POINTS OF VIEW FROM A LOGICAL PERSPECTIVE (I)

Marie DUŽÍ – Bjørn JESPERSEN – Pavel MATERNA

In the paper we offer a *logical explication* of the frequently used, but rather vague, notion of *point of view* We show that the concept of point of view prevents certain paradoxes from arising. A point of view is a means of *partial* characterisation of something Thus nothing is a *P* and at the same time a non-*P* (*simpliciter*), because it is a *P* only relative to some point of view and a non-*P* from another point of view. But there is a major, complicating factor involved in applying a logical method that is supposed to provide a formal and rigorous counterpart of the intuitively understood notion· 'point of view' is a homonymous expression, and so there is not just one meaning that would explain points of view Yet we propose a common scheme of the logical type of the entities denoted by the term 'point of view'. It is an empirical *function*. when applied to the viewed object in question, it results in a (set of) evaluating proposition(s) about the object. If there is an agent applying the criterion, the result is the agent's *attitude* to the respective object. The paper is organised into two parts. In Part I we first adduce and analyse various examples of typical cases of applying a point of view to prevent paradox. These cases are examined according to the type of the viewed object· a) the viewed object is an individual and b) the viewed object is a property or an office. In Part II we then show that the method described in Part I can be applied also to the analyses of agents' attitudes. We thus explain how an agent can believe of something that it is a *P* and at the same time a non-*P* the agent applies different viewpoint criteria to the viewed object. The inversion of perspective consisting in the perspective shifting from the believer on to the reporter in the case of attitudes *de re*, and from the reporter to the believer in the case of attitudes *de dicto*, is also analyzed. We show that there is no smooth logical traffic back and forth between such attitudes and prove that they are not equivalent. By way of conclusion, we explicate the notion of *conceptual* point of view and analyze cases of viewpoints given by conceptual distinction We show, finally, that the proposed scheme of the type of point of view can be preserved, this time, however, in its extensional version.

1. Problem and methodological preliminaries

This paper offers a *logical explication*[1] of the notion of point of view. The method of such an explication is to replace a contentual but vague notion by a formal but rigorous simulacrum and then distil various charac-

[1] It was Carnap who has introduced the term 'explication' See Carnap (1962)

ORGANON F **13** (2006), No. 3, 277 – 305

teristics of this formal notion. The idea behind this method of indirect illumination of a notion is that since the original notion is not logically tractable, at least its formal counterpart will be. The philosophical task accompanying the logical work therefore consists in maintaining and arguing for the intuitive link between the contentual and the formal notion, and will be successful if the link is established. If the link is torn asunder, the findings relating to the formal notion will have no bearing on the informal notion and the explication will have failed. Obviously, there can be no such thing as a proof of the adequacy of the explication, since the link can be nothing other than intuitive. However, what speaks in favour of a given explication is whether it can give a principled and unified account of the host of phenomena that the informal notion gives rise to.

Our problem then is to characterize at least some facets of the notion of point of view and to do so through the prism of logic. It is important to stress that the enterprise is not the much more ambitious one of providing an "actual logic" of points of view. This would require, inter alia, a set of axioms and inference rules together with a regimented vocabulary. Such an undertaking is far beyond the scope of this paper, which instead intends to indicate the general direction that a logical inquiry into points of view should head into while committing itself to a logical analysis of various key notions by providing formalizations.

Our problem-solving approach is the one characteristic of contemporary analytic philosophy. Rather than attempting a direct analysis of a notion we provide a semantic analysis of expressions within which terms denoting or expressing the notion occur. I.e., we do not directly ask what points of view are but what the expression 'point of view' denotes and means. The link between the meaning of 'point of view' and the intuitive notion of point of view is established by the fact that we make the linguistic meaning part of the explication of the intuitive notion.

But there is a major complicating factor involved. Points of view obviously cannot be brought under just one hat. In linguistic terms, 'point of view' is a homonymous expression, and so there is not just one linguistic meaning that would explain points of view. Further, we need to introduce a not inconsiderable measure of flexibility as for the objects that may serve as points of view. Some cases call for fairly crude individuation, as when a point of view is simply a criterion which divides a domain into those elements that are in and the rest which are out. Other cases demand a very fine-grained separation of points of view, as when a point of view results in an attitude.

The semantic analyses will be cast in terms of one particular theory called Transparent Intensional Logic (TIL). Our principal reason for choosing TIL is that of the current theories in formal semantics it arguably provides one of the most worked-out and rigorous conceptions of various linguistic meanings. The architecture of TIL consists of *extensional* objects on the ground-floor, *intensional* objects on the first floor and so-called *constructions* (*hyper-intensional* objects) on the second floor and up. Extensional and intensional objects (not involving constructions) together form the type of first-order objects, while constructions (and objects involving constructions) are higher-order objects. The intensional objects are those of possible-worlds semantics, i.e., mappings defined on a logical space of possible worlds. In keeping with more recent trends in possible-worlds semantics, instants of time are included. An *intension* then becomes a mapping from worlds onto chronologies (strings of times) of objects. The resulting objects may be extensional ones like sets, individuals or truth-values, but also other intensions. For instance, the intension *the Pope's noblest property* takes properties, not sets as its values. Intensions whose ranges are themselves intensions are still first-order objects but of a higher degree than those intensions whose ranges are extensions.

The key *constructions* are four in number[2], two of which qualify as *structured hyperintensions*. Hyperintensions, because their principle of individuation is finer than necessary equivalence. Structured, because they literally contain (invariably abstract) constituents arranged in particular ways, which altogether specify an intellectual itinerary whose destination is some particular object. A leading idea in forming TIL is that *function*, unlike relation or set, is the fundamental primitive notion in any explication. Functions are the 'movers' of logic; though they do not actually move objects around, of course, they take one particular object, or pool of objects, to another particular object by mapping a pre-image to its image. Functions allow you to depict how different objects hook up

[2] Pavel Tichý who founded the TIL system actually introduced six constructions. Besides the four types that we are going to use, namely a variable, trivialisation, application and a lambda abstraction he also defined execution and double-execution. For details see Tichý (1988).

with each other (as arguments and values). So do relations, but the logic of relations has no room for the notions of the procedure of setting up a relation from such-and-such to such-and-such and the procedure of moving from a relation to one of its terms. So thanks not least to functional abstraction and application, the logic of functions (i.e., the lambda-calculus) is more powerful. Besides it is able to account for relations and sets as functions. A set is a characteristic function (akin to a *Begriff* in Frege), and a relation is a function from an n-tuple to a truth-value.

The vertical architecture of TIL offers a wide array of points of view. Both intensions and constructions can be made to serve in the roles of points of view. For instance, we may say that:

(*a*) As a musician Charles is an amateur, but as a rocket scientist he is a true professional.
(*b*) From the viewpoint of their ability to amuse people, Charles and Paul are similar, but otherwise they are quite dissimilar.
(*c*) From the point of view of logical behaviour, man is a rational being, but from the point of view of social behaviour man is a wild beast.
(*d*) The wolf is useful from the viewpoint of natural balance, but from hare's point of view he is destructive.
(*e*) Under the aspect of being a Quaker, Richard Nixon is a pacifist; under the aspect of being a Republican he is militant.
(*f*) What is Charles's viewpoint of the problem of nuclear power usage? He believes that it is indispensable, but he doubts whether it is safe.

We can see that points of view provide some *partial* characteristics of the viewed object; (*e*) is an example of two incompatible perspectives on the same sort of state-of-affairs. Under the aspect of *Quaker*, Richard Nixon is a pacifist; under the aspect of *Republican*, Nixon is militant. This is a standard example from the literature on non-monotonic reasoning, which deals with defeasible inferences. Here the example serves as one of mutually exclusive properties (pacifist, militant) being applied to the same individual simultaneously. (*a*) makes Charles both an amateur and a professional, but not with respect to the same property (instead: musician, scientist). (*b*) brings Charles and Paul on an equal footing by having them share the same property (the ability to amuse people), but makes them otherwise dissimilar.

We will show that the concept of point of view prevents certain paradoxes from arising. E.g., without the option of falling back on points of view, either the question whether Charles is an amateur or a professional could not be asked, since nobody is an amateur or a professional *simpliciter* but only relative to some point of view, or else Charles is both an amateur and a professional (*simpliciter*). Hence we avoid the following sort of arguments:

Charles is an amateur from the point of view of music, so Charles is an amateur. Charles is a professional from the point of view of science, so Charles is a professional. Therefore, Charles is an amateur and Charles is a professional.

Our point of departure is that what everything which qualifies as a point of view at all has in common is the following. A point of view is a view of something from a particular vantage-point, and the vantage-point is a means of characterisation. An agent who avails himself of the characterisation makes it his perspective. We wish to stress that views need not be perceptual. In fact, we shall have nothing in particular to say about perception or vision[3]. And the object which is at the receiving end of the point of view need not be concrete. It may be an individual like Lulu, Lulu's carnivorous pet, Lulu's house, Lulu and Lola, but also an abstract object of any kind, like the property of being a spider, the individual concept of the Pope, a mathematical proposition, a chemical or political problem, etc.

A few examples using questions and answers may help fix ideas. When asking,

What is Lulu's most characteristic property?

we expect the answer to cite a particular property, as in "Lulu is two metres tall." Another question is more interesting to ask,

What is Lulu's most characteristic property from the point of view of her social relations with other people (or her hobbies or her political convictions)?

As we continue to ask and answer questions about Lulu's properties, including relational ones, we are thereby applying criteria that, as it were, send Lulu into various sets and keeps her out of other. As we go along we build up a partial picture of Lulu. We can also compare an individual to another individual by asking,

[3] See Does, Lambalgen (2000)

Is Lulu similar to Lola?

Again, several answers are possible. E.g., yes, they are similar from the viewpoint of their age and the colour of their hair, but otherwise completely dissimilar in terms of social background.

The underlying schematic question is,

What is the most characteristic property of X from the viewpoint of A?

If X is not Lulu now but the property of being a spider, then A might be, for instance, the maximal size, the typical activity, the favourite habitat, the number of limbs, etc, of those individuals who are spiders. By specifying the values of these criteria for spiders we get the characteristic properties of spiders, or even the necessary conditions (*requisites*) for being a spider. If *all* those criteria are evaluated for the property of being a spider we get the *essence* of the property of being a spider (the necessary and sufficient condition for being a spider). Formulating the essence of an intension is equivalent to *ontologically defining* the intension.

Now we can try to generalize particular cases of using notions like *criterion, viewpoint, with respect to,* as mentioned above:

There is always some *object* in the broadest sense of the word, be it an individual, a tuple of individuals, but also a property (or, in general, some intension), or even a problem, theory, etc., and a *point of view* (or: a *viewpoint*) from which the object is viewed; any of the possible characteristics of the object is (or is not) relevant *with respect to* such a viewpoint. We can observe a common feature of the entities denoted by the term 'point of view': It is the *functional character* of such an entity (criterion). Being applied to the viewed object, it results in a (set of) evaluating proposition(s) about the object. If there is an agent applying the criterion, the result is the agent's *attitude* to the respective object.

Hautamäki in his (1986) presents a thorough analysis of the notion point of view using the apparatus of modern logic; informally, he exploits the term *determinable* and shows that determinables "represent objective aspects of real entities" (p.23). He also states that determinables can be handled as Chen's *attributes*, i.e., as functions that map an entity set into a value set. In our opinion a more precise conception of attributes has been worked out in the "HIT conceptual model" (see Duží (2001) where attributes are *empirical functions*, i.e., functions whose domain is the logical space (possible worlds) and whose range is a chronology of some values). What is important, however, is that determinables (attributes) are a sort of *functions*. Verbally they are expressed as *the X of.*

Thus we have *the age of, the colour of,* even *the colour of the hair of,* etc. Hautamäki defines (1986, p.65) points of view as follows:

> A point of view is a finite (proper) subset of I, where I is a set of determinable indices (i.e., of particular 'aspects', particular attributes).

Thus while the members of I are particular (empirical) functions, the points of view are particular selections of such functions (members of 2^I).

Hautamäki's work is a good example of applying logic to the analysis of a frequently used and particularly useful notion. The contemporary tools of logic and mathematics (in particular, theory of lattices) are exploited and interesting results obtained. The purpose of the present paper is a similar one; TIL, however, differs from Hautamäki's approach by (at the least) its explicit intensionalisation (attributes are *empirical* functions).

2. Transparent Intensional Logic (TIL)

Here we first offer an informal sketch of the philosophy of TIL and of its approach to the analysis of (natural) language and then introduce definitions of key notions. Some of the central semantic principles of TIL are the following.

Transparency. Every expression, without exception, is referentially transparent in the sense that its semantic properties are insensitive to the sort of linguistic context which the expression occurs in. In TIL, semantics is fully *a priori* and completely anti-contextualist. What an expression means and what it denotes can be ascertained prior to the deployment of the expression in a context.

Compositionality. Meaning of a complex expression is a function of the meanings of the involved subexpressions (together with the logical structures of the complex expression and its subexpressions).

Principle of subject-matter. An expression is about all and only those objects which receive mention by subexpressions occurring in the overall expression[4].

Linguistic sense is procedural. The sense of any expression, whether atomic or molecular, is a procedure, of one or several steps, for intellectually arriving at the object, which is the denotation of the expression.

---

[4] A forerunner of the principle may be found in Frege (1884): "Ueberhaupt ist es unmöglich, von einem Gegenstande zu sprechen, ohne ihn irgendwie zu bezeichnen oder zu benennen". See also Carnap (1947), §24.2, §26 and Materna, Duží (2005).

All four principles turn on the same conception of language. A piece of language serves to point to a logical structure beyond itself, its sense, and that's it. Which object, if any, an empirical expression determines in the actual world at the present moment is a matter of contingent fact and cannot be 'calculated' *a priori*. E.g., a language-user will be able to determine the condition for being the King of France, but his linguistic competence won't suffice for fixing the individual, if any, who is actually and presently the King of France. Similarly, a language-user will be able to calculate the empirical truth-*condition* of a sentence but never its actual and present truth-*value*. The contrast is between conditions and satisfiers, concepts and their instances.

There is common agreement that reasonable (empirical) sentences denote propositions (not just truth-values), but it is much disputed what a proposition is. Frege's problem with the difference in cognitive value between "a = a" and "a = b" (where 'a' and 'b' are names) is still puzzling for some logicians, while leading others to a conception of Russellian structured propositions (see Russell (1956)), and still others to hyperintensional objects and structured meanings (see Cresswell (1985)). It is also commonly agreed that empirical definite descriptions are not rigid designators in Kripke's sense, but that (names and) definite descriptions contribute only their denotation to the proposition denoted. Still, there is a lot of dispute what the denotation actually is. Our system is based on the possible-worlds semantics conception, and particular denotations of (empirical) expressions are on our conception *intensions*—functions from possible worlds (ω) and time-points (τ). Thus a *proposition* is a function from possible worlds to a chronology of truth-values (o): $\omega \rightarrow (\tau \rightarrow o)$. The (individual) description '$(\iota x)\Phi$' denotes an *individual office*—a function from possible worlds to a chronology of individuals: $\omega \rightarrow (\tau \rightarrow \iota)$, etc. When d(t) is the denotation of a term t, an atomic sentence of the form 'Rab' denotes a proposition the extension of which takes the truth-value T at a world w and time t iff the extensions of d(a) and d(b) at w, t are members of a relation, viz. the extension of d(R) at that w, t. This is in a good accordance with a general characterisation of a sentence: it denotes some *truth-condition* that is or is not fulfilled in a state-of-affairs. For example, the two singular propositions denoted by 'Rab' and 'Ra$(\iota x)\Phi$' will be identical only if 'b' and '$(\iota x)\Phi$' denote the same object—intension. This conception, however, does not solve Frege's problem of 'b = $(\iota x)\Phi$' being informative unlike 'b = b'. Such intensions are *set-theoretical* objects (mappings), they are *not structured complexes*, though they consist of elements. According to Zalta (1989), Russellian propositions play the desired role of *complexes* that result by 'plugging' objects into the 'gaps' of properties and relations. In our opinion, however, properties, relations, or functions in general, have no gaps; particular objects simply are members of (the arguments of) these unstructured mappings[5]. But we can accept the possible-world semantics of propositions, while the demand for *structured meanings* is met by another entity: Between an expression and the denoted 'flat' object there is a structured *mode of presentation* (*construction* in our terminology) of the object, i.e., a meaning (perhaps the Fregean sense) of the expression. It is a complex, a (declaration of a) *procedure* that consists in the *creation of a function* by abstracting over objects and/or in *applying the function* to its arguments. But particular (physical/abstract) objects cannot be 'plugged' into such a (conceptual) procedure; they must always be presented in an (albeit primitive) way, i.e., their concepts are constituents of the procedure. There are two such primitive modes of presentations that insert the objects into the construction: *variables* and *trivialisations*. The other two kinds of constructions working over these ones are more complex; they are *closure* (creating a function by abstraction) and *composition* (applying a function to an argument).

The whole TIL conception can be illustrated by an adjusted Frege-Church schema:

$$\begin{tikzcd} \text{expression} \arrow[r, "\textit{expresses}"] \arrow[rr, bend right, "\textit{denotes}"'] & \text{sense} \arrow[r, "\textit{identifies}"] & \text{denotation} \end{tikzcd}$$

The *sense* of an empirical expression EE is a construction C of an intension Int, and the *denotation* of EE is Int. The *reference* of EE at some WT-index is the extension, if any, of Int at WT.

The *sense* of a non-empirical (i.e., logical or mathematical) expression NEE is a construction C either of an extensional object or else of another

[5] True, Russell's propositions are the sort of things you can plug objects in and out of; those propositions are *structured* entities, no mappings, but then the notion of mapping is missing.

construction C*. The *denotation* of NEE is the object, if any, which is constructed by C.[6]

Hence linguistic senses are constructions. The sense of a sentence is a construction of a proposition denoted by the sentence. Sub-sentential expressions, like definite descriptions or predicates, have senses, too. There are no syncategorematic terms in TIL. The relation between the first-order object (intension) and that what is, in most semantic theories, considered to be the reference of an expression (for instance, an individual in space and time, a set of individuals, etc.) does not have a semantic character; it is influenced by an empirical factor (a state of affairs), and thus it is not directly the subject of a semantic investigation. Hence the co-reference of expressions is from the TIL point of view a contingent, empirical matter. Expressions can be equivalent when they denote one and the same object, but do not have (even in this case) to have the same sense, i.e., do not have to be synonymous.

Empirical expressions are never empty, for they always denote an intension. But an empirical expression may fail to take a reference at a world-time pair. Non-empirical expressions are empty in rare cases, for not every construction is proper in the sense of constructing an object. A classical example: The construction of the largest prime is improper, and therefore the mathematical expression 'the largest prime' has no denotation. It is vital to understand that the realm of reference is beyond the realm of semantics. It is irrelevant to the semantic properties of an empirical definite description, say, whether it is Dick, Tom, Harry or no-one who is the King of Canada. This is also why the issue of co-reference of empirical expressions is an extra-semantic one.[7]

PSW propositions are in fact nothing but truth-values-in-intension, nothing but truth-values distributed across worlds and times. Apart from simply identifying propositions with truth-values, as in classical propositional logic, the PWS conception of proposition is the shallowest possible. Such propositions merely depict the modal profile of an empirical sentence. This fairly crude differentiation of sentences is required, for we want to be able to say that 'a is heavier than b' denotes one and the same proposition, or state-of-affairs, as 'b is lighter than a'. Why? Because reality looks the same. One overstretches the capacity of PWS

[6] Only in a degenerate sense does NEE have a reference, since its reference, if any at all, is the same for all worlds and times

[7] Jespersen (2006)

propositions by turning them into sentential meanings, though. For surely 'a is heavier than b' should not mean the same as 'b is lighter than a'. But then we must part company with truth-conditional semantics. Sentential senses are not truth-conditions. Instead sentential senses are procedures whose products are truth-conditions.

The TIL 'language of constructions' is transparent as well. In contrast to standard formal theories, which start out with a naked syntax and only subsequently proceed to a semantic interpretation in a model, in TIL the notion of a naked formal expression as a pure graphic shape can be arrived at only through abstracting its sense away from it. In terms of conceptual priority, TIL starts out with sense-endowed expressions, which is to say that the 'symbolic language' of TIL-constructions constitutes an 'interpreted formalism'. Every factor that is semantically salient is explicitly present in the respective formalism. This is evident, for instance, in the explicit typing of the theory, the types of TIL being exclusively objectual. So what qualifies the 'formal language' of TIL as inherently interpreted is that a naked shape can be introduced as an expression only if it is paired off with a construction constructing an object of a particular type.

Definition 1 (***Simple types over a base***):

An objectual base is a collection of mutually disjoint nonempty sets.

i) Every member of the base is a *type over the base*

ii) Let α, $\beta_1,\ldots,\beta_m$ be *types over the base.* Then $(\alpha\,\beta_1\ldots\beta_m)$, i.e., the set of all m-ary total and partial) functions with an argument (i.e., a tuple) $\langle b_1,\ldots,b_m\rangle$, where b_i $(1 \leq i \leq m)$ is a member of the type β_i, and at most one value of type α, is a *type over the base.*

iii) Nothing is a *type over the base* unless it so follows from i) - ii).

An object O (that is a member) of a type α will be denoted an α-object, O/α.

An *epistemic* base consists of sets of objects of four basic categories: ι, o, ω, τ, where $\boldsymbol{\iota}$ is a type (set) of *individuals,* $\mathbf{o}$ is the type (set) of *truth-values* {T, F}, $\boldsymbol{\omega}$ is the type (set) of *possible worlds,* and $\boldsymbol{\tau}$ is the type (set) of *time moments,* or real *numbers* (playing the role of their surrogates).

TIL is an open-ended system. The above epistemic base $\{o, \iota, \tau, \omega\}$ was chosen, because it is apt for natural-language analysis, but in the case of mathematics a (partially) distinct base would be appropriate; for instance, the base consisting of natural numbers, of type ν, and truth-values. Derived types would then be defined over {ν, ο}.

TIL language of constructions can be viewed as a typed λ-calculus whose terms denote not the functions constructed but the constructions themselves. They are, as it were, transparent windows to constructions, and it is superfluous to mention the terms, since the symbolic language of the λ-calculus allows us to talk about the constructions. Observe that the Frege-Church schema does not apply to lambda-terms. These terms denote directly without the mediation of another construction. Did the lambda-terms not directly denote, their denotational schema would have to account for how the construction which some lambda-term expresses constructs the construction that is the expression's denotation. Thus, e.g., instead of claiming that '$\lambda x\ [^0\!> x\ ^0 0]$' denotes the construction $\lambda x\ [^0\!> x\ ^0 0]$ which constructs the class of positive numbers, we simply say $\lambda x\ [^0\!> x\ ^0 0]$ is the construction that constructs the class of positive numbers.

Definition 2 (*Constructions*):

i) *Variables* are constructions. Variables and constructions involving variables construct objects dependently on a valuation v, they v-construct.

ii) If X is an entity whatsoever, even a construction, then ^{0}X is a construction called *trivialisation*. Trivialisation ^{0}X constructs X without any change.

iii) If X_0 is a construction that v-constructs a function F, i.e. an $(\alpha\, \beta_1 \ldots \beta_n)$-object, and $X_1,\ldots,X_n$ are constructions that v-construct $\beta_1,\ldots,\beta_n$-objects $b_1,\ldots,b_n$, respectively, then $[X_0\ X_1 \ldots X_n]$ is a construction called *composition*. If F is defined on the argument $\langle b_1,\ldots,b_n\rangle$, then composition $[X_0\ X_1 \ldots X_n]$ v-constructs the value of F on $\langle b_1,\ldots,b_n\rangle$; otherwise it does not construct anything; it is v-*improper*.

iv) Let $x_1,\ldots,x_n$ be pair-wise distinct variables that range over types $\beta_1,\ldots,\beta_n$, and let X be a construction that v-constructs an α-object for some type α. Then $[\lambda x_1 \ldots x_n\ X]$ is a construction called *closure* (abstraction). It v-constructs the following function F of the type $(\alpha\ \beta_1 \ldots \beta_n)$: Let v' be a valuation that differs from v at most by assigning objects $b_1,\ldots,b_n$, (of the respective types) to variables $x_1,\ldots,x_n$, respectively. Then the value of the function F on the argument $\langle b_1,\ldots,b_n\rangle$ is the object v'-constructed by X. If X is v'-improper, then F is *undefined* on the given argument.

v) Nothing is a construction unless it so follows from i) – iv).

Notes:

1. The simplest constructions are variables; they are open constructions that *v*-construct objects. They are no letters, no characters.
2. Trivialisation is a one-step procedure which consists in grasping an object and its "delivering" without any change. If X is an entity, then ^0X is a presentation of X without a 'perspective' and without going via any properties of X. The term '^0X' might be likened to a constant of a formal language; but unlike such a formal constant, which can be interpreted in many ways so as to denote different entities and thus actually is not a constant but a 'variable construction', '^0X' rigidly denotes construction ^0X that constantly constructs X.

   A possible objection against such a conception might be: Well, your transparent approach may be accurate enough, but you lose the expressive power of model theories enabling us to examine common properties and relations holding in all the particular models. Not at all; The TIL transparent approach is more precise, particular 'models' are rendered by valuations of (higher-level) variables.

   If ^0X constructs X without any change, it sounds like trivialization is just a paper-shuffling bureaucrat, but trivialization is by no means as innocent or idle as it may seem. E.g., when prefixed to a construction C, C becomes *mentioned* rather than *used* so that C itself can become a subject of predication (for the use/mentioned distinction as applied to constructions, see Materna (1998), §5.5). A historical forerunner of trivialization would be Bolzano's singular ideas; see Casari (1992), §II.7, and Bolzano (1837), § 101. The basic idea is that to every object *o* corresponds an idea (*Vorstellung an sich*; construction) which exclusively picks out *o*. Similarly, any object can be presented via a trivialization of it. For instance, the trivialization ^{000}X is constructed by ^{0000}X. The assumption of trivialization is a strong one, of course, and restraint in its use in an analysis is called for. Ideally, the best analysis of an expression should trivialise only logically non-composite objects, unless a construction is mentioned in a hyper-intensional context.
3. Composition corresponds to the traditional operation of *functional application* of a function to an argument. It should be borne in mind, however, that a composition is the very procedure of applying a function to an argument and not the product, or result, of the application.

Of course, by using a composition $[X_0\ X_1 ... X_n]$ we obtain the product of this procedure. If we want to obtain the very procedure itself ('to talk about it'), we can use trivialisation $^0[X_0\ X_1 ... X_n]$. Composition is the only construction that may be (v-)improper, namely in two cases: *First,* the component X_0 constructs a function F and components $X_1,...,X_n$ construct $\langle b_1,...,b_n\rangle$, but F is not defined on this argument (remember, F is a partial function). *Second,* some or more of the components $X_0, X_1,..., X_n$ fail to construct an object (are v-improper).

In case some of the components $X_0, X_1,...,X_n$ do not construct an object of the proper type so as to create an argument for F, the expression '$[X_0\ X_1 ... X_n]$' is not a well-formed term, and it does not denote a construction.

4. λ-Closure corresponds to the traditional operation of *functional abstraction.* It enables us to construct a function, and thus also to analyse talking about the whole function ('mentioning'), not only talking about a particular value on a given argument ('using' the function). Closure can never be v-improper; it always constructs a function, even if the constructed function is not defined on any of its arguments, like, e.g., $[\lambda x\ [^0\mathrm{Div}\ x\ ^00]]$.
5. Quantifiers—universal $\forall_\alpha$ and existential $\exists_\alpha$—are no expressions but functional objects of type $(o(o\alpha))$. We will write $(\forall x)A$, $(\exists x)A$ instead of $[^0\forall_\alpha\ \lambda x A]$, $[^0\exists_\alpha\ \lambda x A]$, respectively. The singulariser[8] ι_α is an object of type $(\alpha(o\alpha))$, and instead of $[^0\iota_\alpha \lambda x\ A]$ we will use $\iota x A$ (*the only x such that A*). We will also use a standard infix notation without trivialisation in case of using truth-value functions ($\wedge$, $\vee$, ...), identities, less than, greater than functions ($=$, $\leq$, $\geq$, ...), but we have to keep in mind that these are just abbreviations that conceal the self-contained meaning of the respective 'logical symbols'.

Our apparatus is very powerful. It enables us, as we saw, to distinguish between various levels of *using* and *mentioning* particular *entities* of our ontology:

- Non-functional and non-procedural objects can only by mentioned, like ^{03}, ^{0}Charles.

[8] We use the same symbol ι for denoting the type of individuals, as well as the function singulariser, since no confusion may arise

- Functions can be used for arriving at their value at an argument, i.e., by composing their construction with an argument; e.g., [λx [^{0}+ x ^{0}1] ^{0}3], which yields the number 4.
- Functions can also be mentioned; we can predicate some φ of a function, like, e.g.: [^{0}Bij λx [^{0}+ x ^{0}1]], where Bij(ection) / (o ($\tau\tau$)) is the class of one-to-one mappings.
- Constructions can be used for constructing (identifying) an entity, for instance the construction λx [^{0}+ x ^{0}1] has been used above for constructing the function of adding number one. Constructions themselves can be mentioned as well. For instance, "dividing by zero does not yield any result for any number" — [^{0}Imp$_1$ ^{0}[^{0}: x ^{0}0]], where Imp$_1$ is a class of constructions that fail to v-construct anything for any valuation v.
- Constructions are mentioned, e.g., in *hyper-intensional* contexts of knowing, believing, etc., where the meaning (i.e., the expressed construction) plays a crucial role. For instance when "Charles knows that dividing by zero does not yield any result for any number" he is related to the construction [^{0}Imp$_1$ ^{0}[^{0}: x ^{0}0]] and not to its truth-value True. Such contexts we refer to as 'constructional', since the attitude relata are constructions and not what they construct.
- But constructions cannot be typed within the simple theory of types, and constructional contexts would therefore be without a logic. The ramified hierarchy of types in TIL serves exactly to type constructions. The hierarchy is ramified because it includes objects of an increasingly higher order.

Definition 3 ***(Ramified theory of types)***

Let B be a *base*, i.e., a collection of mutually disjoint non-empty sets.

(T$_1$) *Types of order 1*
Defined according to Definition 1.

(C$_n$) *Constructions of order n*

i) Let α be a type of order n over B. If ξ is a variable that ranges over α, then ξ is a *construction of order n* over B.

ii) If X is a member of a type of order n over B, then ^{0}X is a *construction of order n* over B.

iii) If X$_0$, X$_1$,..., X$_m$ are *constructions of order n* over B, then [X$_0$ X$_1$...X$_m$] is a *construction of order n* over B.

iv) If distinct variables $x_1,...,x_m$, as well as X, are *constructions of order n* over *B*, then $[\lambda x_1...x_m X]$ is a *construction of order n* over *B*.

v) Nothing is a *construction of order n* over B unless it so follows from ($\mathbf{C}_n$ i-iv).

($\mathbf{T}_{n+1}$) *Types of order n+1*

Let $*_n$ be the collection of all constructions of order *n* over *B*. Types of order *n*+1 over *B* are defined as follow:

i) $*_n$ and all the types of order *n* are *types of order n+1* over *B*.

ii) If $\alpha, \beta_1, ..., \beta_m$ are types of order *n*+1, then the set $(\alpha\, \beta_1...\beta_m)$ of all *m*-ary (total and partial) functions from $\beta_1 \times ... \times \beta_m$ to α is also a *type of order n+1* over *B*.

iii) Nothing is a *type of order n+1* over *B* unless it so follows from ($\mathbf{T}_{n+1}$ i) and ($\mathbf{T}_{n+1}$ ii).

Notational conventions: Empirical expressions denote *intensions*, where α-intensions are functions of type $(\alpha\omega)$; since α-intensions are often of type $((\alpha\tau)\omega)$, we abbreviate this type by $\alpha_{\tau\omega}$. We use variables $w, w_1, ...$ as ranging over ω, and variables $t, t_1, ...$ as ranging over type τ. If X is a construction that constructs an intension of type $\alpha_{\tau\omega}$, then instead of $[[Xw]t]$ we write X_{wt}.

Examples of intensions:

- *Individual roles (offices)* (e.g., the president of USA) are objects of type $\iota_{\tau\omega}$;
- *properties of individuals* (e.g., being a spider) are objects of type $(o\iota)_{\tau\omega}$;
- binary *relations-in-intensions* between individuals (being in love, kicking, …) are objects of type $(o\iota\iota)_{\tau\omega}$;
- *propositions* are objects of type $o_{\tau\omega}$;
- *magnitudes* (like the temperature in Prague) are objects of type $\tau_{\tau\omega}$.

To illustrate the principles of TIL analyses of (natural language) expressions, we now adduce an example of an analysis of the sentence

(S) *The richest man (in the world) is in danger.*

a) First, we have to perform a *type-theoretical analysis* of the objects the sentence (S) talks about, i.e., of the objects denoted by semantically self-contained sub-expressions of (S) ('$\rightarrow$' means: denotes an object of a type α, '/' means: the object belongs to the type α):

(S) $\rightarrow$ RMD / $o_{\tau\omega}$
richest $\rightarrow$ Richest / $(\iota\,(o\iota))_{\tau\omega}$
man $\rightarrow$ Man / $(o\iota)_{\tau\omega}$

the richest man $\rightarrow$ RM / $\iota_{\tau\omega}$
being in danger $\rightarrow$ Danger / $(o\iota)_{\tau\omega}$

Comments: The whole sentence denotes, of course, a proposition. The *richest* denotes a function Richest that (dependently on worlds/types) selects an individual from a set of individuals, namely the richest one, Richest of type $(\iota\ (o\iota))_{\tau\omega}$. *Man, being in danger* denote properties of individuals, *the richest man* denotes an individual office.

b) *Synthesis*. Now we have to compose constructions of particular subdenotations in order to obtain an adequate analysis of the sentence, i.e., a construction of the denoted proposition RMD. When doing so, we have to follow the *principle of subject matter*, in other words, we can combine *only* constructions of the objects the sentence talks about. Moreover, to obtain *the* most accurate analysis, we have to use *all* the objects the sentence talks about (we must not 'omit anything').

The simplest possible analysis might be ^{0}RMD, which is just the simplest trivial presentation of the denoted proposition without any structural perspective (the trivialisation analysis fails to reveal any sort of constructional structure). A more accurate analysis, revealing the structure of the meaning of (S), can be obtained by realising that (S) claims that the individual who occupies the office of the richest man (at some world/ time couple) has the property of being in danger:

$\lambda w \lambda t\ [^0\text{Danger}_{wt}\ ^0\text{RM}_{wt}]$.

Anyway, such an analysis is not fine-grained enough. It does not enable us to perform all the adequate inferences, e.g., to deduce that there is a man who is in danger, etc. We have to construct the office RM of the richest man by composing ^{0}Richest and ^{0}Man:

$\lambda w \lambda t\ [^0\text{Richest}_{wt}\ ^0\text{Man}_{wt}]$,

and we obtain *the* analysis of the sentence (S), which is identical to the meaning of (S):

$\lambda w \lambda t\ [^0\text{Danger}_{wt}\ [\lambda w \lambda t\ [^0\text{Richest}_{wt}\ ^0\text{Man}_{wt}]]_{wt}]$.

To make the construction easier to read, we can now perform an innocent[9] λ-conversion (β-reduction):

$\lambda w \lambda t\ [^0\text{Danger}_{wt}\ [^0\text{Richest}_{wt}\ ^0\text{Man}_{wt}]]$.

[9] 'Innocent' because such a λ-conversion is always an equivalent transformation we just substitute variables w, t for variables w, t

Note that the sentence does not say anything about Bill Gates, and so the analysis must not and does not contain a construction of Bill Gates. Without an additional premise that the richest man at some WT couple is Bill Gates we cannot logically derive from (S) that Bill Gates is in danger. Construction $[\lambda w\lambda t\ [{}^0\mathrm{Richest}_{wt}\ {}^0\mathrm{Man}_{wt}]]$ constructs the office RM / $\iota_{\tau\omega}$, not a ι-object. However, the property of being in danger cannot be ascribed to an office, but to an individual. Hence the construction $[\lambda w\lambda t\ [{}^0\mathrm{Richest}_{wt}\ {}^0\mathrm{Man}_{wt}]]$ is subjected to the *intensional descent* with respect to *w, t,* and the office RM serves as a pointer to an *unspecified* individual that now is just contingently Bill Gates.

Constructions can be used in two distinct suppositions. Using a medieval terminology, we distinguish the *de dicto* and *de re* supposition. We apply this terminology only to analyses of empirical sentences, though the extension into analytical/mathematical contexts would be straightforward. To adduce a simple example, consider the following sentences:

(R) *The president of the Czech Republic is a tennis player.*

(D) *The president of the Czech Republic is eligible.*

Both the sentences talk about the office of the president of the Czech Republic (PCR); the sentence (R) talks also about the property (of being a) tennis player, the sentence (D) about the property (of being) eligible. But, whereas the property eligible is a property of an office (that its holder, whoever it may be, must be elected), we cannot ascribe the property of being a tennis player to an office, but to an individual. Hence, whereas in (D) the office PCR is *mentioned*, in (R) this office is *used* to point at an individual, the construction of the office PCR has to be subjected to the intensional descent. Type-theoretical analysis and synthesis reveal these facts:

President (of) / $(\iota\ \iota)_{\tau\omega}$ – an empirical function

CR (Czech Republic) / ι – for the sake of simplicity, let CR be an individual

TennisP(layer) / $(o\iota)_{\tau\omega}$

Eligible / $(o\ \iota_{\tau\omega})_{\tau\omega}$

To construct the office PCR / $\iota_{\tau\omega}$, we have to compose ^{0}President and ^{0}CR: $\lambda w\lambda t\ [{}^0\mathrm{President}_{wt}\ {}^0\mathrm{CR}]$. Our sentences are analysed as follows:

(R′) $\lambda w\lambda t\ [{}^0\mathrm{TennisP}_{wt}\ [\lambda w\lambda t\ [{}^0\mathrm{President}_{wt}\ {}^0\mathrm{CR}]]_{wt}]$

$[\lambda w\lambda t\ [{}^0\mathrm{President}_{wt}\ {}^0\mathrm{CR}]]$ used *de re*

(D′) $\lambda w\lambda t\ [{}^0\mathrm{Eligible}_{wt}\ [\lambda w\lambda t\ [{}^0\mathrm{President}_{wt}\ {}^0\mathrm{CR}]]]$

$[\lambda w\lambda t\ [{}^0\mathrm{President}_{wt}\ {}^0\mathrm{CR}]]$ used *de dicto*

It may be the case that a construction that has been subjected to an intensional descent is still *de dicto*, when occurring in a *de dicto* context, because the *de dicto* context is dominant. In our example the construction ^{0}President occurs *de dicto* in (D′) though it is composed with w and t, because it is a constituent of the $[\lambda w \lambda t\ [^0\text{President}_{wt}\ ^0\text{CR}]]$ construction that is used with the *de dicto* supposition in (D′). This is due to the fact that ^{0}President is not composed with the left-most occurrences of w, t; α-renaming the variables reveals these facts:

(R′) $\lambda w \lambda t\ [^0\text{TennisP}_{wt}\ [\lambda w^* \lambda t^*\ [^0\text{President}_{w^*t^*}\ ^0\text{CR}]]_{wt}]$
$[\lambda w^* \lambda t^*\ [^0\text{President}_{w^*t^*}\ ^0\text{CR}]]$ used *de re*

(D′) $\lambda w \lambda t\ [^0\text{Eligible}_{wt}\ [\lambda w^* \lambda t^*\ [^0\text{President}_{w^*t^*}\ ^0\text{CR}]]]$
$[\lambda w^* \lambda t^*\ [^0\text{President}_{w^*t^*}\ ^0\text{CR}]]$ used *de dicto*

We have seen that speaking about the *de dicto / de re* supposition of a construction C is reasonable in case C is a constituent (i.e., a subconstruction that is used, not mentioned) of another construction C' of an intension (most frequently a proposition). Hence C' is usually of a form $\lambda w \lambda t\ [\ldots]$. Calling the state-of-affairs constructed by the left-most w, t pair 'reporter's perspective', we can roughly characterise the *de re / de dicto* distinction as follows:

A construction C of an intension Int occurs with the *de re* supposition in C' iff the intension Int is used as a pointer to its instance with the reporter's perspective, i.e., C is subjected to the intensional descent with respect to the left-most w, t; otherwise C occurs *de dicto* in C'.[10]

3. Analysing points of view

So far we have argued that the expression 'point of view' is homonymous. Since it expresses more than one meaning, it denotes distinct objects of different types dependently on the context in which the expression is used. Therefore, when attempting at an analysis, we have to take into account the context of (at least) the whole sentence. Moreover, the expression is rather vague and it is used in many dissimilar contexts. Still, there is a common feature shared by all the possible denotations: It is, in general, an intensional *function*, which dependently on worlds /times when being applied to a criterion, (i.e. an attribute, a property, or whatever) and to a viewed object (an individual, or another intension or

[10] For more information on the *de dicto / de re* distinction see Duží (2004).

extension), results in a proposition or a set thereof. Expressions which may denote functions of particular distinct types that behave in the same way are called *polymorphic*. In computer science polymorphic algorithms are frequently used without any problems, and programming languages based on the *typed* λ-calculus making polymorphic functions tractable are highly valued by programmers. Explicit typing and type-checking during the compilation process prevents writing programs that would fail during the run-time, and the programmer does not have to write a special piece of program for each type of argument.

A paradigmatic example is the function Cardinality or, more precisely, functions of particular types denoted by 'cardinality'. These functions are of type $(\tau\ (o\alpha))$. A programmer simply writes a program that counts the number of α-elements of a set, where α is a type *variable* (type *parameter*), the value of which is supplied during the run-time.

When analysing natural language expressions, we come across many ambiguities, but explicit typing is a significant semantic feature that cannot be neglected. Yet in some cases the need for typing can become a restrictive factor. For instance, an analysis of the simple question—*What is Charles thinking about?*—is impossible even within the ramified type system, because Charles can be thinking of an object of *any* type, even of the type, the type of a type, etc. An analogy with the programming language is not helpful here, for there are no such things as the compilation-time or run-time of an analysis. We cannot introduce 'type variables' or constructions constructing types, for types are simply 'pre-concepts' of the TIL system. A possible solution within a 'super-ramified' theory of types has been outlined in Duží (1993), but this is out of the scope of the present study.

Still we know the schematic type of the object denoted by a particular polymorphic expression 'think'. The type-theoretical analysis terminates in a relation-in-intension between an individual and an arbitrary α-object: Think / $(o\ \iota\ \alpha)_{\tau\omega}$. Using this polymorphic expression in a sentence usually disambiguates α. (But not always: the sentence *Charles is thinking of the richest man* is inherently ambiguous; Charles can be thinking of the individual who occupies the office of the richest man, $Think_1$ / $(o\ \iota\ \iota)_{\tau\omega}$, or of the office itself—$Think_2$ / $(o\ \iota\ \iota_{\tau\omega})_{\tau\omega}$, or even of the concept of the richest man—$Think_3$ / $(o\ \iota\ {*_1})_{\tau\omega}$.)

The term 'point of view' is also vague and type-polymorphic. In general, as we already said, there is always a *viewed object* and a *criterion* with respect to which the object is evaluated and thereby characterised and classified. Thus we can propose a schematic type for the object denoted:

'Point of view' denotes a function *V*, which—when applied to a 'viewing criterion' in the broadest sense, i.e., an individual, office, property, or even a scientific theory, and to the viewed object (again in the broadest sense)—returns a proposition: V / $(o_{\tau\omega}\ \beta\ \alpha)$ or V / $(o_{\tau\omega}\ \beta\ \alpha)_{\tau\omega}$, where β is the type of the 'viewing criterion', α the type of the object that is viewed (by an agent, who is not typed individually).

In what follows we classify particular types of points of view with respect to the type α of the viewed object. The list is not, and actually cannot be, exhaustive (due to the infinite hierarchy of the objects of our ontology).

3.1 The viewed object is an individual

The simplest way of characterising an individual is by using its *attributes*. Example:

(S1) *From the point of view of age, Charles is an old man, but from the point of view of his activities, Charles is a vital, youthful man.*

First, we have to explicate the term 'attribute of an individual' (for details see Duží (2001)). In a common use this term denotes a property of individuals, but in the last two decades—due to the development of database conceptual theories—the term 'attribute' has been used in a broader sense, denoting an (empirical) function of any type. Thus, for instance, the age of ... , the salary of ... are attributes of the type $(\tau\ \iota)_{\tau\omega}$, the address of ... is of type $(\alpha\ \iota)_{\tau\omega}$ (where members of α are tuples (state, town, street + number, ZIP code)), parents of ... / $((\iota,\iota)\ \iota)_{\tau\omega}$, etc[11]. That attributes are intensions is obvious: The address of a person can change over time (temporal parameter τ), and it is not logically necessary that the person lives at the given address A; he/she could have lived anywhere else (modal parameter ω).

An analysis of (S1):

1. *Type*-theoretical analysis.

[11] In Conceptual Modelling we often need to model attributes that return an object that is a *tuple*, which consists of other simpler objects Hence we use an adjusted definition of types and constructions—enriched with tuple types or sequences, and constructions of tuples and projections. See, e g., Muller (2006, 2006a)

Charles → Ch / ι
point of view 1 → View[1] / $(o_{\tau\omega}(\tau\iota)_{\tau\omega}\,\iota)_{\tau\omega}$
age of → Age / $(\tau\,\iota)_{\tau\omega}$
old man → $\mathrm{Old}^{\mathrm{Age}}$ / $(o\iota)_{\tau\omega}$
point of view 2 → View[2] / $(o_{\tau\omega}((o(o\iota)_{\tau\omega})\,\iota)_{\tau\omega}\,\iota)_{\tau\omega}$
activities of → Act / $((o(o\iota)_{\tau\omega})\,\iota)_{\tau\omega}$
(for the sake of simplicity, activities assigned to an individual are analysed as sets of properties)
vital, youthful man → $\mathrm{Young}^{\mathrm{Act}}$ / $(o\iota)_{\tau\omega}$

2. *Synthesis*.

(S1′) $\lambda w\lambda t\,[([{}^0\mathrm{View}^1_{wt}\,{}^0\mathrm{Age}\,{}^0\mathrm{Ch}] = \lambda w\lambda t\,[{}^0\mathrm{Old}^{\mathrm{Age}}{}_{wt}\,{}^0\mathrm{Ch}]) \wedge ([{}^0\mathrm{View}^2_{wt}\,{}^0\mathrm{Act}\,{}^0\mathrm{Ch}] = \lambda w\lambda t\,[{}^0\mathrm{Young}^{\mathrm{Act}}{}_{wt}\,{}^0\mathrm{Ch}])]$

Here the equality = / $(o\; o_{\tau\omega}\; o_{\tau\omega})$ is a relation between propositions (identity of propositions).

Remarks:

a) Note that the sentence (S1) does not talk about the *way* in which the obtained characteristics (i.e., that Charles is an old man, etc.) have been arrived at. In particular, (S1) does not specify which values of Charles' attributes were taken into account, and what these values are. E.g., Charles may be an 80-year old active musician, but there is no trace of this fact in (S1). Our analysis (S1′) respects the *principle of subject matter*[12]: Particular constituents ^{0}Age, ^{0}Act, as well as ${}^0\mathrm{Old}^{\mathrm{Age}}$ and ${}^0\mathrm{Young}^{\mathrm{Act}}$ are used *de dicto*, i.e., their values do not matter.

b) Some apparent paradoxes are often explained away by referring to points of view: Don't we often say something like "Charles is both old and young" tacitly meaning just (S1), namely "Age-viewing Charles—he is an old man, whereas activity-viewing Charles—he is a young man"? We think so.
Our analysis (S1′) blocks undesirable, flawed arguments like:

From the point of view of age, Charles is an old man, so Charles is an old man.
From the point of view of activities, Charles is a young man, so Charles is a young man. Therefore, Charles is old and young.

[12] For details see Materna, P – Duží, M (2005).

A schema of such a *flawed argument* is as follows. Let *Criterion* be a construction of the viewing criterion, *Object* a construction of a viewed object, *Property* a construction of a property ascribed to the object:

$$\frac{[{}^0\!= [{}^0\mathrm{View}_{wt}\ \mathit{Criterion}\ \mathit{Object}]\ [\lambda w_1 \lambda t_1\ [\mathit{Property}_{w_1 t_1}\ \mathit{Object}]]]}{[\mathit{Property}_{wt}\ \mathit{Object}]}$$

Construction $[\lambda w_1 \lambda t_1\ [\mathit{Property}_{w_1 t_1}\ \mathit{Object}]]$ occurs *de dicto* in the premise, i.e., the constructed proposition is just mentioned, which does not justify the truth (in a given w, t) of the value v-constructed by $[\mathit{Property}_{wt}\ \mathit{Object}]$ in the conclusion. Properties ascribed to a viewed object relative to a particular viewpoint cannot be ascribed to the object 'absolutely'.

Viewing an individual may be performed also by an *explicit* agent. Consider the sentence

(S2) *Marie likes this flower from the viewpoint of its colour.*

1. *Type*-theoretical analysis.
   point of view $\rightarrow$ View / $((o_{\tau\omega}((o\iota)_{\tau\omega}\,\iota)_{\tau\omega}\,\iota)_{\tau\omega}$
   colour (of an individual) $\rightarrow$ Colour / $((o\iota)_{\tau\omega}\,\iota)_{\tau\omega}$
   like $\rightarrow$ Like / $(o\iota\iota)_{\tau\omega}$
   Marie, this flower $\rightarrow$ Mary / ι, This-Flower / ι[13]

2. *Synthesis.*

(S2′) $\lambda w \lambda t\ [[{}^0\mathrm{View}_{wt}\ {}^0\mathrm{Colour}\ {}^0\text{This-Flower}] =$
$= \lambda w \lambda t\ [{}^0\mathrm{Like}_{wt}\ {}^0\mathrm{Marie}\ {}^0\text{This-Flower}]]$

Remark: Again, the sentence (S2) does not talk about the *way* in which Marie likes the flower. Marie may like pink colour and the flower may be pink, but our analysis (S2′) respects the *principle of subject matter*: The particular constituents ^{0}Colour, ^{0}Like are used *de dicto*.

3.2 The viewed object is an intension (property or office)

Consider the following sentences:

(S3) *Spiders have 8 limbs.*
(S4) *Whales are mammals.*

[13] Indexicals fall outside of the scope of this paper; *this flower* is simply treated as denoting an individual here.

(S5) *Birds can fly* (unless they are penguins or ostriches or …).
(S6) *Swans are white* (unless they are born in New Zealand or Australia or …).
(S7) *God is omniscient.*
(S8) *The King of France is a ruler of France.*

It might seem that these sentences characterise individuals, but it is not so. *Having 8 limbs, being a mammal, having the ability to fly, being white, being omniscient* are properties of individuals. However, these sentences characterise particular *properties (or offices*—S7, S8) by specifying some other properties which their instances (bearers) necessarily (S3, S4, S7, S8) or typically (S5, S6) have.

We need to define two important notions: *a requisite (Req) of a property (an office)* and a *characteristic, typical property (TP) assigned to a property (an office)*. Let p, q be variables of 1st order, ranging over properties $(o\alpha)_{\tau\omega}$, c a variable ranging over offices $\alpha_{\tau\omega}$, x a variable ranging over α. We define:

$\mathrm{Req}^{pr} =_{df} \lambda pq\ \forall w \forall t\ \forall x\ [[q_{wt}\ x] \supset [p_{wt}\ x]]$ (p is a requisite of q)

$\mathrm{Req}^{of} =_{df} \lambda pc\ \forall w \forall t\ [[{}^0\mathrm{E}_{wt}\ c] \supset \forall x\ [[c_{wt} = x] \supset [p_{wt}\ x]]]$
or equivalently:

$\lambda pc\ \forall w \forall t\ [[{}^0\mathrm{E}_{wt}\ c] \supset [p_{wt}\ c_{wt}]]$ (p is a requisite of the office c)

$\mathrm{TP}^{pr} =_{df} \lambda pqe\ \forall w \forall t\ \forall x\ [\neg[e_{wt}\ x] \supset [[q_{wt}\ x] \supset [p_{wt}\ x]]]$
(p is typical of q, unless e)

$\mathrm{TP}^{of} =_{df} \lambda pce\ \forall w \forall t\ [[{}^0\mathrm{E}_{wt}\ c] \supset \forall x\ [\neg[e_{wt}\ x] \supset [[c_{wt} = x] \supset [p_{wt}\ x]]]]$
or equivalently:

$\lambda pce\ \forall w \forall t\ [[{}^0\mathrm{E}_{wt}\ c] \supset [\neg[e_{wt}\ c_{wt}] \supset [p_{wt}\ c_{wt}]]$
(p is typical of the office c, unless e)

where $\mathrm{E}/(o\ \alpha_{\tau\omega})_{\tau\omega}$ is the *property of existence*, e ranges over some 'exceptions', 'the *unless properties*' $/(o\alpha)_{\tau\omega}$, the so-called *guards* of the rule in Artificial Intelligence. The property E is in want of explanation. In his (1979) Tichý makes a strong case for the claim that existence is not a property of individuals, but a property of intensions, namely the property of being instantiated, or occupied, in a world/time. Claiming, e.g., that the President of the Czech Republic does not exist, we do not want to say that a particular individual does not exist (which 'non-existing' one would it be?), but we simply make a claim about the state-of-affairs in which the office of the president of Czech Republic is not occupied.

Now more precise way formulating statements S3 – S7 can be either in terms of requisites (e.g., that having eight limbs is a requisite of the property of being a spider), or using the necessary status of ascribing the implied property—requisite to an object on the assumption that the object has the antecedent property assigned (for instance, that necessarily all spiders have eight limbs).

The respective analyses are (denoted properties: S(pider), 8L–having 8 limbs, W(hale), M(ammal), B(ird), F(ly), P(enguin or ostrich), Sw(an), W(hite), ZA–born in New Zealand or Australia, RF–being a ruler of France, Omni(scisent), all of them of type $(o\iota)_{\tau\omega}$, and God, K(ing of) F(rance) of type $\iota_{\tau\omega}$):

(S3′) $[{}^0\mathrm{Req}\ {}^0\mathrm{8L}\ {}^0\mathrm{S}]$ or $\forall w \forall t\ \forall x\ [[{}^0\mathrm{S}_{wt}\ x] \supset [{}^0\mathrm{8L}_{wt}\ x]]$
(Necessarily, all spiders have eight limbs)

(S4′) $[{}^0\mathrm{Req}\ {}^0\mathrm{M}\ {}^0\mathrm{W}]$ or $\forall w \forall t\ \forall x\ [[{}^0\mathrm{W}_{wt}\ x] \supset [{}^0\mathrm{M}_{wt}\ x]]$
(Necessarily, all whales are mammals)

(S5′) $[{}^0\mathrm{TP}\ {}^0\mathrm{F}\ {}^0\mathrm{B}\ {}^0\mathrm{P}]$ or $\forall w \forall t\ \forall x\ [[\neg {}^0\mathrm{P}_{wt}\, x] \supset [[{}^0\mathrm{B}_{wt}\ x] \supset [{}^0\mathrm{F}_{wt}\ x]]]$
(Necessarily, if something is not a penguin or an ostrich, then if it is a bird then it flies)

(S6′) $[{}^0\mathrm{TP}\ {}^0\mathrm{W}\ {}^0\mathrm{Sw}\ {}^0\mathrm{ZA}]$ or $\forall w \forall t\ \forall x\ [[\neg {}^0\mathrm{ZA}_{wt}\, x] \supset [[{}^0\mathrm{Sw}_{wt}\ x] \supset [{}^0\mathrm{W}_{wt}\ x]]]$
(Necessarily, if something is not born in New Zealand or Australia, then if it is swan then it is white)

(S7′) $[{}^0\mathrm{Req}\ {}^0\mathrm{O}\ {}^0\mathrm{G}]$, or $\forall w \forall t\ [[{}^0\mathrm{E}_{wt}\ {}^0\mathrm{God}] \supset \forall x\ [[{}^0\mathrm{God}_{wt} = x] \supset$
$\supset [{}^0\mathrm{Omni}_{wt}\ x]]]$
or
$\forall w \forall t\ [[{}^0\mathrm{E}_{wt}\ {}^0\mathrm{God}] \supset [{}^0\mathrm{Omni}_{wt}\ {}^0\mathrm{God}_{wt}]]$
(Necessarily, if God exists then He is omniscient)

(S8′) $[{}^0\mathrm{Req}\ {}^0\mathrm{RF}\ {}^0\mathrm{KF}]$, or $\forall w \forall t\ [[{}^0\mathrm{E}_{wt}\ {}^0\mathrm{KF}] \supset \forall x\ [[{}^0\mathrm{KF}_{wt} = x] \supset [{}^0\mathrm{RF}_{wt}\ x]]]$
or
$\forall w \forall t\ [[{}^0\mathrm{E}_{wt}\ {}^0\mathrm{KF}] \supset [{}^0\mathrm{RF}_{wt}\ {}^0\mathrm{KF}_{wt}]]$
(Necessarily, if the King of France exists then he is a ruler of France)

Now we can again apply particular points of view with respect to some criteria. We can, for instance, analyse

(S9) *From the point of view of the number of limbs, spiders are eight-limbed.*

(S10) *From the point of view of the ability to move, birds can usually fly.*

(S11) *From an epistemic point of view, God is perfect.*

Type analysis:

number of limbs → NL / $(\tau\ \iota)_{\tau\omega}$
point of view S9 → View⁹ / $(o_{\tau\omega}\ (\tau\ \iota)_{\tau\omega}\ (o\iota)_{\tau\omega})_{\tau\omega}$
mobile ability of → MA / $((o\iota)_{\tau\omega}\ \iota)_{\tau\omega}$
point of view S10 → View¹⁰ / $(o_{\tau\omega}\ ((o\iota)_{\tau\omega}\ \iota)_{\tau\omega}\ (o\iota)_{\tau\omega})_{\tau\omega}$
knowledge of → Know / $((oo_{\tau\omega})\ \iota)_{\tau\omega}$
point of view S11 → View¹¹ / $(o_{\tau\omega}\ ((oo_{\tau\omega})\ \iota)_{\tau\omega}\ (\iota_{\tau\omega}))_{\tau\omega}$

(S9′) $\lambda w\lambda t\ [[{}^0\mathrm{View}^9{}_{wt}\ {}^0\mathrm{NL}\ {}^0\mathrm{S}] = \lambda w\lambda t\ \forall x\ [[{}^0\mathrm{S}_{wt}\ x] \supset [{}^08\mathrm{L}_{wt}\ x]]]$
(S10′) $\lambda w\lambda t\ [[{}^0\mathrm{View}^{10}{}_{wt}\ {}^0\mathrm{MA}\ {}^0\mathrm{B}] = \lambda w\lambda t\ \forall x\ [[{}^0\mathrm{B}_{wt}\ x] \supset [{}^0\mathrm{F}_{wt}\ x]]]$
(S11′) $\lambda w\lambda t\ [[{}^0\mathrm{View}^{11}{}_{wt}\ {}^0\mathrm{Know}\ {}^0\mathrm{God}] = \lambda w\lambda t\ [[{}^0\mathrm{E}_{wt}\ {}^0\mathrm{God}] \supset$
$\supset [{}^0\mathrm{Omni}_{wt}\ {}^0\mathrm{God}_{wt}]]]$

And so on.

The well-known example of a paradoxical sentence claiming that Richard Nixon is both a pacifist and militant can now be solved as follows. The claims

(S12) Being a Quaker, Nixon is a pacifist; being a Republican, he is militant

can be understood in two different ways:

a) Under the aspect of being a Quaker, Nixon is a pacifist; under the aspect of being a Republican, he is militant.
b) Since a typical property of being a Quaker is being a pacifist and Nixon is a Quaker, he *might be* a pacifist; since a typical property of being a Republican is being militant and Nixon is a Republican, he *might be* militant.

The reading ad a) does not lead to paradox as we have shown in Section 3.1. Nixon is simply neither an 'absolute' pacifist nor an 'absolute' militant. Only from particular viewpoints may he appear to be a 'partial' pacifist and a 'partial' militant.

The reading ad b) makes use of typical properties, making the claim rather vague. An obvious way to avoid paradox consists in considering Nixon as the exception of both (or just one of the) rules: as a result he is neither a pacifist nor militant, or he is either not a pacifist or not militant.

Note that up to now the use of points of view has not led to a total, exhaustive characteristic of the viewed object. We usually provide just some *partial characteristics* even if we characterise a property or an office by means of some of its requisites, which its instances must have necessarily. To obtain *total characterisations*, we would have to use *all* the requisites of an intension. The collection of all the requisites of an intension q is called the *essence* of the intension q, Ess / $((o\ (o\alpha)_{\tau\omega})\ (o\alpha)_{\tau\omega}))$:

^{0}Ess $=_{df}$ $\lambda q\, \lambda p\ [^0\text{Req}\ p\ q]$.

An essence is a function which when applied to a property q returns the set of all the requisites of q. (Analogously for an office c.)

The property q is then *defined* by its requisites in the following manner:

$q =_{df} \lambda w \lambda t\, \lambda x\ (\forall p\ [[^0\text{Ess}\ q]\ p] \supset [p_{wt}\ x])$

q is the property of α-objects of having all the properties that belong to the essence of q. In other words, an α-object x has the property q if and only if it has all the properties belonging to the essence of q:

$[q_{wt}\, x] \equiv [\forall p\ [[^0\text{Ess}\ q]\ p] \supset [p_{wt}\ x]]$.

If the number of requisites $p^1,\ldots,p^n$ belonging to the set [^{0}Ess q] is finite, we can exhaustively *define* the intension q in the following manner:

$q =_{df} \lambda w \lambda t\, \lambda x\ [[p^1_{wt}\ x] \wedge [p^2_{wt}\ x] \wedge \ldots \wedge [p^n_{wt}\ x]]$.

The question whether using (general) points of view belonging to a particular theory (e.g., biology, logic, physics, ...) might lead to *equivalent definitions* (total characterisations) of the viewed intension and the problem of definitions in general will be handled in details in Chapter 5 of Part II of this study.

Marie Duží
Katedra informatiky FEI
VŠB-technická universita
17 listopadu 15
708 00 Ostrava-Poruba
Česká republika
marie duzi@vsb.cz

Bjørn Jespersen
Delft University of Technology
Julianalaan 134
2628 BL Delft
The Netherlands
b t f.jespersen@tbm.tudelft nl

Pavel Materna
Filosofický ústav
Akademie věd České republiky
Jilská 1
110 00 Praha 1
Česká republika
materna@lorien.site cas.cz

ACKNOWLEDGEMENTS

This work has been supported by the following grant projects:
GACR 401/04/2073 "Transparent intensional logic (a systematic exposition)" (Duží, Materna)
1ET101940420 „Logic and Artificial Intelligence for multi-agent systems" supported by the program „Information Society" of the Czech Academy of Sciences (Duží).

REFERENCES

ALMOG, J – PERRY, J – WETTSTEIN, H (eds) (1989): *Themes from Kaplan* Oxford University Press.

BOLZANO, B (1837) *Wissenschaftslehre* Sulzbach

CARNAP, R (1947). *Meaning and Necessity* Chicago UP.

CARNAP, R (1962): *Logical Foundations of Probability* Chicago UP

CASARI, E. (1992): An interpretation of some ontological and semantical notions in Bolzano's logic In George, R. (ed): 55 – 105

CHILDERS, T (ed) (2000): *The Logica Yearbook 1999* FILOSOFIA, Prague

CHURCH, A (1954) Intensional Isomorphism and Identity of Belief *Philosophical Studies,* Vol 5, 65 – 73.

CRESSWELL, M J (1985): *Structured meanings* MIT Press, Cambridge, Mass

CRESSWELL, M. J. (1978): Prepositions and points of view. *Linguistics and Philosophy,* Vol. 2, 1 – 41

van der DOES, J. – LAMBALGEN, M (2000): A logic of vision. *Linguistics and Philosophy,* Vol. 23 (1), 1 – 92.

DUŽÍ, M. (1993): Frege, Notional Attitudes, and the Problem of Polymorphism In: Max, I – Stelzner, W. (eds.). *Logik und Mathematik* Frege-Kolloquium Jena 1993, de Gruyter, Berlin, 314 – 323.

DUŽÍ, M. (2000): Existential quantification into intentional contexts In: Childers, T (ed) 2000, 258 – 272

DUŽÍ, M (2001): *Logical Foundations of Conceptual Modelling* Habilitation thesis. Retrievable at: http://www cs vsb.cz/duzi/

DUŽÍ, M. (2004) Intensional logic and the irreducible contrast between *de dicto* and *de re, ProFil,* vol. 5, 1 – 34. Retrievable at http://profil muni cz/01_2004/duzi_de_dicto_de_re.pdf

FOLEY, R. (1986) Is it possible to have contradictory belief? *Midwest Studies in Philosophy* X, 327 – 355.

FREGE, G (1884): *Die Grundlagen der Arithmetik* W Koebner, Breslau

GEORGE, R (ed.) (1992): *Bolzano's Wissenschaftslehre* 1837-1987 Leo S Olschki, Florence.

HAUTAMAKI, A (1986): *Points of View and their Logical Analysis* Acta Philosophica Fennica 41, Helsinki.

JESPERSEN, B. (2001) Ortcutt, Oedipus and other puzzles *de re* In: Majer, O (ed.) 2001, 175 – 192.

JESPERSEN, B. (2002): Why the tuple theory of structured propositions isn't a theory of structured propositions. *Philosophia* Vol. 31: 1 – 2 (2003), 171 – 183

JESPERSEN, B (2006): The Superman substitution puzzles. Invited paper read at Department of Logic, Comenius University of Bratislava, and Philosophical Institute, Slovak Academy of Sciences, 2 May 2006

JESPERSEN, B – MATERNA, P. (2002): Are wooden tables necessarily wooden? Intensional essentialism versus metaphysical modality. *Acta Analytica* Vol 17, No. 28, 115 – 150.

MAJER, O (ed) (2001): *The Logica Yearbook 2000.* FILOSOFIA, Prague

MARSCH, R (ed) (1956): *Logic and Knowledge* Essays 1901-1950 London: George Allen & Unwin, Ltd New York: The MacMillan Company

MATERNA, P (1997) Rules of Existential Quantification into 'Intensional Contexts' *Studia Logica* 3, Vol 59, 331 – 343.

MATERNA, P. (1998): *Concepts and Objects* Acta Philosophica Fennica Vol 63, Helsinki

MATERNA, P (2000): Simple concepts and simple expressions In: Childers, T. (ed.) 2000, 245 – 257

MATERNA, P. – DUŽÍ, M (2005) Parmenides Principle. *Philosophia*, Vol. 32, Nos 1 – 4, 155 – 180

MULLER, J (2006): *TIL language for Jade*. Retrievable at http //labis vsb.cz

MULLER, J (2006a): Transparent Intensional Logic and Multiagent Systems Paper presented at the International MultiConference of Engineers and Computer Scientists 2006, Hong Kong, June 2006 Retrievable at
http //www.iaeng org/IMECS2006/IWC2006 html

RUSSELL, B. (1956): The Philosophy of Logical Atomism. In: Marsch, R. (ed) 1956, 177 – 281.

TICHÝ, P. (1979): Existence and God. *The Journal of Philosophy*, vol LXXVI, 8, 1979. Also in Tichý (2004), 353 – 372

TICHÝ, P (2004): *Pavel Tichý's Collected Papers in Logic and Philosophy*. Svoboda, V. – Jespersen, B. – Cheyne, C. (eds), Filosofia, Prague – University of Otago, Dunedin.

TICHÝ, P. (1988): *The Foundations of Frege's Logic*. De Gruyter: New York et al

ZALTA, E N. (1988) A comparison of two intensional logics *Linguistics and Philosophy*, Vol. 11, 59 – 89.

ZALTA, E. N. (1989): Singular propositions, abstract constituents, and propositional attitudes. In: Almog – Perry – Wettstein (eds.) 1989, 455 – 478.