Carnap and Newton: Two Approaches to the Method of Theory Construction (Part II)

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Abstract: The paper, as a continuation of the paper Hanzel (2009), provides a methodological generalization of Newton's method of theory construction as applied in Book I and Book III of his *Principia*. It reconstructs also the method of measures applied in those books. Finally, it shows how the term "harmonic law" changes its meaning in the *Principia*.

Keywords: Newton's method of theory construction, method of measures, meaning change of the term "harmonic law."

Drawing on our analysis of the method of theory-construction as given in the so-called "Standard Conception" of scientific theories and on our reconstruction of the internal structure of Book I and Book III of Newton's *Principia*, both given in part I of this paper,[1] we will now try to make some generalizations about this structure. We will, first, reconstruct three types of measure used by Newton in his thought-movement from the phenomena-effects to their cause. Second, we will reconstruct the specific characteristics of this movement. Third, based on this we will show, by means of transparent intensional logic, how the so-called harmonic (or third Kepler's) law changes its meaning in the framework of Book I and Book III of the *Principia*.

1 Part I of this paper was published as Hanzel (2009).

3 Metareflections

A The Three Dynamic Measures of Force

From an examination of propositions 6 and 7 in Book I, one can distinguish three ways of determining the force by means of its effects. Our previous analysis yielded the result that $F \propto PX/\Delta t^2$. It was based on the idea that *uniform rectilinear accelerated motion* can provide us the quantitative data enabling us to find the quantity of the force causing it. According to I. B. Cohen: "This is Newton's dynamical measure of a force [...] It is a dynamical measure of force because it measures the force by its dynamical effect, the rate at which the action of the forces causes the moving object to deviate from a linear path" (1999, 321), while J. Brackenridge labels this measure as the *linear dynamic ratio* (1995, 7, 171).

In proposition 6 Newton draws not only on uniform linear motion, but also on uniform circular motion. He uses the following figure (1999, 454) (Q is any point on the curve; R is a point on the tangent so that QR is a parallel to SP and QT is a perpendicular to SP):

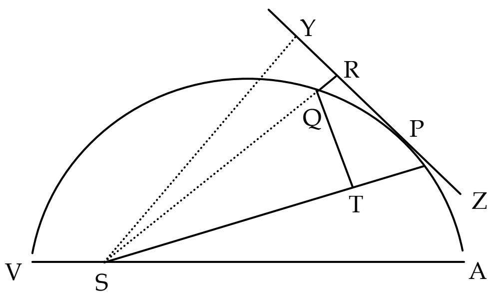

Fig. 1 Newton's dynamical measure of force for uniform circular motion

where SY is a perpendicular to the tangent YZ and passes through the center S of force. In corollary 2 he states that $F \propto QR/(SY^2 \times QP^2)$. The proof is as follows. If Q approaches P, QP approaches RP and then, because the triangles TQP and YSP are similar, $SY{:}SP = QT{:}QP$. But if $SP \times QT = SY \times QP$, the result form proposition 6 that $F \propto QR/(SP^2 \times QT^2)$ is equivalent with $F \propto QR/(SY^2 \times QP^2)$.

As a next step one should notice the line PV in the figure above. It is the chord of a circle that approximates the curve APQ in point P. By a combination of Figure 1 with Figure 4, reproduced in Part I of our paper, we obtain the following figure (X is here the intersection of QQ' and VP):[2]

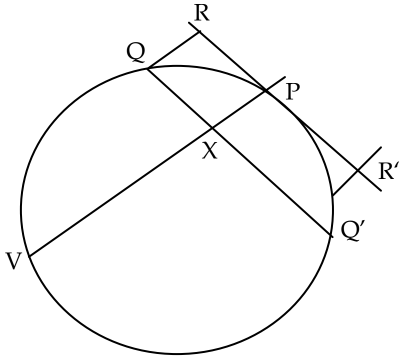

Fig. 2 Determination of the circular dynamic ratio for the centripetal force

It holds[3] $QX \times XQ' = VX \times XP$. But because $XP = QR$ we obtain $QX \times XQ' = VX \times QR$. If Q approaches P, then VX approaches PV and XQ and XQ' approach QP, and we have $QP^2 = PV \times QR$. Because, by corollary 2 of proposition 6, $F \propto QR/(SY^2 \times QP^2)$, we obtain, finally, $F \propto 1/(SY^2 \times PV)$. J. Brackenridge labels $1/(SY^2 \times PV)$ as the "circular dynamic ratio" (1995, 37), which serves Newton as yet another *measure of the centripetal force*.

Finally, in corollary 2 and 3 of proposition 7 Newton employs, as an alternative solution, a third measure of force, namely, the so-called "comparison theorem" (Brackenridge 1995, 173).[4] It grows out of the linear dynamics ratio, but here Newton considers the ratio of a force by which a body P revolves in an orbit around the center S to a force by which this body revolves in the same orbit around any other center R.[5]

2 We draw here partially upon the figure in Brackenridge (1995, 195).

3 It holds by proposition 35 of Book 3 of Euclid's *Elements*.

4 This third measure is applied in the alternative proof of proposition 11 of Book I.

5 The whole computation is given in Brackenridge (1995, 172 – 174).

B The Derivation of Forces from the Phenomena of Motion

The above-mentioned three measures enable Newton to derive forces from their effects. Let me now deal with these effects, with the place of the measures of force in these derivations, and, finally, with the cyclical "end" of these derivations.

An important characteristic of the effects-phenomena from which derivation starts is that, with respect to the character of the thought-movements which take their course from them, that they are idealized phenomena.[6] On the one hand, some of the idealizations are "short-lived" in Book I (idealization of the orbit, so that initially it has a circular character, then abolished in favor of an elliptical orbit; the initial supposition that the center of force is non-accelerated, then abolished in favor of an accelerated one, etc.) On the other hand, Newton holds to the following, more "long-lived" idealizations which span several propositions in the *Principia*:

- (i) the center of force, orbited by a body, is devoid of any mass, therefore, it is not attracted by the orbiting body (up to proposition 57, Book I);[7]
- (ii) there is only one body orbiting the center of force, or, if there are several of them, then they do not mutually interact;[8]
- (iii) the orbiting body is devoid of any dimensions, i.e., it is a mass-point (up to proposition 19, Book III);
- (iv) the central body is devoid of any dimensions, i.e., it is a mass-point (up to Section 12, Book III).

Based on these idealizations Newton employs his strategy of deriving from the knowledge of quantity given in the idealized phenomena-effects the knowledge about the quantity of the forces causing them. Four cases of applying this strategy are worth noting. The first involves the derivation (proposition 2 of Book I) of the centripetal character of the force acting on a body from the fact that the latter's

6 For a list of these idealizations see Harper (1993, 148 – 153).

7 From proposition 57 to proposition 63 Newton considers the case when the center of force has mass and thus both the central body and the other body are orbiting a common center of force.

8 For this see, e.g., propositions 14 and 15, Book I.

orbital motion satisfied Kepler's area law, that is, in the same time it will describe the same areas, or, that its area rate is constant. In corollary 1 of proposition 2 Newton then states that if the area rate decreases, the force is directed off the center in the direction of motion, and if the area rate in increasing, the force is directed off the center against the direction of motion. Proposition 2 together with corollary 1 thus give us "systematic dependencies, which make a constant area rate measure the centripetal direction of force maintaining a satellite in its orbit" (Harper 1999, 77).

The second is the derivation (proposition 4 of Book I) of the inverse-square character of the centripetal force from (Kepler's) harmonic law (i.e., from the fact that the periodic time of a group of bodies orbiting the same center are as the 3/2 power of their distances from that center). In corollary 7 of proposition 4 Newton broadens the relation between the orbital characteristics of the bodies and the centripetal force to which they are subjected, so that if the periodic times are as the n-th power of the distances, the centripetal force will be inversely as the 2(n-1)th power of the distances. Here again it is readily seen that the area law measures the power-of-the-distance-dependence of the centripetal force.

The third case involves derivation (proposition 45 of Book I) of the power-of-the-distance-dependence of the centripetal force from the precession characteristics of the orbit under the impact of this force. Newton proves here that "zero orbital precession measures inverse square law for distances explored by orbit" (Harper 1999, 87), and generally that "if the centripetal force is as any power of the radius, that can be found from the motion of the apsides" (1999, 87).

The fourth case is a specific form of inference of the quantitative characteristic of the centripetal force, namely, not from one but simultaneously from two phenomena-effects. Such a type of derivation was accomplished by Newton in proposition 2 of Book III, where he derived the inverse-square character of the centripetal force for the primary planets from (Kepler's) harmonic law holding for these planets and simultaneously from the zero-orbital precession of their orbits. In a similar manner Newton accomplished also his famous first moon-test in Book III. Here he used two phenomena: the fall of a body on the Earth and the acceleration of the moon toward the Earth. By computations for the moon Newton arrives at "data" which "measure a force producing accelerations at the surface of the earth. These accelerations are equal and are equally directed toward the center of the earth" (Harper 1993, 161). Thus, for the fourth case of derivation holds in general that (Harper 1993, 159)

> there is a special advantage to inferences to a proposition from alternative phenomena. Each such inference is a measurement of the value of the relevant magnitude specified in the proposition. An inference to this same proposition from another phenomenon is an independently agreeing measurement of this same magnitude.

From the above-reconstructed four cases of thought-derivation of forces from idealized phenomena it follows that "the phenomenal parameters *measure* corresponding values of the theoretical parameters that are inferred" (Harper 1999, 74), and that "values of the phenomenal magnitude carry the information that corresponding values of the theoretical magnitude obtain" (Harper 1993, 147). That Newton consciously uses this method of measuring the respective cause is readily seen from the following claim of his: "The representatives of times, spaces, motions, speeds and forces are any quantities whatsoever proportional to things represented" (Newton 1965, 312).

C *The Cyclical Method of Theory Construction and the Change of Meaning of the Harmonic Law in the* Principia

Newton, as shown in 2.4.B, derives the centripetal nature of the force acting on the orbiting body as well as the dependence of the size (quantity) of this force on the distance from the center of force by drawing on certain idealized phenomena. This has, as we will show now, a very interesting "feedback" consequence on what we today label as "Kepler's third (or harmonic) law" of planetary motion which is one of the starting points of Newton's movement *to* the characterization of centripetal force (in Book I) and of the force of gravity (in Book III).

Let us start with Book I. Here, in proposition 4, corollary 6, as shown above in 2.2.A, Newton derives from the claim that "the periodic times are as the 3/2 powers of radii" (1999, 451) the claim that the centripetal force will be inversely as the square of the radii, while the term "radius" is here understood as the *line from the orbiting body, viewed as mass-point, to the center of force, viewed as a point without mass and any spatial dimensions.* Accordingly, we can give the following concise representation of the harmonic law (T_1, T_2 stand for the periodic times of two bodies orbiting the same center of force, r_1 and r_2 are their respective distance from this center):

(1) $T_1^2/T_2^2 = r_1^3/r_2^3$

If we consider the case of just one orbiting body we can state the harmonic law as follows ("$\propto$" stands for "is proportional to"):

(2) $T^2 \propto r^3$

In a next step Newton at the very beginning of Section 11, Book I states the following (1999, 561):

> Up to this point, I have been setting forth the motions of bodies attracted toward an immovable center, such as, however, hardly exists in the natural world. For attractions are always directed toward bodies, and—by the third law—the actions of attracting and attracted bodies are always mutual and equal; so that if there are two bodies, neither the attracting nor the attracted body can be at rest, but both (by corol. 4 of the laws) revolve about a common center of gravity is if by mutual attraction. […] For this reason I now go on to set forth the motion of bodies that attract one another, considering centripetal forces as attractions, and where, according to the third law, by mutual actions of these bodies "equal changes occur [… in their] motions" (1999, 417), and where motion is understood as arising "*from the velocity and quantity of matter jointly*". (1999, 404)

By bringing in the masses (quantities of matter) of the orbiting body and of the other body which we label here, tentatively, as "Sun", he then proceeds in proposition 60 (Book I) to the following statement of the harmonic law (1999, 564):

> If two bodies S and P, attracting each other with forces inversely proportional to the square of the distance, revolve about a common center of gravity, I say that the principal axis of the ellipse which one of the bodies P describes by this motion about the other body S will be to the principal axis of the ellipse which the same body P would be able to describe in the same periodic time about the other body S at rest as the sum of the masses of the two bodies S + P is to the first of two mean proportionals between this sum and the mass of the other body S.

So, we have here a change in the initial statement of the harmonic law so that now holds:[9]

(3) $T_1^2(S + P_1)/T_2^2(S + P_2) = r_1^3/r_2^3$,

where P_1, P_2 are the masses of the orbiting bodies and S is the mass of their common "Sun". In the case of just one body with mass P orbiting its "Sun" the following harmonic law can be derived:[10]

(4) $T^2(1 + P/S) \propto r^3$.

What has to be emphasized here is, first, that the radii are still understood as the lines connecting the "dimensionless" bodies with their dimensionless "Sun." Second, Newton does not draw here on the harmonic law as one of the presuppositions of his thought-movement *to* the force, but already on proposition 15 (Book I), where the harmonic law is already the "end"-point of the derivation starting *from* the inverse-square-distance dependence of the centripetal force.

Let us now move to Book III, where the harmonic law appears for the first time in the statements for the Phenomenon 1 (for the satellites of Jupiter), Phenomenon 2 (for the satellites of Saturn), and Phenomenon 4 (for the primary planets) (1999, 797 – 800):

> The circumjovial planets [...] their periodic times [...] are as the 3/2 powers of their distances from that center [the center of Jupiter]. [...] The circumsaturnian planets [...] their periodic times [...] are as the 3/2 powers of their distances from that center [the center of Saturn]. [...] The periodic times of the five primary planets [...] are as the 3/2 powers of their mean distances form the sun.

Unlike the understanding of the term "distance" in the harmonic law in propositions 6 and 60 in Book I, in Phenomena 1 and 2 "distance" is understood as the line connecting the satellites with the center of the respective planet (i.e., the latter are viewed as already having spatial dimensions), and in Phenomenon 4 as the "mean distance" from the Sun, i.e., as the semimajor axis of the elliptical path of the planet. A concise representation of the harmonic law in Phenomena 1, 2 and 4 thus is as follows (R_1, R_2 and R stand here for the semi-major axises):

9 We draw here on Wilson (1989, 260).

10 We draw here on Cohen (1980, 224).

(5) $T_1^2/T_2^2 = R_1^3/R_2^3$
(6) $T^2 \propto R^3$.

Newton then modifies the harmonic law in proposition 15, where his aim is "[*t*]o *find the principal diameters of the* [planetary] *orbits*" (1999, 819), and then states (1999, 819 – 820):

> These diameters are to be taken as the 2/3 powers of the periodic times by book 1, prop. 15; and then each one is to be increased in the ratio of the sum of the masses of the sun and each revolving planet to the first of two mean proportionals between that sum and the sun, by book 1, prop. 60.

This means that similar to the status of the harmonic law in proposition 60, Book I, where it was already the "end"-point of the derivation starting *from* the centripetal force, here the harmonic law is viewed already as the "end"-point of the derivation proceeding *from* the force of gravity. The following expressions can then be derived:

(7) $T_1^2(S + P_1)/T_2^2(S + P_2) = R_1^3/R_2^3$,
(8) $T^2(1 + P/S) \propto R^3$.

The transformation of the harmonic law in Book I (from proposition 6 to proposition 60) and Book III (from Phenomena 1, 2 and 4 to proposition 15) of the *Principia* can thus be represented as follows (the content of the frames stands for the context *via* which the harmonic law has to be transformed):

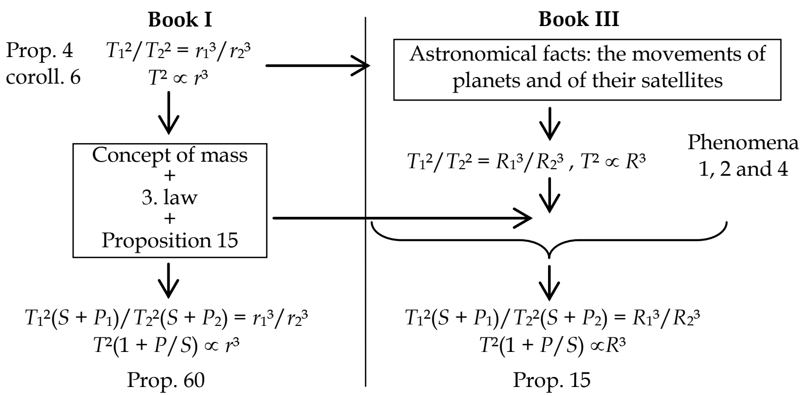

Fig. 3 Transformations of the harmonic law in Book I and Book III of the Principia

From this it is readily seen that once Newton, starting from idealized phenomena, derived in Book I the characteristics of centripetal force and in Book III the characteristics of the force of gravity, he had to take into account these forces as a perturbing factor, e.g., of the orbiting body (bodies) acting on the central body, and thus had to *return* to those phenomena and correct, modify them. This feature of the *Principia*, together with our previous reconstruction of its internal built-up, shows that it is, contrary to the view of the so-called "Standard Conception", built by a *cyclical method of theory construction*. In fact *Principia* contradict that conception also in yet another important aspect. While according to the "Standard Conception" one should end up at the level of statements referring to the observable state of affairs, in case of Book I and Book III of *Principia* it is readily seen that the statements pertaining to the harmonic law (proposition 60 from Book I and proposition 15 from Book III) contain in an irreducible manner the term "mass", thus a term referring, from the point of the *Principia*, to something *unobservable*.

What has to be emphasized is that *Principia*'s cyclical return to the phenomena from which its construction initially started and their successive correction does invalidate neither Newton's thought-movement from phenomena of motion to the forces causing them, nor this method of theory construction as such. "So long as the corrections are perturbations attributable to other forces – whether other components of gravitational force or even foreign forces – the inferences to the original phenomena can be construed as components of the perturbed motion" (Harper – Smith 1995, 144). In respect to Kepler's laws, which initially served as a basis of derivation of the inverse-square character of the centripetal force, this means that (Harper 1993, 156)

> the formula [...] for [...] a perturbed orbit is properly conceived as a formula for a composition of motions one of which is the Keplerian orbit that fits the [initial] law and the other is the perturbation produced by the interaction. According to such a conception the Keplerian phenomenon [from which the inference initially started] is there to be found [...] It is, however, transformed from a claim about the total motion to a claim about that component of the total motion caused by the inverse square centripetal force.

Figure 3 above now leads to the following question. Does the term "harmonic law" transform its meaning in the course of theory-construction as given in Book I and Book III of the *Principia*? In order to exclude beforehand any possible misunderstanding with respect to this question we emphasize that we are here reflecting only into the possible meaning-changes of this term as given in the framework of Book I and Book III of the *Principia* and not into changes of the meaning of this term as it was initially stated by Kepler.[11]

In order to discern the meaning of the term "harmonic law" one could apply transparent intensional logic[12] (or TIL for short) to the above given reconstructions (2), (4), (6) and (8) of this "law." So as TIL views constructions represented by language expressions as the latter's meanings, by means of it one could prove that TIL-constructions correspond exactly to the meanings of those reconstructions and thus that they have different meanings.

From this we draw the conclusion that Book I and Book III of the *Principia* can be viewed as a hierarchically organized sequence of gradually shifting constructions, and thus that the term "harmonic law" inside Book I and Book III of the *Principia* gradually changes its meaning.

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11 On Kepler's approach to his "laws" see, e.g., Aiton (1969), Aiton (1973), Aiton (1975a), Aiton (1975b), Davis (1992), Donahue (1994), Russell (1964), Stephenson (1987), Voelkel (2001), Whiteside (1974), Wilson (1968), Wilson (1970, 92 – 106), Wilson (1972a), Wilson (1972b), and Wilson (1975).

12 This term was coined in Tichý (1988). On the application of TIL see, e.g., Materna (1998).

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