rutgers University Press.

VICKERS, P. (2008): Frisch, Muller and Belot on an Inconsistency in Classical Electrodynamics. *British Journal for the Philosophy of Science* 59, 767–792.