The Nature of Assertoric-Force and the Truth in Logic: An Elucidation of Fregean Truth in the Light of Husserl's Theory of Doxic-Modification

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Abstract: The unique relation between logic and truth (*protorelation*) is crucial for understanding Fregean conception of logic. Frege has an insight that the nature of logic resides in the "truth", which he finally locates in the *assertoric-force* of a sentence. Though Frege admits that *assertoric-force* is ineffable in ordinary language, he coins in his conceptual notation for such a force a much-disputed sign, i.e., *judgment-stroke*. In this paper, I will try to demonstrate that judgment-stroke is not adequate for the task its inventor has assigned to it. Accordingly, it is misconceived and inconducive to clarify Frege's vague insight into the *protorelation*. The mistake of judgment-stroke for the sign of assertoric-force has its root in Frege's ignorance of the significant difference between "judgment" and "assertion", which will be elucidated at length in the light of Husserl's theory of "doxic-modification". In the end, based on a further elucidation of the activity of assertion, I will advance a tentative interpretation of the vague insight Frege has concerning the *protorelation*.

Keywords: assertoric-force; doxic-modification; Frege; judgment-stroke, Husserl, logic, truth.

1

The unique relation between logic and truth (in what follows I shall call it the *protorelation*), which fascinates Frege so deeply, is crucial for understanding his conception of logic. Unfortunately, it turns out that the discussions on *protorelation* is the least clear part in Fregean text,

which on most occasions might be regarded as the model of perspicuity. He complained himself of failing to articulate it:

> Logic is the science of the most general laws of being true. One may find that he can form no very precise impression from this description of what is meant. The author's inadequacy and the awkwardness of language are probably to blame for this. (Frege 1969, 139)

As shown below, Frege does impute this to the awkwardness of ordinary language, and, as a result, insofar as ordinary language is involved, he prefers to describe the *protorelation* by comparison and analogy. For example, in order to highlight the uniqueness of the *protorelation*, he manages to contrast it to the one between natural science and truth:

> All sciences have truth as their goal; but logic is also concerned with it in a quite different way. Logic has much the same relation to truth as physics has to weight or heat. To discover truths is the task of all sciences; it falls to logic to discern the laws of being true. (Frege 2003, 35)

Scientists engage themselves in the searching for various truths; by contrast, instead of being interested in the concrete scientific truths (e.g., the law of gravity), logicians as such aim at the laws *of* being true. The difference between logicians' treatment of *truth* and scientists', is quite similar to the difference between scientists' treatment of *heat* and ordinary people's: everyone can feel the change in temperature and react to it properly (e.g., take off or put on clothes), physicists beyond this also study the nature of temperature for acquiring the laws of thermodynamics. Bearing this in mind, we come to a conclusion which needs to be further developed: logic should not approach to the truth in the same way as other sciences do.

Probably influenced by the teaching of Neo-Kantianism that Truth, Good and Beauty are three basic values, in the famous article "Der Gedanke", Frege compares the *protorelation* to the relation of ethics to good and aesthetics to beauty as well: "Just as 'beautiful' points the way for aesthetics and 'good' for ethics, so do words like 'true' for logic" (Frege 2003, 35). But in a posthumous writing entitled "Meine grundlegenden logischen Einsichten", dated about three years before those words, we find a significant restriction to the parallelism among "truth-logic", "good-ethics" and "beauty-aesthetics":

There is no doubt that the word "beautiful" actually does indicate the essence of aesthetics, as does "good" that of ethics, whereas "true" only makes an abortive attempt to indicate the essence of logic, since what logic is really concerned with doesn't lie in the word "true" at all but in the *assertoric-force* with which a sentence is uttered. (Frege 1969, 272; italics mine)

At first sight, it seems that by these words Frege denies the unique correlative between truth and logic, but a more careful reading will reveal that what Frege really denies herein is only the association of the *word* "true" to logic. In fact, what he has in mind meanwhile is that the *word* "true" is redundant at all, so that it cannot express the truth which is the utmost concern of logic. What really manifests the truth in question is the assertoric form of a sentence, or the assertoric-force, so to speak. I shall return to this in the next section.

If this posthumous text should be taken seriously, as proven everywhere in Frege's writings,[1] then the assertoric-force definitely provides us with a valuable clue to clarify the vague insight that Frege has into the *protorelation*. Then what is the nature of assertoric-force? Although, according to Frege, assertoric-force is of great significance to logic, it seems that he can't find a proper way to describe it. However, in the relevant texts of Frege we find a notable phenomenon: Though Frege admits that assertoric-force, so long as it is supposed to reveal the truth which could not be signified by the word "true", is ineffable in ordinary language, he has coined in his conceptual notation for such a force a much-disputed sign, the so called *judgment-stroke*. As an artificial language, conceptual notation is aimed to fully unearth the structure of thought obscured by ordinary language, therefore, it seems that nothing could say against its assignment of a proper symbolic expression to the core conception of logic, for the incapability of ordinary language in this regard may just be the one of the most important motives which promoted Frege to develop such a formal language. However, is Fregean conceptual notation competent to the job which cannot be accomplished by ordinary language anyway? In this paper, I will try to demonstrate that judgment-stroke, though retained by Frege till the final stage of his career since has been introduced in *Begriffsschrift*, is not adequate to the task its inventor has assigned to it. As a result, it

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[1] Cf., Frege (1969, 140). Even in "Der Gedanke" we find a significant comment, cited below, which confirms this text. Cf., Frege (2003, 41).

is misconceived and inconducive to clarify Frege's vague insight into the *protorelation*. The mistake of judgment-stroke for the sign of assertoric-force has its root in the Frege's ignorance of the significant difference between "judgment" and "assertion", which will be elucidated at length in the light of Husserl's theory of "doxic-modification" in section 4. In the end, based on a further elucidation of the activity of assertion, I will advance a tentative interpretation of the vague insight Frege has concerning the *protorelation*.

2

Many Frege's commentators unconsciously confuse *assertion* with *judgment*.[2] As a result, little attention has been paid to the uniqueness of the former. In the last analysis, this confusion has its origin in Frege's own texts.

The explicit differentiation between judgment and assertion shows itself in the article "Der Gedanke", in which Frege, based on "thought", clearly delimited thinking, judgment and assertion (Frege 2003, 41):

(1) the grasp of a thought – thinking,
(2) the recognition of the truth of a thought – judgment (*Urteilen*),
(3) the manifestation of this judgment – assertion (*Behaupten*).

To take this delimitation literally, it seems that what concerns truth is the judgment alone; the assertion just plays the role of "manifesting" the result of judgment, and is of nothing productive in itself. The emphasis Frege puts on the judgment, as it were, also reflects in the fact that the definition of judgment is supplied by a long note, however, it is in this very note which seemingly devoted to judgment that what we read is almost all about assertion:

> It seems to me that thought and judgment have not hitherto been adequately distinguished. Perhaps language is misleading. For we have no particular part of *assertoric* sentences which corresponds to *assertion*; that something is being *asserted* is implicit rather in the form of *assertoric* sentences… (Frege 2003, 41; italics mine)

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2   As will be indicated in the beginning of section 3, many English translators simply render *Urteilstrich* as "assertion sign" (Frege 1960, 34, 35, 38; see also, Frege 1972, 111). Wittgenstein, despite his perspicacious criticism of Fregean judgment-stroke, reformulates this term as *Behauptungszeichen* in *Philosophische Untersuchungen* § 22.

It is quite remarkable that while Frege distinguishes between judgment and assertion in the main text, he turns his back on this distinction, confusing them in the footnote without the least hesitation.

In the first sentence Frege complains that his idea has been ignored. Early in the *Begriffsschrift* in 1879 he has already provided a special formal sign, the judgment-stroke, for distinguishing the act of judgment from the thought to be judged. In the notation " $\vdash$ A" the judgeable conceptual content " $-$ A", which would later on be called "thought", is explicitly distinguished from the left vertical stroke signifying the element of judgment. Frege believes that his conceptual notation so effectively uncovers the logical structure obscured by ordinary language that by making use of it we can discern the content of thought and the act of judgment intuitively.

But, as the quotation shows, after mentioning the "judgment" in the first sentence, Frege turns to the "assertion" immediately. One sound reason might be that what Frege wants to analyze is the way in which language (more precisely, ordinary language) induces one to confuse the thought with judgment, since judgment is "manifested" in ordinary language by the form of assertoric sentence, the turning to the assertion might be well defended by taking into consideration the fact that Frege of course wants to trace the source of confusion in logic into the realm of ordinary language.

The advantage of Fregean conceptual notation becomes visible as long as being compared with ordinary language: What is absent in the latter, a word or sign corresponding to the assertoric-force, is formulated explicitly by the judgment-stroke, which is an in-built element of the logical language. We cannot find in the ordinary assertoric sentence "sea-water is salt" the corresponding part of assertion, "there is no word or sign in language whose function is simply to assert something" (Frege 1969, 201), yet it might be added purposely, the sentence mentioned above is accordingly transformed into "it is true that sea-water is salt." In doing so, we, as it were, represent the logical structure " $\vdash$ A" in ordinary language.[3] It is precisely in this back-translation of

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[3]   Two caveats: Firstly, Frege really has said in *Begriffsschrift* that "the sign $\vdash$ is its common predicate for all judgments", except that this common predicate is read at that time as "is a fact" rather than "is true" (Frege 1993, 4), but wording like "common predicate" is just a compromise with those who are bound by the traditional logic: "… if one wishes, he can distinguish subject and predicate" (Frege 1993, 3). Frege in fact rejects the frame of subject-

logical language into the ordinary one, however, the issue we are concerned about shows up.

In "Meine grundlegenden logischen Einsichten" Frege believes that the sentence "it is true that sea-water is salt" asserts nothing more than "sea-water is salt" (Frege 1969, 271). In other words, the phrase "it is true" is completely redundant. Similar statements can be found elsewhere, as:

> We express recognition of truth in the form of assertoric sentence. We do not need the word "true" for this. And even when we do use it the authentic assertoric force does not lie in it, but in the form of assertoric sentence; and where this form loses its assertoric-force the word "true" cannot put it back again. (Frege 2003, 41; see also, Frege 1969, 140)

Right in the next sentence Frege points out the situation where the assertoric-force is absent, "this happens when we are not speaking *seriously* (*im Ernste*)" (Frege 2003, 42). He further explained that assertion on the stage or in fictions is merely sham assertion, and the thoughts expressed have only the *form* of assertoric sentence, but by no means *actually* (*wirklich*) contain assertion. The form of assertoric sentence is here further distinguished from the *actuality* of the assertoric-force, whether a sentence really has the assertoric-force or not now depends solely on whether the speaker is *serious* or not (Frege 2003, 42). If, while saying "… is true", the speaker doesn't take it *seriously*, then the word "true", no matter how many times it might be uttered, could not make the sentence in which it appears to have the assertoric-force; on the contrary, if he takes what has been said *seriously*, then the word "truth" need not be uttered at all. It is in this sense that Frege maintains a kind of "Redundancy Theory of Truth".

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predicate, "is a fact" should not be placed in the same category with, for example, "is red", the former should not be regarded as a real predicate, as Frege would not regard "is true" as a real predicate later on as well. Secondly, here Frege doesn't read this "common predicate" as "is true". Although "is a fact" is nothing other than "is true", yet to take Fregean thoughts as a whole, the special predicate "is true" should be more appreciated than the predicate "is a fact", which is more fit to the Correspondence Theory. The absence of the term "true" in fact indicates that at that time "truth" as a key word to logic was still out of Frege's sight. Cf., Sluga (2000, 75).

Let's call the situation described above *The Introductory Context of Assertoric-Force*. The assertoric-force is in a sense first of all characterized by this *seriousness*. Accordingly, as will be further developed, the ineffable truth which is supposed to indicate the essence of logic should be understood in the sense of *seriousness* (truthfulness) which, in turn, could not become the sense (*Sinn*) of any word, for any word, after all, including the word "true", can be uttered without being taken seriously. It is in this context that what we have cited above—"true" only makes an abortive attempt to indicate the essence of logic, since what logic is really concerned with doesn't lie in the word "true" at all but in the assertoric-force with which a sentence is uttered (Frege 1969, 272)—comes to its full light.[4]

3

Despite widespread criticism, the judgment-stroke, which has been introduced in the very beginning of *Begriffsschrift*, was retained by Frege till the end. This insistence on judgment-stroke is sharply contrasted to his endorsement of the Redundancy Theory of Truth, for the latter is nothing other than the maintenance of the ineffability of the assertoric-force. But the author, strangely enough, explicitly takes the judgment-stroke to be the sign which has the assertoric-force: "In the *Begriffsschrift* I have a special sign with assertoric-force: the judgment-stroke" (Frege 1969, 214; see also, Frege 2003, 76). In accordance with it, many English translations of Frege's writings simply render the judgment-stroke (*Urteilstrich*) as "assertion sign" (Frege 1960, 34, 35, 38; see also, Frege 1972, 111). Even among critics of judgment-stroke we find testimonies to this. For example, when Wittgenstein attacks Fregean judgment-stroke in his *Philosophische Untersuchungen*, he reformulates this term as *Behauptungszeichen* rather than *Urteilstrich* (Wittgenstein 2009, 14).

From this there arises a question: How could the assertoric-force be expressed by a special sign—the judgment-stroke—in logical language

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[4] Though the word "truth", the sign as such, also could not express the significance of seriousness, it nevertheless is distinctive in that it has a special "sense that contributes nothing to the sense of the whole sentence in which it occurs as a predicate," so "seems fitted to indicate the essence of logic." (Frege 1969, 272)

if it is utterly ineffable in ordinary one? To be sure, Frege believes that ordinary language is incapable of expressing the assertoric-force, for the word "true" which seems most qualified to this task only "makes an abortive attempt" to do so. But does it mean that it is impossible to assign a sign to this force even in an artificial language? To answer this question we need first of all cut off the association of judgment-stroke and assertoric-force in the above texts, turning back to the context into which the judgment-stroke has been introduced.

As shown by the definition of judgment, there is nothing obscure about the motive that leads Frege to introduce the judgment-stroke: to try to distinguish the recognition of the truth of a thought (the act of judgment) from the mere grasp of the same thought (thinking or, use another term which will frequently be mentioned below, assuming):

> This separation of the act of judgment from what is judged seems to be indispensable; for otherwise we could not express a mere *assumption* (*Annahme*) – the *positing* (*setzen*) of a case without a simultaneous judgment as to its arising or not. We thus need a special sign in order to be able to assert something. (Frege 1994, 32; italics mine)

In *modus ponens*, the major rule of inference in *Begriffsschrift*, this special sign seems most imperative:

     1. If P, then Q
     2. P
     ——————————
     3. Q

In the above inference both P and Q appear twice, but the first time as mere assumptions, the second time they are judged as true. Using Fregean notations, we could reformulate it as follows:[5]

     1. $\vdash (P \rightarrow Q)$
     2. $\vdash P$
     ——————————
     3. $\vdash Q$

In the first step „$\vdash (P \rightarrow Q)$", it is „$P \rightarrow Q$" as a whole instead of P or Q that is judged as true. According to Frege, without the judgment-stroke

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[5]   It is a compromise to reformulate the inference in this way, for Fregean style of notation is too complicated to be strictly followed.

functioning as an affirmation, every letter will represent a true thought, P → Q will be read as "the affirmation of Q follows from that of P", therefore the step 2 and 3 will be redundant. With mainly this in mind, Frege felt the necessity to introduce judgment-stroke into his conceptual notation for distinguishing the thought merely assumed from the same thought judged as true. We call this situation *The Introductory Context of Judgment-Stroke*.

4

It must be admitted that since Frege in most cases does not distinguish between assertion and judgment, the two contexts discussed respectively in the last two sections are often lumped together in his texts. In fact, it is evident that what Frege has in mind is that the assertoric-force is absent where 1), a thought is not seriously put forth, or where 2), a thought is just assumed. Example of the former case is that the thought in question is uttered on stage; of the latter is that it appears in subordinate clauses or as a scientific hypothesis (I shall turn back to the further differentiation in the latter case in section 5), and in both cases one should not use the judgment-stroke (Frege 1969, 214, 271, 272).[6]

In what follows, I will argue that the difference between the two contexts is so essential that the Introductory Context of Judgment-stroke should never be put on a par with that of Assertoric-force, and that even if judgment-stroke as a sign could be used to distinguish the thought judged as true from the same one merely assumed (i.e. applicable to Introductory Context of Judgment-stroke), it nevertheless cannot effect the assertoric-force (i.e. inapplicable to that of Assertoric-force). As a main part of this article, I will show in this section how Husserl's theory of "doxic-modification" could support us in this argument.

Husserlian phenomenology is known for its elaborate differentiation of various modes of consciousness. According to Husserl, an important characteristic of consciousness is that it could take divers attitudes towards the same thought or state of affairs. Varying with different degrees of fulfillment of its intention, those attitudes could be *certain belief* and its modifications (modalizations) *deeming possible*, *deeming likely*, *questioning* and *doubting* etc., which, termed by Husserl *belief-* or

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[6]   The sentence on p. 272 has been crossed out in the manuscript by Frege himself.

doxic-modalities, are in the noetic side. Correlatively, their parallel no-ematic *being-modalities* characterize the thought or state of affairs as *actual* and its modifications (modalizations) *possible*, *probable*, *questionable* and *doubtful* respectively. Each of those modalities normally manifests a certain *position-taking* on the thought or state of affairs, consequently, those modalities are all of *positionality*, and the consciousness-acts in question are called "*being-*'*positing*,' '*thetic*' acts" (Husserl 1976, 239).

In Husserl's view one can freely exercise a very special modification, among others, to those doxic-modalities, removing their belief-characters and modifying them to modalities of *non*-positionality which do not posit any more. Husserl calls such modification *neutrality* modification.[7] Note that Neutrality modification is by no means the *negation* (which, along with *affirmation*, is itself a *positional* modification of the higher level) of those doxic-modalities. While the negation of, e.g., doubting turns out to be undoubting which is still a *positing*, Neutrality modification (or *Neutralization*) keeps doubting as doubting, only the "neutralized" doubting is no longer positional, no longer an *actual* doubting. Neutralization is not a modification within the range of positionality, as, for example, modification from certain belief (the so called *Urdoxa*) to doubting, or the latter to undoubting, etc.; rather, it is a modification of *positionality itself*. If we contrast neutrality with positionality, then all of those modifications under the title of positionality will have their neutralized counterparts. For example, corresponding to the positional, actual (*wirklich*) doubting there is its neutrality modification which neutralizes it to *inactual* one. Husserl describes the neutralized doxic-modalities as follows:

> [Neutrality is] a modification which, in a certain way, completely annuls, completely *renders forceless* (*entkräften*) every doxic modality to which it is related… Believing is now no longer *serious* (*ernstlich*) Believing, Deeming likely is no longer *serious* Deeming likely, Negating is no longer *serious* Negating, etc. (Husserl 1976, 247f; italics mine)

It is not so hard to see from this description, especially from those key words given in German (*entkräften*; *ernstlich*; *wirklich*), that what Husserl's "neutrality" corresponds to is nothing but the situation in which the assertoric-force is absent. It is an in-*actual* situation in which

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7   For an elaborative analysis of neutrality modification, see Ni (1999, 189ff).

the *force* has been eliminated and the *serious* attitude is lacking. The question facing us now is whether this situation is the same as the one concerning assumption in which judgment-stroke should be left out?

As if a special answer to this question, Husserl emphasizes that the neutrality modification "has never been scientifically elaborated", so that "where it had been touched upon it had been confused with other modifications," among which that of *assumption* comes first (Husserl 1976, 248).[8] Therefore, after introducing the neutrality modification in § 109, he forthwith distinguishes it from assumption in § 110.

Assumptive consciousness does not judge any thought, neither affirm nor negate, nor even doubt it. It merely *supposes*. To this extent, "it is a modification… entirely of its own sort standing over against and apart from the principal series dealt with above" (Husserl 1976, 249). Put in this way, assumption is easily confused with neutralization which does not posit at all. However, Husserl points out definitely that assuming is still "something like positing", modification of assuming is still a "modification of doxic positing" (Husserl 1976, 249).

The difference between neutralization which does not posit anymore and the assumption which still posits becomes visible first in the light of a general criterion of differentiation between *positionality* and *neutrality*: "[All] *genuinely* non-neutralized noeses are subject to the '*legitimating of reason*,' whereas *the question about reason and unreason makes no sense for the neutralized noeses*" (Husserl, 1976, 249).

This criterion, characterized by legitimating of reason (*Rechtsprechung der Vernunft*), is of great interest, I shall return to it in the concluding section. At present, we must realize that the assuming is still a kind of posting if in a very modest form, it is sup-posing. Husserl describes the suppositional character of assumption as follows:

> It can enter into the unity of posita to be judged about rationally as a member (the supposed as hypothetical "antecedent" or consequent) and hence itself be subject to rational valuation. It is not of a mere undecided (*dahinstehend*) thought, but rather of what is hypothetically supposed that it can be said that it is correct or not. It is a fundamental error to confuse the one with the other… (Husserl 1976, 249)

As shown above, the fact that on the level of "legitimating of reason" there is, among others, a situation of assumption obviously motivates

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8   Another one is phantasy.

Frege to introduce the judgment-stroke. Taking this into consideration, the last sentence cited above, which is more likely to be Husserl's self-critique of his early thought,[9] in fact constitutes a critique of Frege: the confusion between neutralization and assumption does correlate to that between the two Introductory Contexts discussed earlier.

To make the issue more accurate, some important amendments should be added. To be sure, the seriousness of Husserlian positionality is not restricted exclusively to the Fregean assertoric-force; it rather represents the "force" itself reflected in all positings. The assertoric-force comes into being only when this "force" joins with a pair of special positings embodied in the act of judgment as decision-making, which, taken as themselves, could be either serious or unserious. This pair of positings, as modifications of the higher level, is affirming and negating. The difference between affirmation and negation on the one hand and the modifications of the lower level (e.g., doubting) on the other "resides in the fact that the former do not weaken belief but, instead, either cancel it or confirm it. Or put differently, the other modalizations leave their correlate hanging in the balance, whereas affirmation and negation decide the matter—either yes or no, with no middle ground" (Brainard 2002, 155).[10]

From the perspective of noetic side, we can call this pair of positings modifications of decision-making as opposed to modifications of the lower level which, to a greater or lesser extent, hang their correlate in the balance. Obviously, at least as far as classic logic is concerned, positings of decision-making alone are at issue. Moreover, given the same decision making character, considering the "economy" as regards a principle of inference (Frege 2003, 78), even negating is not an inde-

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9   In his pre-phenomenological period, Husserl has conflated the "assumption" of hypothetical judgment with "neutralization" in the context of the polemic over the "objectless representations". However, since he understood the "assumption" mainly in the sense of "reservation", the actual contradiction has been avoided. Detail in this aspect could be found in Husserl (1979, XXXVIII ff).

10   It must be noted that, as Husserl once emphasized, affirmation *qua* modification of higher level should not be confused with certainty *qua Urdoxa*. The former "'confirms' a position by 'assenting' instead of 'annulling' it as in Negation" (Husserl 1976, 244) whereas the latter is not a matter of decision, but is a naive certainty.

pendent modality, it rather ultimately resolves itself into thought in the form of the sense of negative sign.[11]

This situation aggravates the confusion between assertoric-force and judgment-stroke in logic. However, if the judgment has been defined only with reference to assumption, as described earlier in the Introductory Context of it, i.e., its function lies in distinguishing the thought judged as true from the same thought assumed, then it could never be the sign which has the assertoric-force. In general, as I will show in the next section, every sign by itself is forceless.

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Classic logic is supposed to be uni-modality. But assumption, as another modality, finds a quite dependent way to enter it. The *modus ponens* described in section 3 testifies to this entrance. In my view, the fact that *modus ponens* is the major rule of inference in *Begriffsschrift* is the most compelling reason why Frege has introduced the judgment-stroke from the very outset. In this section, I shall demonstrate that this introduction is at best unnecessary, and at worst, misconceived. Let's begin with the best part.

Wittgenstein (and others follow him) has already pointed out that "' ⊢' is no more a component part of a proposition than is, for instance, the proposition's number" (Wittgenstein 1922, 94f). This remark does strike home. We need no such sign as long as the propositions are numbered, or even only if we can distinguish the lines of inference from each other. For if each line of inference in *Begriffsschrift* is prefixed with " ⊢" as their "common predicate", then this sign precisely can make

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[11] It is not to say, however, that negation is not an independent modification, for as two-valued logic clearly shows, the same thoughts of course can be affirmed as well as negated. Moreover, if we confine ourselves on the level of logic as such in which every thought must already be decided (true or false), then negating is even the only substantive modification. In this connection the particularity of decision-making modification looms large. Different as they are with respect to modification, affirming and negating share the same degree (i.e., maximum) of belief, whereas deeming possible, deeming likely, questioning and doubting are different in degree of belief from each other. Therefore, though negation posits in opposite direction of affirmation, they belong to the same belief-modality.

no difference at all, and, as a result, can be omitted altogether. The role judgment-stroke plays in inference is further clarified by Wittgenstein in *Philosophische Untersuchungen*: "It distinguishes the whole period from a clause within the period" (Wittgenstein 2009, 14). Generalized to such an extent, the function of judgment-stroke has little to do with the tension between judgment and assumption. Its remainder in modern notation could be discerned—besides from the line of reference—from the bracket, as, for example, the bracket in „A∧(A→B)→B" or in „(A∨¬A)".

But Frege surely doesn't conceive his conceptual notation only as *calculus ratiocinator*, his aspiration is to develop a kind of *characteristica universalis*, in which the judgment-stroke is invested with more signification than that reflected in inferential calculus. It is abstracted from inferential procedure, and, as its name shows, used to generally distinguish judgment from assumption (or other modalities) in ordinary sense.[12] It is in this extra-inferential usage of judgment-stroke that we encounter the worst part of this story: judgment-stroke is mistaken as the sign that has the assertoric-force.

To examine this point, let's take an example from Frege's text:

With
├── 2 + 3 = 5
we assert that 2 + 3 equal 5. Thus here we are not just writing down a truth-value, as in
    2 + 3 = 5,
but also at the same time saying that it is true. (Frege, 1994, 32)

To be sure, unlike in the case of *modus ponens*, leaving out the judgment-stroke in this context does make difference. But a more careful investigation reveals that in such context the function for which judgment-stroke is intended is being introduced rather than performed. Or as Dummett puts it, the sign in question is used to only *describe* rather than *effect* the act of assertion (Dummett 1973, 333ff).

David Bell, in an account independently of, but in many respects similar to that given by Dummett, regards the judgment-stroke as a *pure performative operator* (Bell 1979, 98). In contrast to the normal performative operator such as "I promise", "├──", understood as having

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[12] Assumption in this sense (as, for example, taking the form of scientific hypothesis) presents itself more independently than in *modus ponens*.

assertoric-force, must in its every occurrence always effect the act of assertion. If this sign is used to only describe or report, like "I promise" used in other tenses than the present and in other persons than the first person singular or plural, it would contribute materially to the sense (*Sinn*) of the sentence in which it occurs, and, as a result, fail to be a kind of force. Taking this into consideration, Dummett and Bell argue that the ordinary language "I assert that–", even stipulate that it is to "suffer change neither of tense nor of subject" is not a suitable render-ing of " ⊢", for the latter "must be incapable of appearing within a subordinate clause" (Bell 1979, 98).

Dummett and Bell are quite right in showing the difficulty or even impossibility of translating judgment-stroke, understood as having as-sertoric-force, into ordinary language, but it seems that they don't ex-plicitly reject that judgment-stroke itself, as a sign of artificial language, could have the assertoric-force Frege himself endows it. In my opinion, this account, valuable as it is, fails to do justice to the common ground of ordinary and artificial language. The point is not that the usage of the latter could be artificially stipulated by its inventor for some special ends, while it is not the case for the former; what's important here is that neither the sign of ordinary language nor that of artificial one, as long as it is a sign, could have any force. To be able to really affect the assertoric-force in its every occurrence, judgment-stroke has to have some kind of magic, compelling the one who has written down a sen-tence prefixed with it in whatever situation to be serious. Such a sign is nothing but a spell, a sign in its normal sense can effect no "force", neither the magic force nor the assertoric one, for the force lies in its use, and, as Baker and Hacker put it, "a sign *is not itself a use*; it must *be used*" (Backer – Hacker 2005, 81).

To further clarify this matter, it is worth pointing out that a judg-ment without assertoric-force is always conceivable. By using the con-clusion drawn in the last section, we could freely extend the range of neutrality modification to judgment as well as to assumption: as there are positional judgment and assumption, there also are their neutral-ized counterparts. Scientists often put forth an assumption seriously, after a long term of hesitation and hard work, draw a positive conclu-sion, and then make a serious judgment. The assumption and judgment involved here are all serious; but if the same procedure happens on stage, then even if the actor who plays the scientist puts forth the as-

sumption or judges it to be true, he will not be regarded as serious in doing so. Frege once was aware of the trouble which the confusion of judgment and assertion brought about:

> The sentence "I smell the scent of violets" has just the same content as the sentence "It is true that I smell the scent of violets". So it seems, then, that nothing is added to the thought by my ascribing to it the property of truth. *And yet* is it not a great result when the scientist after much hesitation and laborious researches can finally say "My conjecture is true"? (Frege 2003, 40; italics mine)

He fails to settle this question, but leaves it to the *sui generis* of the predicate "true". According to our analysis, however, it can be answered as follows:

The situation described before *And yet* involves assertion, as long as the speaker is serious, the phrase "it is true" is surely redundant. On the other hand, the situation after *And yet* involves judgment, if the assumption (or conjecture) and the judgment later on are all made seriously, then the result may be great, but, as shown below, is of no logical significance.

After all this, one may suspect that judgment probably has nothing to do with logic as such. In the "Logik," the manuscript of "Der Gedanke," we find a support on this: "When we *inwardly* recognize that a thought is true, we are making a judgment: when we manifest this recognition, we are making an assertion" (Frege 1969, 150; italics mine). This paragraph contains an additional word "inwardly" (*innerlich*) when compared with the related text in "Der Gedanke" cited above. What Frege wants to express with judgment-stroke is, authentically, the interior act of recognition. The introduction of judgment-stroke into logic must be considered as unwise when taking into account that Frege himself manages to distinguish in his entire career the logical from the psychological, ascribing all occurrences in the inner world, under the title "representation" (*Vorstellung*), to the latter (Frege 2003, 47).[13] Logical inferences consist of object steps which are by no means the descrip-

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13   Perhaps taking into account the function of his judgment-stroke in logic, Frege does consciously exclude the judgment *qua* decision-making (*Entschlüsse*) from the list of representation, while giving no excuse for this.

tion of psychological process, so the system of logical language should not contain such a sign representing an interior modification.

Let's try to reconstruct the connection between judgment and logic which Frege might have in mind, conjecturing how this unwise move happens: The basic Fregean ideas are that logic is concerned with truth, and the thought (as *Sinn*) is the way by which the truth-value (as *Bedeutung*) is given. So the reason why logic cares about thought lies in that one can access truth only through thought. Taking this into consideration, it seems that the step from the assumed thought to the judging of the same thought to be true would be necessary. For example, Frege says that "in every judgment, no matter how trivial, the step from the level of thoughts to the level of meaning (the objective) has already been taken" (Frege 1994, 49), and that "the step from thought to truth value—more generally, the step from sense to meaning—has to be taken" (Frege 1969, 133).

But, as the quotation shows, this step is the one which is necessary only for arriving at the logical level; it is a pre-logical step which need not, and should not, appear in logic as such. If logic relates to truth through judgment, then it is concerned with truth in the same way as ordinary science does. What the Fregean definition of judgment represents is precisely the correspondence theory of truth which characterizes the ordinary scientific truth. But logic should be concerned with truth "in a quite different way" (Frege 2003, 35).

It must be admitted that in a sense logic is surely at work in the step from assumption to judgment, since every important scientific judgment is made logically.[14] Let's take a simplest example of judgment procedure, first assume that "Socrates is mortal", then demonstrate through the famous syllogism that this hypothesis is true. The interior process of judgment seems here precisely the unfolding of the logic connections between thoughts. But if we write down the process from assumption to affirmation in the help of Fregean judgment-stroke as follows:

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[14] That means every such judgment is made seriously. But serious judgments occur, as the concluding section will show, only when the one who makes it is ready for the potential intersubjective controversy concerning Truth, therefore, is bound to the forthcoming assertion. In this sense, serious judgment presupposes assertion, not *vice versa*.

1. Socrates is mortal. (assumption)
2. ├─ (x) (x is human being → x is mortal). (affirmation of major premise)
3. ├─ Socrates is human being. (affirmation of minor premise)

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4. ├─ Socrates is mortal. (affirmation of conclusion)

We will find that the first step precisely doesn't belong to logical inference, and after deleting it the judgment-stroke could still be omitted from the rest steps of this inference.

In the procedure of judgment shown above, the motive of making a judgment even in the sheerest case is not the logical connections as such between thoughts, rather it is the evident insight, of the one who judges, into this connections, or put it another way, it is the fulfillment of a vague intention in the concrete carrying out of logical inference. Judgment is not an objective process from premise to conclusion; rather it is a subjective experience from assumption to affirmation, intention to fulfillment. In short, the conception of truth related to judgment is "correspondence", which is rejected precisely by Frege as an interpretation of "truth" as soon as he has an insight into the *protorelation* (Frege 2003, 36f).

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After all those preparations, we are ready for an elucidation of what Frege says concerning the *protorelation* that "what logic is really concerned with… lies in the assertoric-force with which a sentence is uttered" (Frege 1969, 272). To begin with, let's turn back to the definition of assertion cited in section 2.

Despite his confusion between judgment and assertion, Frege nevertheless points out an important feature of the later: it is the manifestation (*Kundgebung*) of judgment.[15] The manifestation in question, however, should not be understood as a mere symbolic expression of the interior act of judgment (Dummett 1973, 311). Moreover, the idea that the sign "├─" alone is already a manifestation of judgment is, in my

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[15] *Kundgebung*, literally "giving message". In Logical Investigations, Husserl used this term and its correlate *Kundnehmung* (literally "receiving message") to illuminate the communicative function of expression. Cf., Husserl (1984, 39f).

opinion, partly the source of mistaking the judgment-stroke as having assertoric-force, and, therewith, of confusion between judgment and assertion in general. Assertion is, at any rate, not the merely unproductive manifestation of the judgment, besides the externalization of the judgment-contents, what is manifested in an assertion is the assertor's commitment to what has been asserted by him. As opposed to the interior private act of judgment, an assertion is an intersubjective practice.[16] To make an assertion is at the same time to make an intersubjective *truth-claim*. It is only in the space of plurality that the assertoric-force arises.

The assertoric-force is primarily a constraining force for those who have made an assertion. An assertion, in the normal sense of the word, is by no means a sheer utterance, the assertor, as far as he could be regarded as the one who has made an assertion, has to taking what he has asserted seriously. This means, as pointed out by Husserl in the characterization of non-neutralized *noeses*, that the assertor has the responsibility of subjecting what he/she has asserted to the "legitimating of reason". Such activity of justification, inspired by legitimating of reason, moving from opinion (*doxa*) towards truth, provides logic with its original horizon.

Taking into consideration the universality of logic, a convenient approach to clarify the relation between assertion and the dimension of truth unfolded in the laws of logic is to start from what is illogic for Frege.

In his refuting to psychologism, Frege compares psychologist position to that of Cretan liar.

> If anyone tried to contradict the statement that what is true is true independently of our recognizing it as such, he would by his *very assertion* contradict *what he had asserted*; he would be in a similar position to the Cretan who said that all Cretans are liars.[17] (Frege 1969, 144; italics mine)

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[16] If judgment could be characterized by the tension between "true" and "what makes it true"; assertion, on the other hand, should be characterized by the one between "true" and "claiming to be true". Cf., Jager (1970, 165).

[17] Strictly speaking, what Frege refutes here is a radical individual relativism rather than the psychologism as a specific relativism (anthropologism). For the difference between individual relativism and psychologism as a specific relativism or anthropologism, cf., Husserl (1975, 122ff).

Similar formulation could also be found in Husserl's battle against psychologism, especially against the individual relativism which he identifies as skepticism:

> The *content of such assertions* rejects what belongs to the *sense or content of every assertion* and what accordingly cannot be significantly separated from any assertion. (Husserl 1975, 123; italics mine)

Here it seems that Husserl, and Frege as well, fails to find suitable terminology in expressing what contradicts each other in question. However, what they want to express is not so hard to understand. Comparing the italicized texts in the above two quotations, the "content of such assertions" in Husserl corresponds to "what he had asserted" in Frege. To use the terminology of Husserl's Fifth *Logical Investigations*, we may call it the *matter* (*Materie*) of the relativist assertion, which is basically the same as what Frege calls *sense* (*Sinn*). On the other hand, the "sense or content of every assertion" in Husserl corresponds to the "very assertion" in Frege, which, in terminology of Fifth *Logical Investigations*, may be titled as the *quality* (*Qualität*) of assertion, and which, characterized by assertoric-force, *qualifies* an act as assertion.[18] Husserl interprets this quality as "to assert, is to claim the truth of this or that content" (Husserl 1975, 129f). Assertion, first of all, is a serious truth-claim.

With the help of suitable terminology, we can reconstruct more clearly the critique of psychologism shared by Frege and Husserl:

1. Relativism is an assertion about "true", which *explicitly* asserts that what is true is only for him who has recognized it to be true. (The *matter* of relativist assertion)
2. But every assertion is also a truth-claim, which *implies* that what has been asserted is true. (The *quality* of assertion in general)
3. Would the relativists like to consistently apply what they have asserted to their own truth-claim? In other words, do they really want to concede that their "assertion" itself is also true only for themselves and not for others? (The *quarrel* between matter and quality)

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18 For the difference between material and quality in Husserl, cf., Husserl (1984, 425).

Except those die-hard relativists, anyone who has carelessly made the relativist "assertion" would immediately give it up as soon as following this reasoning to the last step. For, as long as he/she knows what means to make an assertion, he/she surely wants whatever has been asserted by himself/herself is also approved by others.

But there also are those die-hard relativists, at least methodologically, who will nevertheless insist that even the relativist "assertion" is also true only for themselves and not for others. In other words, they decide to give up the general *quality* of assertion in favor of the specific *matter* of relativist "assertion". And that is what Husserl already anticipates:

> He will not bow to the ordinary objection that in setting up his theory he is making a claim to be convincing to others, a claim presupposing that very objectivity of truth which his thesis denies. He will naturally reply: My theory expresses my standpoint, what is true for me, and need be true for no one else. (Husserl 1975, 123).

But, as Frege points out, the big problem of the die-hard relativist lies in that:

> To be consistent, any person holding this view would have no right whatever to contradict the opposite view; he would have to espouse the principle: *non disputandum est*. He would not be able to assert anything at all *in the normal sense*… (Frege 1969, 144; italics mine)

Since the two essential parts of the relativist "assertion", its matter and quality, contradict with each other, it is self-cancelling, or strictly speaking, it cannot be put forward as an assertion, "even if its utterances had the form of assertions."

A more careful study will reveal that the conflict between the matter and quality of relativist "assertion" is in effect a conflict about "true", about the sense of the word "true" that makes *explicit* in the content of relativist "assertion" and the one which is *implicitly* and *ineffable* as precondition of every assertion, which everyone must already tacitly grasp in order to be able to "assert anything at all *in the normal sense*" (Frege 1969, 144). The relativist "assertion" does not, in its content, do justice to what makes every assertion in the normal sense possible.

As to the sense of the word "true", Frege argues that "[i]f anyone seriously and sincerely defended the view we are here attacking, we

should have no recourse but to assume that he was attaching a different sense to the word 'true'" (Frege 1969, 144). In a similar way Husserl also resorts to the normal sense of the word "true": "Alternatively, such beings use the words 'true' and 'false' in some different sense, and the whole dispute is then one of words" (Husserl 1975, 126).

We can find two explicit interpretation of what kind of normal sense of the word "true" here is referred to in the relevant texts of Frege and Husserl:

1. As the opposite of relativist assertion, the normal sense of true would be that what is true is true "in itself" (Husserl 1975, 123), is "independence of being recognized as true" (Frege 1969, 144).
2. Frege once said that "[t]he meaning of the word 'true' is unfolded in the laws of being true" (Frege 2003, 36). In almost the same tone, Husserl said that "[w]e saw that the principles of contradiction and excluded middle tell us what belongs to the mere sense of the words 'true' and 'false'" (Husserl 1975, 125).

The first interpretation leads ultimately to a platonic conception of truth which is in principle insusceptible to any human activity. This is what Frege and Husserl aim at in their fight for an independent logic realm against the invasion of psychologism. But to resort to such conception would be too much for an argument. To blame the relativist for attaching a different sense to the word "true" would be to make the fallacy of begging the question, for what relativist argued against us is precisely the sense of the word "true". Given the first interpretation, the normal sense of the word "true" would at best be a consequence of *reductio ad absurdum*, rather than itself constitute an anti-relativist argument. By the way, to say this normal sense is that "true is independent of being recognized as true" is to assert too much, for the antithesis of relativism would be that *it is not the case* that what is true only for him who recognized it to be true. And the platonic conception of truth, which claims that what is true is independent of recognition *in general*, is not a necessary consequence of it.

What in effect deconstructs the relativist conception of truth is the *normal sense*[19] of the word "true" embedded in every serious assertion as their quality, which, reflected in assertoric-force, is in essential con-

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19 To be sure, the "sense" in this context should not be taken as the Fregean terminology *Sinn*.

nection with those intersubjective activities such as claiming, justifying, refuting and approving. And this sense, in my opinion, as Frege and Husserl formulate in the second interpretation, is precisely the one *formally* unfolded in the laws of logic.

The laws of logic by no means presuppose a platonic conception of truth. The sense of "true" formally unfolded in them is the general implicit conventions governing the activity of assertion to which any serious assertor, as long as he/she could be regarded as an assertor, has to accord. For example, the principles of contradiction could be regarded as the formalization of one of such implicit conventions that the assertion, aiming at a general understanding and approval, consists of a "*disputandum est*" (Frege 1969, XXI). A serious assertor is in principle able to claim that his/her own opinion is more justified even in the eyes of others than the opposite opinion, otherwise "there would be no science, no error and no correction of error; properly speaking, there would be nothing true *in the normal sense of the word*" (Frege 1969, 144; italics mine).

An elaborative reconstruction of the logic laws from the implicit conventions of assertion, though interesting, would be beyond the scope of this paper. Here we are only content with the conclusion that the *protorelation* Frege has in mind should be elucidated in terms of the serious assertion, which, understood as an intersubjective linguistic practice, goes beyond the *Sinn* of any sign, whether it is that of ordinary language or artificial one.

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