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Prior's Definition of Creative Definitions

(Sobociński-Prior-Lejewski's Discussion on the Leśniewskian Definitions)

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ABSTRACT: The article introduces Prior's paper *Definitions, Rules and Axioms* which deals with Leśniewski's creative definitions. It presents the origins of Prior's paper and the discussion which is linked with its final form. Prior's aim in this paper was to present the Leśniewskian definitions in comparison with Russell's concept of definitions, demonstrating their advantages and disadvantages. The main source of Prior's knowledge about the Leśniewskian definitions were Sobociński's papers and letters, which are stored in the Bodleian library. Although the paper *Definitions, Rules and Axioms* is a unique attempt at approximating creative definitions, it contains several mistakes. Lejewski identified them in his letter to Prior and also described how they arose. Lejewski's critique was not severe, however, and Prior coped with it in the introductory page of his paper.

KEYWORDS: Arthur N. Prior – Bolesław Sobociński – Czesław Lejewski – Stanisław Leśniewski – the creative definitions.

0. Introduction

Arthur Prior's work deals with numerous problems and his papers cover a broad spectrum of logics. A number of his papers have been discussed intensively (e.g. Prior 1955a), while other have been somewhat neglected. The paper *Definitions, Rules and Axioms* belongs to the latter category, even though

it is not without interest (see Prior 1976). Prior presented an ambitious attempt to approximate the Leśniewskian definitions to logicians who were only familiar with the Russellian definitions in it, long before Rickey (1975) paper was published. This Prior's effort was quite brave but not entirely successful. It induced Lejewski's (1956) comment on Prior's paper which has the form of an autonomous paper, even though, as far as I know, Lejewski never published it.

The aim of my paper is to present an analysis of the paper *Definitions, Rules and Axioms* in order to identify the problematic points and explain the principles following Prior's correspondence with Lejewski and Sobociński. In particular, I would like to argue that at least some of them have roots in Prior's adoption of Sobociński's theory, which was not entirely in accordance with Leśniewski's theory. Furthermore, I would like to illustrate the broader problem of spreading Leśniewski's ideas among logicians, who were more familiar with Russell's system of logic, using this example. Firstly, Leśniewski's papers were not easily available (which is not the case any longer). Secondly, Leśniewski's system of logic was dissimilar to Russell's system of logic. This situation was known to Leśniewski's students and they discussed these dissimilarities intensively. They were at home, however, in Leśniewski's system and therefore sometimes failed to explain clearly all the troublesome features. This could lead logicians who based their knowledge about Leśniewski on his student's papers into misinterpretations.

Prior was acquainted with Leśniewski's system of logic through works of Łukasiewicz. Łukasiewicz used Leśniewski's axioms in his system of logic, which Prior used and praised for a certain period of his life (see, e.g., Prior 1952). It might have been Łukasiewicz who encouraged Prior to contact Sobociński and Lejewski. There is no written evidence in Łukasiewicz's letters[1] that he did so, however, thus it might also have been Prior's own idea.

In Prior's archive, we find Sobociński's and Lejewski's letters but not Prior's responses. In 1953, Prior contacted Sobociński and the following year Prior also received Lejewski's first letter. Sobociński's letters contained a detailed expression of Leśniewski's system of logic, including several proofs.

[1] Notwithstanding, Sobociński claimed in his first letter that Łukasiewicz asked him to send some offprints of his papers to Prior and Łukasiewicz (1953) mentioned Sobociński, when he claimed that Sobociński, his former student, was the editor of the Journal of Computing Systems.

Lejewski's letters are in general shorter and focused on a discussion of several features of Leśniewski's, Lejewski's or Prior's systems of logic. The Bodleian Library stores Sobociński's letters from 1953 to 1955 and then two letters from 1965, which Sobociński wrote as the editor of the Notre Dame Journal of Formal Logic. Lejewski and Prior exchanged several papers and sent comments to one other. Their letters are also more personal since they met several times and were also colleagues at Manchester University.

1. Creative definitions

Rickey (1975, 273-274) points out that Leśniewski seems to be the first philosopher to have introduced the idea of creative definitions. Leśniewski did not discuss this topic in his papers, however, but presented his ideas in his lectures and used them in his Mereology, Ontology and Protothetic. This fact is not significant for Prior. He merely read the secondary sources of Leśniewski's ideas. However, it might have affected the different understanding of the concept, which arose between Sobociński and Lejewski. Namely, Sobociński developed Protothetic during Leśniewski's lifetime but also after Leśniewski's pre-mature death (see Sobociński 1998, 70-74). In his papers as well as in his letters to Prior, he did not differentiate between Leśniewski's ideas and his inventions, even though, he clearly expressed Tarski's contribution.

Leśniewski did not consider definitions to be abbreviations. Urbaniak (2014, 152) asserts that they are more axioms than definitions. Sobociński (1953a) claimed that the new semantical category could be introduced into theory via definitions. Since the variables of the newly introduced semantical category are contained in the creative definition, the semantical category can be used in theory. Sobociński maintained that this feature made Leśniewski's system of logic a growing system, to which new semantical categories could be added. He further pointed out that these definitions had to be based on the rules of a system.

The Leśniewskian definitions are creative, but as was mentioned before, there are established rules, which have to be fulfilled. In his second letter to Prior, Sobociński (1953b) demonstrated to Prior that the contradiction, which Prior encountered, was accounted for by his violation of certain rules. The definition had to specifically have the form of equivalence, in which the position of definiens and definiendum was strictly settled. Definiens is on the

right side and definiendum is on the left.[2] The newly introduced term or entire category is consequently situated on the left side of the definition. There are also specific rules for protothetical and ontological definitions. Sobociński claimed:

> 1) In the protothetical definitions (in prototheticc, ontology, a.s.o.) the first sign of definiendum *must be* a defined constant.
> 2) In the ontological definitions (in ontology, mereology a.s.o.) the fourth sign of definiendum must be a defined constant. Any of theses constants can be followed by a row of the different pairs of parentheses. In which parentheses there are included only the variables. Each of these variables must be different from the others and all must occur in the definiens and in the main quantifier of the definiendum. (Sobociński 1953b)

The form and the use of creative definitions are also presented in Sobociński's paper *An Investigation into Prototheticc* (see Sobociński 1998) which brought additional information about the Leśniewskian definitions to Prior (cf. Prior 1955-1956, 199). Sobociński used creative definitions for introducing operators, for instance:

> If the symbolic expression 'p ⅆ q' equivalent to '~ (p ∨ q)', is introduced, so that
>
> [pq]: p ⅆ q.≡.~ (p ∨ q)
>
> becomes a valid theorem, the following theses of prototheticc can be established … (Sobociński 1998, 76-77)

Sobociński did not discuss the theory connected with creative definitions and its rules in detail in his paper. He did not even mention that he had handled creative definitions, but briefly presented that this was the way the new terms could be introduced to Prototheticc.

Although Prior based his paper about the Leśniewskian definitions on the information provided by Sobociński's letters and paper, Lejewski (1958) also wrote a paper about the Leśniewskian definitions. It was Lejewski's paper *On Implicational Definitions*, which consisted of part of his dissertation. Lejewski provided there a propositional calculus based on the implication as a sole prim-

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[2] For a detailed expression of Leśniewski's definitions, see Miéville (2009, 29-59).

itive function by the use of implicational definitions, which have certain features of Leśniewski's creative definitions.[3] Nonetheless, this paper was published two years after Prior's *Definitions, Rules and Axioms*.

2. Definitions, Rules and Axioms

Prior discussed certain features of Leśniewski's system of logic in several of his papers (see, e.g., Prior 1952; 1953; 1955b; 1957; 1967). There is, however, a paper which deals exclusively with Leśniewski's theory of definitions, *Definitions, Rules and Axioms*. Prior introduced there two examples of the Leśniewskian definition (cf. Prior 1955-1956, 202 and 206).

He discussed the Leśniewskian definition for the first time in Prototheticc. He chose the following formula as an example:

$$\forall p \forall q \{(p \wedge q) \leftrightarrow (\forall \delta \{\delta qp \leftrightarrow [\delta p (\forall p (p \leftrightarrow p))]\})\}$$

which is Sobociński's definition of the conjunction. Discussing the form of Leśniewski's definitions, Prior argued that the essence of the theory of definition lies in the form of the definitions, i.e. that variables are bound by a universal quantifier and it is an equivalence. He claimed:

> It is, in brief, the theory that definitions are universal equivalences which we lay down in the form of axioms whenever we wish to introduce a new expression. (Prior 1955-1956, 203)

Prior additionally asserted that Leśniewski also suggested the usage of other operators in his definitions, e.g. an equivalence could be replaced by an exclusive disjunction. Prior also reformulated the definition of conjunction by the use of the exclusive disjunction. Although the second form of the definition is far more complicated than the first one, Prior admitted that it is still a permissible variant of Leśniewski's definition of conjunction.

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[3] They are namely creative and their introduction is limited by rules. In contrast to the Leśniewskian definitions, the primitive operator which is used here is not an equivalence or an exclusive disjunction but an implication (see Lejewski 1958, 189-193).

He also discussed the ontological definition. Prior introduced the role of creative definitions in Ontology using the example of the formula (cf. Prior 1955-1956, 204-206):

I. For all *a*, *b*, *c* and *d*, if the *c* is an a and the *b* is a *c* and the *d* is a *c*, the *b* is a *d*[4]

and maintained that this example is important from a historical point of view since: "This theorem expresses the individualising force of the word '[t]he'" (Prior 1955-1956, 204). One could deduce from this:

II. For all *a*, *b* and *c*, if the *c* is an *a* and the *b* is a *c*, then the *b* is an *a*

due to the addition of a creative definition:

III. For all *a*, *b* and *c*, the *c* is a star-*ab* if and only if the *c* is an *a* and the *b* is a *c*

Prior (1955-1956, 205) emphasized that formula I cannot be proved from theorem II unless definition III is added, even though neither formula I nor theorem II contain "star-*ab*."

Prior (1955-1956, 207-208) further raised objections to this concept from the Russellian point of view. He initially pointed out that Russellians might have objected that the proper sign used in the definitions should be "=" instead of "↔" and consequently that the Russellian definitions as abbreviations has their place in the theory. They simplified the notation. The Leśniewskian definitions in contrast multiplied axioms. They did not respect the gulf between axioms and definitions and handled it with terms where the meaning was not explained.

Prior demonstrated that the creativeness of definitions can lead to a contradiction. Namely, if "the" is replaced by "every" in the formula III, then the following formula is obtained:

IV. For all *a*, *b*, *c*, every *c* is a star-*ab* if and only if, every *c* is an *a* and every *b* is a *c*.[5]

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[4] The numerals of the formulas are different than in Prior's paper. They were changed for the sake of unity in my paper.

[5] The formula was changed in accordance with Prior's corrigenda.

This seems to be false in every possible meaning of "star-ab". Prior maintained that it is easy to prove:

V. For no *d* is it the case, that for all *a*, *b* and *c*, every *c* is a *d* if and only if every *c* is an *a* and every *b* is a *c*,

which could be reformulated as:

VI. For every *d* there is some *a*, *b* and *c* such that "Every *c* is a *d*" is not equivalent to "Every *c* is an *a* and every *b* is a *c*."

According to Prior, these three formulas cannot be all true at the same time but V and VI follow from IV and, hence, IV is contradictory.

In response to the Russellian objections, Prior (1955-1956, 208-210) claimed that if the Russellian definitions were to be consistent, they have to be formulated as rules of inference. If the definitions are neither axioms nor the rules of inference, they cannot be consistent with other postulates. This inconsistency could lead to a contradiction, as Prior demonstrated further.

Prior (1955-1956, 211-212) also coped with the objection that the meanings of the terms, which the Leśniewskian definition deals with, were not explained. He asserted that there is no settled procedure to identify the meaning of an expression. This is not just the case of Leśniewskian definitions but also Russellian definitions, axioms and theorems. There is no need, however, for a definition which could entirely cover the meaning. Prior added:

> The expressions are then 'defined' in the sense that the logician knows as much about them as he needs to know for his particular purposes; and ordinary definitions 'define' in this sense too. (Prior 1955-1956, 212)

Prior (1955-1956, 214-215) finally formulated two objections, which might arise among Russellian logicians, but which were in all probability Prior's. Firstly, he claimed that in intensional contexts, e.g. by formalizing beliefs, the Russellian type of definition is syntactically stronger than the Leśniewskian. The Russellian definitions are also more flexible in dealing with this context. He had to admit, however, that neither Leśniewski nor Russell favoured intensionality and therefore this objection was not significant for them. Secondly, Prior pointed out that the Leśniewskian definitions are not quite intuitive. Namely, Sobociński's definition of conjunction did not really correspond to the explanation of the word "and" in ordinary language. From a logical point

of view, the Leśniewskian definition is, however, more informative than Russellian. It guarantees that everything which is provable about

$$\forall\delta\ \{\delta qp \leftrightarrow [\delta p\ (\forall p\ (p \leftrightarrow p))]$$[6]

is also provable about $p \wedge q$.

To sum up, Prior attempted to introduce Leśniewskian definitions to Russellian logicians. Although he found certain objections, which could be formulated by Russell's proponents, he seemed to appreciate certain features of the Leśniewskian definitions. Notwithstanding, the two objections, the inconsistency of IV and the disadvantage of the Leśniewskian definitions in intensional logic, were not solved satisfactorily. As will be demonstrated further, Lejewski responded to both of them.

3. Lejewski's comment

Lejewski's comment consists of the commentary part and a friendly critique of Prior. Although it contained a certain criticism, Lejewski seemed to appreciate Prior's paper. The first part was meant to be a supplement to Prior's paper in which he explained features of Leśniewski's definitions, which Prior did not mention but tacitly presupposed. I entitled the second part "a friendly critique", since Lejewski himself introduced it:

> The second part includes some criticism which – I am sure – you will find not very difficult to answer. (Lejewski 1956)

Lejewski's introductory objections focused on Prior's example of the Leśniewskian definition in Protothetic. Prior chose Sobociński's definition of conjunction which Sobociński invented as the example after Leśniewski's death. It is consequently based on Sobociński's rather than Leśniewski's theory. As Lejewski stressed:

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[6] This definition has, by no means, a correct form of the Leśniewskian definition, since, in his system of logic, every variable is to be bound (see Woleński 1989, 150). However, it is admissible as a part of previously mentioned formula $\forall p\ \forall q\ \{(p \wedge q) \leftrightarrow (\forall\delta\ \{\delta qp \leftrightarrow [\delta p\ (\forall p\ (p \leftrightarrow p))]\})\}$, where all variables are bound.

> His theory allows for definitions such as the one produced by Sobociński but it does not stipulate them. (Lejewski 1956)

Prior (1955-1956, 204) also maintained that the protothetical definitions were the Leśniewskian definitions, but that they were not creative definitions since creative definitions only appeared in Ontology but not in Protothetic. He was mistaken at that point as follows from the previous introduction of creative definitions. He had already encountered creative protothetical definitions in Sobociński's (1998) paper, but since Sobociński did not claim clearly that the system of introducing a new operator, which he used here, were creative definitions, Prior apparently did not recognise them. Lejewski was aware of Prior's mistake and wrote in his answer:

> On page 7, Professor Prior says, that 'in general we do not have 'creative' definitions in the pure theory of truth-functions (what Leśniewski called 'protothetic')'. I find it difficult to agree with this statement, because in the systems of protothetic constructed by Leśniewski one begins the deductions from the axioms by introducing definitions, which are required exclusively for their 'creative' properties. (Lejewski 1956)

It is not clear, however, why Prior maintained that there are no creative definitions in Protothetic. This claim has no support in Sobociński's papers or his letters. Sobociński in contrast introduces ontological as well as protothetical definitions. Prior might have been misled by the fact that Sobociński (e.g. 1953a), while introducing the Leśniewskian definition in Protothetic, did not write directly that they were "creative". He also did not write that they were not.

Another comment concerns a different understanding of definitions in Leśniewski's and Russell's systems of logic. Lejewski pointed out that the Leśniewskian definitions are meant to be definitions only within specific theory – there is nothing there as a definition in the absolute sense of the word. In addition, the formula which is a definition in one system could only be a theorem in another, or in a different stage of the same system. Prior's objections in the sense of intensional logic do not consequently entirely fit with the Leśniewskian definition. Leśniewski's system of logic was strictly extensional.

Lejewski additionally stressed that V and VI could not be obtained from IV. He demonstrated that V and VI are empirical statements. He specifically maintained that from Prior's formulas the formula:

VII. for all *d*, for some *a*, *b*, and *c*, it is not the case that (every *c* is a *d* if and only if (every *c* is an *a* and every *b* is a *c*))

could be obtained which is equivalent to

VIII. for some *a* and *b*, it is not the case that every *a* is a *b*

This statement is empirical and hence cannot be demonstrated in a system of logic. Statement IV, which Prior consequently criticised in his paper, was not inconsistent.

As a reaction to Lejewski's letter, Prior coped with two major objections in one page which preceded the entire paper. He admitted that protothetical definitions are also creative and suggested another statement which could demonstrate the inconsistency of Leśniewskian definitions. This statement was also, however, not in accordance with Leśniewski's Ontology, as Ontology did not assure the accessibility of empty terms. Therefore, even a reformulated proof did not actually harm the Leśniewskian definitions. Everything added to the paper is actually regularly overlooked regardless of how important a part of the paper it is. This page was not consequently included in the reprint of the paper in the book *Papers from Logics and Ethics* (see Prior 1976, 39-55).

4. Conclusion

When dealing with Leśniewskian definitions, Prior demonstrated a great deal of courage since those differed considerably from Russell's definitions, which he was more familiar with. Although it provoked Lejewski's detailed comment, Prior's mistakes were easy to correct as Lejewski had also predicted. They were mostly caused by combining Leśniewski's theory with Sobociński's later inventions and by the fact that Sobociński did not explain certain features of Leśniewski's theory to Prior which might have seemed trivial to him.

Among the objections which Prior formulated against Leśniewskian definitions, the one which neither Leśniewski nor Russell would have supplied, appeared to be crucial. From the publication of the paper *Definitions, Rules and Axioms* up to his death, Prior primarily worked with intensional logic. Leśniewski's system of logic is extensional and his definitions were adapted to

this purpose. As Prior stressed, they are disadvantageous in intensional logic. Prior did not consequently make substantial use or discuss Leśniewskian definitions further. Notwithstanding, he did not abandon Leśniewski's logic entirely.

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