WHITEHEAD AS A LOGICIAN

Augustin RISKA

This historically oriented article describes Whitehead's life-long concern with logic. Although his creative phase in logic seemed to have ended with the publication of *Principia Mathematica*, he remained continuously interested in logic of relations and theory of events and even as a speculative metaphysician appreciated logic as - in his words - his "first love".

The distinguished coauthor of *Principia Mathematica*, Alfred North Whitehead, started his professional career as a mathematician and ended it as a speculative metaphysician and a historian of ideas. He was interested in logic throughout his entire life, although it seems that the creative phase in this area ended with the completion of the three volumes of *Principia Mathematica* (1910-1913), while the promised 4th volume, which was supposed to treat geometry, never appeared. After the completion of this monumental work, Russell and Whitehead terminated their fruitful cooperation and moved into their own - philosophically so different - territories of research, teaching and publishing. The Introduction to the second edition of *Principia Mathematica* (1927) seemed to have been written exclusively by Russell, although in the name of both authors.[1] Here the contributions of H. M. Sheffer, J. Nicod, L. Wittgenstein, L. Chwistek, and others, were duly noted. At the same time, Whitehead - now a professor of philosophy at Harvard University - was busy at work on his speculative cosmology which culminated in the publication of *Process and Reality* in 1929. A young, enthusiastic logician at Harvard University, Willard Van Quine, was thus disappointed when the aging classic in logic could not offer any novel ideas for Quine's doctoral dissertation - a take-off from *Principia Mathematica*.

Yet even in his speculative philosophical phase, Whitehead did not cease to talk about logic with great appreciation. He remained concerned with the applicability of logic and with its philosophy. Frequently quoted are Whitehead's statements in the very first section of *Process and Reality* to the effect that "...the philosophical scheme should be coherent, logical, and, in respect to its interpretation, applicable and adequate...".[2] Here he understands 'logical' in its common sense, as including 'logical' consistency or lack of

contradiction and also the principles of inference, among other things. In spite of very few references to *Principia Mathematica*, relegated merely to a footnote status, one may assume that logical systems and apparata endorsed by Whitehead are those of *Principia Mathematica*, 2nd edition. In Whitehead's comments on logic and its applicability certain themes recur as dominant and worthy of attention. Let us mention some of them: (1) the emphasis on logic of relations accompanied by the criticism of the traditional subject-predicate logic; (2) the elaboration of a very important theory of events in connection with Whitehead's well-known method of extensive abstraction; (3) the recognition of equivalent sets or systems of axioms and postulates in various areas of science, especially in geometry which used to be in the foreground of Whitehead's attention; [3] (4) an ambivalent attitude toward logical inconsistencies (in his Preface to *Process and Reality* Whitehead considers "belief that logical inconsistencies can indicate anything else than some antecedent errors" as one of the myths which are to be repudiated);[4] (5) distrust of language as "an adequate expression of propositions" (another myth to be repudiated, as he states in the above-mentioned Preface). These selected five problem areas will now be given a more detailed treatment.

1. LOGIC OF RELATIONS

Nobody who looks at the first volume of *Principia Mathematica* can miss the numerous sections and passages in which the traditionally overlooked category of relation has been brought under a careful logical scrutiny (especially Part I, Sections C and D). The extension of formal logic beyond the logic of classes (monadic predicates) was certainly something to be proud about, even if some preliminary work had been done by De Morgan, C. S. Peirce, E. Schröder and, of course, G. Frege. Whitehead frequently utilized the apparatus of logic of relations in his works published during his middle - philosophy of science - period: in *An Enquiry Concerning the Principles of Natural Knowledge* (1919), *The Concept of Nature* (1920) and *The Principle of Relativity* (1922). I think that these books are logically clearer and more transparent than Whitehead's later, highly speculative works in which it is so difficult to penetrate through the maze of the author's esoteric language. With increased awareness Whitehead underscored his belief in "the relatedness of nature"[5] or rather, as we might say, in the "interrelatedness of nature", to emphasize his organismic philosophy. Thus in the controversy between Russell and F. H. Bradley concerning external and internal relations, Whitehead joined the Hegelian-Bradleyan camp. In his metaphysico-cosmological speculations the focus of attention turned toward an ontology of relations,

objects, events, and other entities. The field of these relations is made by 'actual entities' or 'actual occasions' (*Process and Reality*) which replaced the neutral 'individuals' of *Principia Mathematica*. The relations themselves - or, should we say, the denotata of n-ary predicates - are what Whitehead terms 'eternal objects'.[6] Formally more elaborate are Whitehead's applications of logic of relations in his theory of events, as well as in his method of extensive abstraction, in which he capitalized on his thorough background in geometry. In *The Concept of Nature* (1920) events are regarded as "the field of a two-termed relation ...of extension...",[7] the properties of which are carefully stated (for instance, its transitivity). We may now turn to Whitehead's theory of events and its logical aspects.

2. THEORY OF EVENTS

Whitehead concentrated his attention more on the ontology of events, reacting to the new developments in the relativistic physics and quantum theory. His approach is that of a working mathematician rather than a logician who pays careful attention to the syntactical and semantical aspects of the language of events. The logic of events had to wait for the contributions of H. Reichenbach, D. Davidson, R. M. Martin and others who have attempted to put it on formally sound grounds. Whitehead needed events as basic entities in dealing with space, time and motion, that is, in his attempts to provide an adequate account of the dynamic properties of the world. This endeavor, which spanned his middle phase (1914-1924), culminated later in his process metaphysics, even theology. While in the works from Whitehead's middle phase, the theory of events was in the forefront of his attention, it became much less pronounced in his later works. Yet the term 'event' did not disappear entirely. For instance, in *Process and Reality* 'event' is defined as "a nexus of actual occasions", and though 'actual occasions' are treated as the building blocks of the actual world, they are nevertheless only "the limiting type of an event with only one member".[8] In another place Whitehead characterized event as "a definite fact with a date".[9] The term 'nexus' has been informally defined as a "particular fact of togetherness among actual entities".[10] One may wonder how much more is expressed by using such term than the recognition of a relation or perhaps a family of relations among the entities in question.

The method of extensive abstraction, accompanied by the underlying theory of events, remained however Whitehead's persistent theme. This topic occupies Part III of *An Enquiry Concerning the Principles of Natural Knowledge* (1919), chapter IV of *The Concept of Nature* (1920), and also

Part IV of *Process and Reality* (1929), which is titled "The Theory of Extension" (he offers there a considerable number of assumptions and definitions).

3. The Recognition of EQUIVALENT SETS OF AXIOMS or POSTULATES

Metatheoretical investigations were not Whitehead's main preoccupation, yet his philosophical and methodological studies offer a rich stock of informal comments and suggestions in this respect. He was very well aware of the alternative systems in geometry and also of the possibilities of employing equivalent sets of axioms or postulates in various scientific and other areas. For instance, in "Analysis of Meaning" (1937) Whitehead stated that "we have not yet arrived at the understanding of arithmetical principles which exhibits them as devoid of alternatives."[11] In an article for *Encyclopedia Britannica* (1910, 11th edition) entitled "Axioms of Geometry", Whitehead claimed that geometry is concerned with "any subject matter to which the formal axioms may apply," while the axioms - mere propositional functions - are neither true nor false.[12] In this article Whitehead discussed also the problem of consistency and independence of axioms. As I have mentioned earlier, Whitehead offered philosophical comments on the problems of logical consistency in his later works. This issue will be therefore the next point of my inquiry.

4. LOGICAL CONSISTENCY (INCONSISTENCY) and the Problem of COHERENCE

When in *Process and Reality* Whitehead belittless the significance of the discovery of logical inconsistency in an investigated text, he seems to side with the Cartesians against the Port-Royal logicians - to use a historical analogy. Like the Cartesians, Whitehead would not worry so much about formal fallacies (logical errors), but rather about material fallacies (factual errors). For his speculative frame of mind *coherence* - hanging together, interlocking of entities, principles, items of experience - was more important. Yet, ultimately, what is coherence but logical consistency in the broadest possible sense? Whitehead talked about 'coherence' and 'consistency' in a rather careless way, which did not help to clarify the meaning of these terms. Sometimes Whitehead even appears to praise philosophers lacking consistency but displaying strength in - what he terms - *adequacy*; for instance John Locke, whose *Essay Concerning Human Understanding* (1690) he regarded as the most important source of his own philosophy of organism.

In the last book published during his lifetime - *Modes of Thought* (1938) - Whitehead returned to the problem of inconsistency defined as "the fact that the two states of things which constitute the respective meanings of a pair of propositions cannot exist together."[13] Now, as a good logician, he is again very well aware of the danger of the ambiguities of words and phrases, for example, of the ambiguity of the word "and". Yet the quasi-hegelian process metaphysician in him has the upper hand: it is by virtue of *process* - he claims - that all inconsistenciesa dissolve, for "inconsistency is relative to the abstraction involved."[14] Again, this proposal seems to be reminiscent of certain attempts to solve, or rather dissolve, the famous Zeno's paradox of the flying arrow.

As a mathematician, Whitehead retained his appreciation of *patterns* even in his discussion of the problem of inconsistency. What is interesting on patterns is how much they exclude rather than include. A paradigm pattern in this respect is Sheffer's notion of inconsistency - Whitehead believed while he was overlooking the distinction between the object language and metalanguage of propositional logic. To base logic on 'pIq' means for Whitehead to base logic upon Spinoza's concept of finitude since "the finite is that which excludes other things comparable to itself."[15] A curious statement coming from a mathematical logician who used to do so much work with Cantor's theory of transfinite numbers![16]

5. DISTRUST OF LANGUAGE as "an adequate expresion of propositions":

The final area of my. inquiry - Whitehead's philosophy and logic of language - is too complex and cannot be treated adequately. I shall therefore offer only few remarks. As to the relationship between logic and language, the following quotation from Whitehead's *Principle of Relativity* (1922) may be very revealing: "Mere deductive logic, whether you clothe it in mathematical symbols and phraseology or whether you enlarge its scope into a more general symbolic technique, can never take the place of clear relevant concepts of the meaning of your symbols, and among symbols I include words ...".[17] Whitehead's message seems to be: before you apply any kind of logic to language, clean first the possible mess in your language, by paying careful attention to the - as he calls them - "immediate facts of observation". Here we have Whitehead's battle cry for clarity and clarification, but how does it square with the final sentence of his 1941 lecture on immortality: "The exactness is a fake" ? A question can be raised whether the criticized inadequacy, ambiguity or feebleness of language is corrigible at all. Whitehead's holistic philosophy

of organism appears to undermine such an ultimate task of clarification, since an analysis of any proposition requires the consideration of the entire context of the actual world; that is, a verification or falsification of any statement may remain forever an unfinished task. It is interesting to note how a logician Quine - for whose first book, *A System of Logistic* (1934), Whitehead wrote an introduction - has tried to escape from a similar holistic-contextual ensnarement.

Whitehead as a logician: final comments

Quine, whom I just mentioned, wrote a very competent article "Whitehead and the Rise of Modern Logic" in 1939, for *The Philosophy of Alfred N. Whitehead* volume of The Library of Living Philosophers (1941). Yet Quine discussed in it mainly the achievements and weaknesses of *Principia Mathematica* and Whitehead's early work in abstract algebra and Boolean algebra of logic (*A Treatise on Universal Algebra* (1898) and articles published between 1900 and 1910). From Whitehead's later phase Quine selected only the 1934 paper "Indication, Classes, Numbers, Validation",[18] which to Quine was only an attempted patchwork on some problems derived from *Principia Mathematica*, of relatively minor importance.

I hope that my account of Whitehead's concern with logic in his later - post *Principia Mathematica* - phase shows his continuous appreciation of his first love,[19] though greatly overshadowed by his speculative interests. While reading *The Concept of Nature* (1920) or *Process and Reality* (1929) - personally I prefer *The Concept of Nature* - we might notice how interesting Whitehead's work is from a logical point of view.

St. John's University,
300 Howard Avenue,
Staten Island, New York 10301, USA,,
e-mail: ztarph@sjumusic.stjohns.edu

NOTES

[1] In **Process and Reality** (1929) Whitehead acknowledges this fact; see Part I, Chapter i, Section III.
[2] **Process and Reality**, corrected edition, ed. by D. R. Griffin and D. W. Sherburne (New York: The Free Press, 1978), p. 3.
[3] See his two works on the axioms of projective and descriptive geometry (Cambridge University Press, 1906 and 1907).
[4] **Process and Reality**, p. xiii.

[5] See, for example, Part I, Chapter II of **The Principle of Relativity** (Cambridge University Press, 1922).

[6] **Process and Reality**, p. 194, etc.

[7] **The Concept of Nature** (Cambridge University Press, 1920; reprint: 1971), p. 75.

[8] **Process and Reality**, p. 73.

[9] **Process and Reality**, p. 230.

[10] **Process and Reality**, p. 20.

[11] In: **Essays in Science and Philosophy** (New York: Philosophical Library, 1948), p. 95.

[12] In: **Essays in Science and Philosophy**, p. 178.

[13] **Modes of Thought** (New York: Capricorn Books - G. P. Putnam's Sons, 1938), pp. 73-74.

[14] **Modes of Thought**, p. 76.

[15] **Modes of Thought**, p. 72.

[16] See Whitehead's articles "On Cardinal Numbers" (1902), "Theorems on Cardinal Numbers" (1904), etc.

[17] **The Principle of Relativity**, p. 39.

[18] Mind 43/July 1934, pp. 281-297; corrigenda, Oct. 1934, p. 543. Reprinted in: **Essays in Science and Philosophy**.

[19] See his "Analysis of Meraning" (originally in **The Philosophical Review**, 1937), reprinted in **Essays in Science and Philosophy**, p. 99.