

## In memory of Pavel Tichý

On October 26<sup>th</sup> it will be 20 years since Pavel Tichý suddenly and tragically passed away in a bush reserve near his home in Dunedin, New Zealand. The obituary in the journal *From the Logical Point of View*, penned by his lifelong friend Pavel Materna soon after the news had reached Prague, reverberated with what can only be called shock.

This short note is devoted to the memory of this outstanding, original and gifted logician, one of the greatest ever born not only in then-Czechoslovakia but also in the world. Tichý was the founder of a theory that he called Transparent Intensional Logic, known today as TIL. Thanks to Tichý, there are a number of TILians (as we call ourselves, tongue-in-cheek) who actively continue Tichý's work by further developing his theory as well as discovering new applications of it, and TIL is becoming a widely known and respected theory in the world of analytic philosophy and philosophical logic. Here we are not going to provide a detailed study of Tichý's work, nor details from his biography.<sup>1</sup> Rather, we want to point out the most important features of his brain-child TIL as of 2014. Indexing the development of TIL to a particular year is essential now that there are more people than ever working on the theory.

The foundations of TIL were laid out in the 1960s when Tichý published two remarkable papers, the first one in Czech ('Smysl a procedura'; see Tichý 1968) and the second one in English ('Intensions in terms of Turing machines'; see Tichý 1969). In these papers he came up with the revolutionary idea of *procedural semantics*. The meaning of an expression is not a set-theoretic object, e.g. a function/mapping. Its meaning is instead an algorithmically *structured procedure* that produces another entity, e.g. a set-theoretic object, or in well-defined cases fails to produce an object.<sup>2</sup> In our opinion, this is the greatest and most revolutionary feature of TIL. Actually, as it often happens to brilliant people of genius, Tichý was ahead of his time. Maybe this was

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<sup>1</sup> A summary of Tichý's logic, his biography and a complete list of his publications can be found in Tichý (2004, 9-34). For an introduction to Tichý's philosophy and logic see also Raclavský (2008).

<sup>2</sup> More precisely, Tichý's semantic schema was simple; an expression *denotes* the procedure as its meaning. Once you have the denoted procedure, you can examine what, if anything, is produced by it, entailed by it, etc.

one of the reasons why TIL was not recognized during his life as much as it would have deserved. At a time when set-theoretical semantics prevailed in the form of model theory, and the theory of algorithms and procedures was still at the dawn of its development, philosophers and logicians were barely in a position to acknowledge the value and power of this approach. Indeed, only almost thirty years later did a similar idea appear, namely in Moschovakis' work (see Moschovakis 1994).

Yet the need for *structured, hyperintensional* meanings had been recognized at least since the time of Carnap's *Meaning and Necessity*. In fact, the topic of hyperintensionality was born out of *negativity*, as it were. Carnap in (1947) noticed that there are attitudes the complement of which is neither extensional nor intensional, because the substitution of logically equivalent expressions fails there. Cresswell was later to define any individuation as hyperintensional that is finer than logically necessary equivalence. But Tichý came with a positive definition of hyperintensional, or fine-grained, individuation. Though he did not use the term 'hyperintension' as it is known today, but 'intension' as it was used before possible-world semantics usurped the term for functions with domain in possible worlds, he actually rigorously defined hyperintensions as TIL *constructions*.

Here we struggle a bit with terminology. The term 'construction' is perhaps not the most fortuitous due to its current connotations, in particular with the sense bestowed upon it by intuitionistic logic which may be considered the logical basis of constructive mathematics. Philosophically, intuitionism differs from logicism by treating logic as a part of mathematics rather than as the foundation of mathematics; from finitism by allowing (constructive) reasoning about infinite collections; and from Platonism by viewing mathematical objects as mental constructs with no independent ideal existence (see Moschovakis, forthcoming). Though TIL has much in common with constructive reasoning and intuitionism, its main paradigms are different. TIL constructions are *abstract procedures* detailing which operations are to be applied to which objects in order to produce a product, if any, of a particular type. Thus constructions are not mental objects and TIL adheres to Platonism. Moreover, for Tichý logic is not a part of mathematics. Rather, logic can lay down the foundations of mathematics. Yet TIL does not deal only with mathematics; rather, it is an overarching framework in which the logical principles governing reasoning about empirical objects are the same as those governing reasoning about mathematical objects.

Tichý's constructions represent our interpretation of Frege's notion of *Sinn* (with the exception that constructions are not truth-bearers; instead some present either truth-values or truth-conditions) and are kindred to Church's

notion of *concept*.<sup>3</sup> While the Frege-Church connection makes it obvious that constructions are not formulae, it is crucial to emphasize that constructions are not set-theoretical functions/mappings, either. Rather, technically speaking, some constructions are *modes of presentation* of functions, including 0-place functions such as individuals and truth-values, and the rest are modes of presentation of other constructions. Thus, with constructions of constructions, constructions of functions, functions, and functional values in TIL's stratified ontology, we need to keep track of the traffic between multiple logical strata. Hence all the entities of TIL ontology receive a type, and the *type hierarchy* is *ramified*. Importantly, constructions may themselves figure as functional arguments or values.

The syntax of TIL is Church's (higher-order) typed  $\lambda$ -calculus, but with the all-important difference that the syntax has been assigned a procedural (as opposed to denotational) semantics, according to which a linguistic sense is an abstract procedure detailing how to arrive at an object of a particular logical type. The TIL construction known as *Closure* is the very procedure of presenting or forming or obtaining or *constructing* a function; the TIL construction known as *Composition* is the very procedure of *constructing* the value (if any) of a function at an argument. Compositions and Closures are both multiple-step procedures, or *constructions*, that operate on input provided by two one-step constructions, which figure as sub-procedures of Compositions and Closures, namely *variables* and so-called *Trivializations*.<sup>4</sup> Characters such as 'x', 'y' 'z' are words denoting variables, which construct the respective values that a valuation function has assigned to them.<sup>5</sup> Trivialization is a special construction that was added to TIL only in 1988. In pre-1988 TIL objects were supposed to construct themselves. Yet Tichý rightly realized that an object that is not a construction cannot be a constituent of a procedure; it cannot be executed. Similarly as the constituents of a computer program must be subprograms, the constituents of a construction must be sub-constructions that occur executed. The object produced by a construction cannot be part of any construction that constructs it; it is beyond the construction. The linguistic counterpart of a Trivialization is a constant term always picking out the same object. An analogy from programming languages might be helpful. The Trivialization of an object *X*

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<sup>3</sup> The TIL theory of concepts has been developed by Pavel Materna, see Materna (1998).

<sup>4</sup> Jespersen (forthcoming) offers a philosophical description of each of the constructions, including *Single* and *Double Execution*, which we have left out above.

<sup>5</sup> See Tichý (1988, §14) on this objectual notion of variables.

(whatever  $X$  may be) and its use are comparable to a *fixed pointer* to  $X$  and the *dereference* of the pointer. In order to operate on  $X$ ,  $X$  needs to be grabbed, or ‘called’, first. Trivialization is such a grabbing mechanism. Another important role of Trivialisation is that it makes it possible to *display* constructions as arguments of other functions. This is needed in hyperintensional contexts, especially mathematical and attitude contexts. To this end we need to type constructions themselves, and the ramified hierarchy of types does just that.

Tichý’s TIL was developed simultaneously with Montague’s Intensional Logic (IL).<sup>6</sup> But TIL differs from IL in at least two important aspects. One is that we  $\lambda$ -bind separate variables  $w_1\dots w_n$  ranging over possible worlds and  $t_1\dots t_n$  ranging over times. This dual binding is tantamount to *explicit intensionalization* and *temporalization*.<sup>7</sup> The other difference is that *functional application* is the logic both of extensionalization of intensions (functions from possible worlds) and of predication. These features make it possible to handle possible-world intensions in the same way as mathematical objects. Moreover, *contra* Frege, Church and Montague, TIL does not embrace reference shift; Tichý denied both that reference shift is a fact of natural language and that a logic or formal semantics of natural language should be contextual due to reference shift. Empirical expressions denote possible-world intensions in all kinds of context, and instead of a reference shift we distinguish occurrence in supposition *de dicto* and *de re*. If the former, the whole intension is the object of predication; if the latter, the value of the denoted intension is the object of predication. Hence the intension in question must be extensionalized, that is, applied to a possible world  $w$  and a time  $t$  of evaluation in order to obtain its value (if any) at this dual index.

A main feature of the  $\lambda$ -calculus is its ability to systematically distinguish between functions and functional values. An additional feature of TIL is its ability to systematically distinguish between functions and modes of presentation of functions and modes of presentation of functional values. What makes TIL suitable for being an overarching theory for the semantics of any language is the fact that the theory construes the semantic properties of the sense and denotation relations as remaining invariant across different sorts of linguistic

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<sup>6</sup> For a critique of Montague’s intensional logic, see Duží et al. (2010, §2.4.3).

<sup>7</sup> Van Eijck – Francez (1995) and Loukanova (2009) both lack explicit intensionalization and temporalization, even though the latter builds on Moschovakis’ extension of Montague’s IL, whereby  $\text{TY}_2$  (in which  $s$  is a regular type) becomes available. Moreover, due to the lack of variables ranging over possible worlds, IL does *not* validate the Church-Rosser ‘diamond’ property. Though it is a well-known fact that an ordinary typed  $\lambda$ -calculus will have this property, Montague’s IL is deviant in this respect.

contexts. The logic of TIL is obtained in a top-down manner by devising a semantics for hyperintensional contexts, and applying the same logical principles to the increasingly less-hard cases of various intensional (modal) and extensional contexts. Thus in TIL we systematically distinguish between three different levels of abstraction. It is (i) the highest level of *constructions* presenting functions, (ii) the intermediate level of the products of constructions, that is, *functions* (including possible-world intensions), and finally (iii) the lowest level of *functional values*.

Tichý issues in (1986, 256; 2004, 654) a warning against inter-defining the notions of extensional (etc.) context and the validity of the rules of substitution of co-referring terms and existential generalization on pain of circularity:

Q: When is a context extensional?

A: A context is extensional when it validates (i) the rule of substitution of co-referential terms (i.e. is transparent) and (ii) the rule of existential generalization.

Q: And when are (i), (ii) valid?

A: Those two rules are valid when applied to extensional contexts.

We steer clear of the circle by defining the above three levels of abstraction, and these three levels are squared off with three kinds of context. Here is a summary of these three kinds of context:

- *hyperintensional context*: one or more hyperintensions/constructions occur *displayed* (though one or more constructions at least one order higher need to be executed in order to produce the displayed constructions)
- *intensional context*: one or more constructions are *executed* in order to produce one or more functions (moreover, the executed constructions do not occur within another hyperintensional context)
- *extensional context*: one or more constructions are *executed* in order to produce one or more particular values of one or more functions at one or more given arguments (moreover, the executed constructions do not occur within another intensional or hyperintensional context).

Tichý's entirely anti-contextual and compositional semantics is, to the best of our knowledge, the only one that deals with all kinds of context, whether extensional, intensional or hyperintensional, in a uniform way. The same extensional logical laws are valid invariably in all kinds of context. In particular, there is no reason why Leibniz's law of substitution of identicals, and the rule of existential generalisation should fail to be valid. What differs according to

the context is not the rules themselves but the types of the objects to which these rules are applicable. In an extensional context they are the values of the functions denoted by the respective expression; in an intensional context they are the denoted functions themselves; in a hyperintensional context they are the displayed procedural meanings themselves. Due to its stratified ontology of entities organised in a ramified hierarchy of types, TIL is a logical framework within which such an extensional logic of hyperintensions has been introduced.<sup>8</sup>

Another important feature of TIL is *partiality*. Unlike almost all the other logical systems, TIL does not avoid working with properly partial functions and improper modes of presentation that fail to produce a product. Partiality is notorious for bringing about technical complications. But Tichý was convinced that the task of a logician was not to circumvent problems stemming from technical complications, but to solve them. Indeed, we need to be able to work with partial functions, unless we rest content with an unmanageable explosion of domains. It is neither possible to restrict the logical space in an *ad hoc* way so as to avoid working with non-referring terms like ‘the King of France’, nor philosophically plausible, though technically possible, to introduce so-called impossible worlds counting ‘individuals’ like the non-existing King of France in their domain.

Moreover, functions typically have more than one argument. Conventional wisdom has it that  $n$ -ary functions can be *represented* by unary composite functions. True, Schönfinkel, in (1924), observed that there is a one-to-one isomorphic correspondence between  $n$ -ary functions and certain unary composite functions. For instance, a two-argument function mapping couples of numbers to numbers can be represented by a unary function mapping numbers to functions from numbers to numbers. However, this isomorphism breaks down when *partial* functions are included, as Tichý showed in (1982, 467–468). One and the same partial  $n$ -ary function may correspond to more than one unary function.

There are many other remarkable features of Tichý’s logic and philosophy of language, to name at least individual anti-essentialism, anti-actualism, a constant domain, and independent treatment of modal and temporal parameters. *Individual anti-essentialism* is the thesis that no individual bears any purely non-constant property by any sort of necessity. This is not to say, though, that TIL

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<sup>8</sup> For an extensional logic of hyperintensions see, for instance, Duží (2012a; 2012b; 2013). The rules for existential quantification into hyperintensional contexts are introduced in Duží – Jespersen (ms.), Duží – Jespersen (2012). Finally, Duží – Jespersen (2013) introduces the substitution method operating on hyperintensional contexts.

rejects essentialism across the board; far from it. Tichý in (1979) introduces *requisites* as necessary relations between intensions. Roughly,  $G$  is a requisite of  $F$  provided, as a matter of analytic necessity, if  $x$  is a/the  $F$  then  $x$  is also a/the  $G$ , because being a/the  $G$  is in the essence of being a/the  $F$ . The *essence* of an *intension* is then the collection of all its requisites. The result is *intensional essentialism*.

*Anti-actualism* is the thesis that the actual possible world has no special status among all the other possible worlds. TIL's possible-world semantics is custom-built for us humans lacking empirical omniscience. We cannot know which of all the possible worlds is the actual one, and we are far from knowing all the actual satisfiers of the various empirical conditions. Yet, nothing of an epistemic nature bars us from being able to apply conditions (modelled as possible-world intensions) and having our discourse revolve around them. Hence empirical expressions denote such conditions/intensions rather than their satisfiers. What Tichý's explicit intensionalization does is to make the satisfiers vanish from the logical-semantic realm altogether and to focus instead on the conditions.

In TIL individuals do not spring into being in some possible worlds and vanish in others; rather, they exist trivially and independently of possible worlds. Hence TIL eschews *possibilia* (possible worlds arguably being the only exception), and the theory operates with a *constant domain* for all worlds and times. What varies are the values that (non-constant) intensions have in different worlds and at different times, and not the domains that different worlds and times have. Non-trivial existence is not a property of individuals but of intensions, to wit, the property of being occupied/instantiated at a particular world-time pair.

Much, much more could be said here concerning TIL and Tichý's logic. Fortunately, there are numerous sources of information, in Tichý's papers, the (1988) and (2004) books, in the work of his followers, and summarizing all his work would be pointless. Instead, the one of us who was fortunate enough to meet Pavel in the flesh would like to finish this short overview with a personal memory. Pavel was sharp-witted, and many considered him difficult to be around. Yet he was a good friend with a very good sense of humor and he loved rational, fair discussions. He always went directly and rigorously to the fundamental questions at the very heart of things. But being deeply involved in his quest for a solution, he would occasionally forego diplomacy. And he had little time for irrational demagoguery devoid of argument. Maybe these were the reasons why people sometimes found him difficult. On the other hand, though a sharp debater, he was ready to accept the other's opinion and even admit his own fault, if only it was supported by a fair and valid argument. True, such

cases were rare, but not because he would not be willing to accept the opposing opinion, but because he was almost always right. I would often feel convinced that I had discovered a mistake in Pavel's arguments, only to realize, upon thorough examination and hard thinking, that Pavel was right, again. Thus I learnt to always read his papers very carefully, and to take into account his brilliant ideas and conclusions. In short, he was a genius.

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