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Truths, Facts, and Liars

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ABSTRACT: A Moderate Anti-realist (MAR) approach to truth and meaning, built around the concept of *knowability*, will be introduced and argued for in this essay. Our starting point will be the two fundamental anti-realists principles that claim that neither truth nor meaning can outstrip knowability and our focus will be on the challenge of adequately formalizing these principles and incorporating them into a formal theory. Accordingly, I will introduce a MAR truth operator that is built on a distinction between being true and being factual. I will show then that this approach partitions propositions into eight classes, on the basis of their knowability. We will then ask the following question: Given the anti-realist principles, what kind of theory of propositional meaning can properly explain the *meaninglessness* of *fully unknowable* propositions? This question will lead us to the claim that the *meaning/content* of propositions should be identified not with the set of possible worlds in which the propositions are *true/factual*, but rather in which they are *known*. This modified approach will then be used to analyze both the *Liar Paradox* and the *Strengthened Liar*. To anticipate the conclusion of this essay, it will be shown that a MAR framework can render definite truth and factuality values to the *Liar sentence* and it will also confirm our intuition that such paradoxical sentences are devoid of proper meaning.

KEYWORDS: Chuch-Fitch paradox – knowability – Liar Paradox – meaning – moderate anti-realism – truth.

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0. Introduction

One standard way of approaching a certain class of semantic paradoxes (as e.g. the *Liar*, the *Knower*, etc.) is to claim that the crucial sentence in the setup of the paradox is *meaningless*. This approach is not without problems: first, the crucial sentences in the setup of the paradoxes (e.g. "this sentence is false" or "this sentence is unknown") do not *seem* to be meaningless. Furthermore, pointing to self-reference as the source of meaninglessness is also problematic as many self-referential sentences (e.g. "this sentence is in English") seem quite fine, and some of the semantic paradoxes can be formalized without self-reference.[2]

Even if it is not without difficulties, and even if it may not be quite fashionable nowadays, this is the approach I will pursue in this essay. The main objective of the essay is to introduce, and argue for, a Moderate Anti-Realist (MAR) framework, based on the verificationist/anti-realist principles that *neither truth, nor meaning can outstrip knowability*. I will start the first section with the Church-Fitch paradox that shows the limits of naïve (or radical) anti-realism. As a response to the paradox, I will introduce a MAR truth operator that defines truth – at least partially – in terms of *knowability*. Some of the relevant logical features of this truth-definition will also be discussed in the first section, among them how truths are different from mere facts (or factual propositions) and how this definition partitions propositions into eight classes, on the basis of their knowability. This division will motivate the question we will ask in the fourth section of the essay: given our anti-realist principles, what theory of propositional meaning can accommodate to our expectations?

The point of introducing a formal truth operator on the one hand, and a possible world interpretation of propositional content/meaning, on the other hand, is not to prove that anti-realism holds. Rather, the point is that such an approach is an adequate and efficient tool to solve a set of semantic paradoxes and other challenges.

I will briefly discuss the basic assumptions our MAR framework relies on in the second section. The individuation of propositions as the sets of possible worlds in which the given propositions are true or factual, as well as the shortcomings of this particular approach, will be discussed in the

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[2] At least this is Yablo's claim (Yablo 1993), although it is not without its detractors.

third section. The fourth section will focus on the challenge of attributing meaning or meaninglessness to *fully unknowable* propositions, and the fifth section will offer a solution to this challenge. I will argue that the set of possible worlds that correspond to the meaning/content of propositions should contain only those worlds in which the proposition is not only factual but *known* as well.

I will apply this MAR framework to the *Liar Paradox* and the *Strengthened Liar* in the last section of the paper. I will demonstrate that our MAR framework can assign definite truth and factuality value to the Liar sentence, and that this truth/factuality assignment allows an explanation of the paradoxical nature of this sentence. I will suggest that the source of the paradox is that we try to attribute content/meaning to a sentence that is – given that it is unknowable – totally devoid of any meaning.

1. The knowability paradox and the MAR definition of truth

The generally agreed upon central tenets of antirealism are that neither truth, nor meaning can outstrip knowability. Somewhat more formally:

(VTP_{inf}) All truths are knowable, and

(VMP_{inf}) All meaningful propositions are knowable.

We will focus on the first of these principles in this section and return to the second one in the fourth section of this essay. The simplest, most straightforward way of formalizing VTP_{inf} is:

(VTP) $\vdash \forall p(p \rightarrow \Diamond Kp)$,

where the operator, K, should be read as "it is known that…".[3] The lesson of the Church-Fitch paradox (Fitch 1963), however, is that this straightforward formalization is inadequate. The paradox shows that, if – besides VTP – the factivity of knowledge and closure under conjunction-elimination in

[3] More formally, $K_{s,t}p$ is the operator that "the epistemic agent, s, knows that p at time, t." Then we can get the above K by generalizing over subjects (epistemic agents) and times: $Kp \leftrightarrow \exists s \exists t K_{s,t}p$

K are also granted,[4] then true propositions are not only knowable, but known as well:

(CFP) $\vdash \forall p(p \leftrightarrow Kp)$,

At the heart of the paradox is the following type of propositions:

(NC) $p \mathbin{\&} \neg Kp$,

i.e. p is an unknown fact. While most of us would agree without hesitation that there are unknown facts, it is *impossible* to single out any of them and hence it is *unknowable* that a fact is unknown.

Another problem with VTP is that it provides only a *necessary*, but not a *sufficient* condition for truth, as the converse of VTP,

(VTP_{conv}) $\vdash \Diamond Kp \rightarrow p$,

does not hold, given that knowability ($\Diamond Kp$) is arguably not a factive.[5] Without a sufficient condition, however, there is a theoretical gap between knowability and truth – the epistemic-metaphysical element that would differentiate between these two concepts is missing.

One way to prevent the paradox, as I argued elsewhere (Marton 2006), is to revise the knowability principles (VTP and VTP_{conv}) in the following way:

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4 Formally: $\vdash \forall p(Kp \rightarrow p)$ for factivity, and $\vdash \forall p(K(p\&q) \rightarrow (Kp\&Kq))$ for conjunction-elimination. Then, here is how the paradox goes: assume, for any arbitrary p, that it is an unknown fact, i.e. $p \mathbin{\&} \neg Kp$. If so, then it is knowable (by VTP), and so $\Diamond K(p \mathbin{\&} \neg Kp)$. Then, in some possible world, $K(p\&\neg Kp)$. Given that knowledge is closed under conjunction-introduction, $Kp \mathbin{\&} K\neg Kp$ also holds. So, given that K is a factive, a contradiction can be derived, and so we can discharge the assumption. Thus, it is $\neg(p \mathbin{\&} \neg Kp)$, for any p, i.e. $\vdash \forall p(p \rightarrow Kp)$ holds. Given that K is factive, we can swiftly derive that $\vdash \forall p(p \leftrightarrow Kp)$.

5 While the factivity of VTP_{conv} was accepted and argued for in the recent past, this principle now seems to be abandoned. See, e.g., Tennant's retraction, for the record (Tennant 2009, 225). Furthermore, even if one does accept the converse knowability principle, $\vdash \Diamond Kp \rightarrow p$, it leads to further paradoxes, as e.g. the modal collapse ($\vdash p \leftrightarrow \Diamond Kp$), if S4 is also granted (Williamson 1992).

(MART) $\vdash Tp \leftrightarrow (p \& \Diamond Kp)$,

where T is a *moderate anti-realist truth operator*. This definition introduces a distinction between truths and facts: while an unknown fact is indeed a fact, it is not a truth, as it is outside of our epistemic reach.

Truth, according to this definition, is essentially two-pronged: the second, epistemic part, $\Diamond Kp$, expresses its anti-realist ideals: truths are more than just facts out there; truths are essentially for us, epistemic agents. The first part, however, acknowledges that truths are not entirely within our realms – at the end, they are determined by how the world is. This, of course, is what "moderates" the anti-realist character of truth. Alternatively, truth is neither purely metaphysical/ontological, nor it is purely epistemic; these two aspects cannot be reduced to either one of them.

Introducing the MAR truth operator obviously preempts the Church-Fitch paradox as NC type sentences, i.e. sentences in the form: $p \& \neg Kp$, are not knowable, and so they are not true either. Our MAR interpretation of VTP_{inf} recognizes that this principle is about *truths*, and not facts in general.

In light of these insights, I will refer to propositions that hold in a given world as being *factual*, preserving the term "true" for propositions in the extension of our newly introduced operator, T. Obviously, all true propositions are factual, however not all factual propositions are true:[6] consider an *unknown contingent* statement, p; then, exactly one of the following two conjunctions must be factual as well: (i) $p \& \neg Kp$ or (ii) $\neg p \& \neg K \neg p$. But neither of them is knowable as they are NC-type propositions. Thus, some factuals are not true. The *logic of factuals* is the standard 2-valued classical logic where e.g. $p \vee \neg p$ is a theorem.

The *logic of truths*, however, is different. First, we can introduce the concept of falsity, mirroring the definition of truth, as follows:

(Def-F) $\vdash Fp \leftrightarrow (\neg p \& \Diamond K \neg p)$.

We can also notice that

[6] In other words, MART restricts *capture* while accepts *release* without any further ado.

$\vdash Fp \leftrightarrow T\neg p$,

as it can be expected. Given that certain propositions are neither true nor false, this system is a 3-valued logic embedded in the more general, bivalent system of factuals.

We can even go one step further: the concepts of truth and falsity were constructed from 3 logically independent elements: a proposition, p; its knowability, $\Diamond Kp$; and the knowability of its negation, $\Diamond K\neg p$. From these three ingredients we can manufacture eight different *classes* of propositions:[7]

(i) two classes for true propositions, i.e. propositions that are factual and their factuality is knowable:

- propositions that satisfy $p \,\&\, \Diamond Kp \,\&\, \Diamond K\neg p$, or
- propositions that satisfy $p \,\&\, \Diamond Kp \,\&\, \neg\Diamond K\neg p$.

(ii) two for false propositions, i.e. propositions that are non-factual and their non-factuality is knowable:

- propositions that satisfy $\neg p \,\&\, \Diamond Kp \,\&\, \Diamond K\neg p$, or
- propositions that satisfy $\neg p \,\&\, \neg\Diamond Kp \,\&\, \Diamond K\neg p$.

(iii) the remaining four for the 3rd value propositions, i.e. propositions whose (non-) factuality is unknowable:

- propositions that satisfy $p \,\&\, \neg\Diamond Kp \,\&\, \Diamond K\neg p$, or
- propositions that satisfy $p \,\&\, \neg\Diamond Kp \,\&\, \neg\Diamond K\neg p$, or
- propositions that satisfy $\neg p \,\&\, \Diamond Kp \,\&\, \neg\Diamond K\neg p$, or
- propositions that satisfy $\neg p \,\&\, \neg\Diamond Kp \,\&\, \neg\Diamond K\neg p$.[8]

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[7] Belnap (1977, 47) considers a structurally similar eightfold division of propositions that also combines epistemic and ontological aspects in a similar way.

[8] There is another way to group these eight basic types:

(i) two of them are not only contingent, but *epistemically contingent*, i.e. both p and $\neg p$ are knowable (propositions that satisfy $p \,\&\Diamond\, Kp \,\&\, \Diamond K\neg p$ and $\neg p \,\&\, \Diamond Kp \,\&\, \Diamond K\neg p$).

(ii) Two of them are *epistemically undisputable* i.e. they are true (or false) but their negation cannot be known (($p \,\&\, \Diamond Kp \,\&\, \neg\Diamond K\neg p$ and $\neg p \,\&\, \neg\Diamond Kp \,\&\, \Diamond K\neg p$) – necessary statements definitely do belong to this category, but arguably there are other propositions in this category as well.

We will soon ask: What kind of theory of propositional meaning can adequately render *meaninglessness* to *fully unknowable* propositions, i.e. to propositions that satisfy p & ¬◊Kp & ¬◊K¬p or ¬p & ¬◊Kp & ¬◊K¬p?

2. Basic Assumptions

We have to pause at this point in our investigation to address the basic underlying assumptions of our approach. First, we assume that only standard possible worlds (i.e. worlds without contradictions or value gaps) are in the set of all possible worlds. We will follow Kripke's approach (Kripke 1980), according to which possible worlds are not discovered, but rather stipulated.

Second, I will take S5 to be the relevant modal system (containing exactly one equivalence class). This choice gives us the comfort of equating possibility with being true/factual in at least one possible world without further specifying the accessibility relation.

Third, the relevant modality to be considered here is *logical* possibility and necessity. This choice of modality is forced upon us by our inquiries into the concept of meaning in the next sections of the essay; to properly represent the content of propositions, all possible worlds (not only those within some narrower concepts of modality such as nomological or metaphysical) must be considered. Arguably, however, the scope of modality is effectively narrowed by our use of the knowledge operator, K, as this operator is limited to (our kind of) epistemic agents.

Fourth, our MAR definition of truth requires a robust concept of knowledge. Unless this theoretical concept of knowledge strongly overlaps with our pre-theoretical, practical concept of knowledge, the truth definition has little use. This concept of knowledge should cover empirical, as well as theoretical knowledge, etc. I also take it for granted that an agent's knowing a proposition, p, implies its factuality (and so, its truth), and that

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(iii) Two of these types are *epistemically disputable* or *falsifiable* (p & ¬◊Kp & ◊K¬p and ¬p & ◊Kp & ¬◊K¬p); while their factuality cannot be known, if they were non-factual, then their non-factuality could be known (p & ¬◊Kp & ◊K¬p and ¬p& ◊Kp & ¬◊K¬p).

(iv) Finally, two of these classes are *fully unknowable* (p & ¬◊Kp & ¬◊K¬p and ¬p & ¬◊Kp & ¬◊K¬p).

p is believed by the agent. A third condition involving some kind of justification, reason, or evidence is also assumed. It may be objected that this assumption is overly optimistic as we have no generally accepted, adequate *theory* of knowledge. But this criticism conflates two different issues; namely, the lack of a theory for a concept with the viability of the concept itself.

Fifth, this essay will avoid treating the concept of knowledge as an essentially modal concept, and accordingly, the knowledge operator, K, as a modal operator. In other words, this essay will not utilize the nowadays popular, formal 2-dimensional approaches, where knowing p in a given world is essentially a function of whether or not p is true in the epistemologically accessible worlds. These formal models, no doubt, have their relevance in certain epistemological investigations. But those models also come with their own limitations and problems (e.g. that any necessary proposition is known, according to the modal interpretation of the knowledge operator). It is also rather doubtful whether these models are consistent with the basic ideals of anti-realism.

Finally, knowledge claims (i.e. that s knows that p at t, $K_{s,t}p$, and the more generalized form, it is known that p, Kp) are epistemic facts, and as such they are parts, or constituents of possible worlds, and can be expressed by propositions. In other words, epistemic facts are facts, and the corresponding propositions can be individuated the same way as any other propositions, i.e. by the corresponding set of possible worlds.

3. On the meaning/content of propositions

It is generally accepted in certain philosophical circles that propositions can be individuated and differentiated by the sets of possible worlds in which the corresponding propositions are *factual*. If two propositions, p_1 and p_2, have different truth (or rather, factuality) values in at least one possible world, then the two propositions are indeed different. However, if there is no such world, then p_1 and p_2 are just two instances of the *same* proposition.

By identifying propositions with sets of worlds, the *meaning* of these propositions is intended to be captured. Indeed, one way to understand propositions – which amounts to capturing their meanings – is to ask: in what circumstances is this proposition true/factual[9] and in what circumstances is it false/non-factual?

This way of identifying the meaning of propositions is not without difficulties. First, there is the threat of circularity: we define the meaning of a particular proposition by referencing some relevant situations which, one can presume, are identified by some other propositions. But those situation-describing propositions (or, truth and falsity conditions) must be individuated and interpreted in some way[10] and that seems impossible to do without referencing – sooner or later – the particular propositions we have started with. However, even if defining meaning this way is circular, it does not necessarily mean that it is *viciously* so.[11]

Second, differentiating propositions by sets of possible worlds results in the fact that there is exactly one necessary proposition. Still, even if a formal individuation leads to the outcome that there should be only one necessary proposition, there should be some differentiation among its instances, according to their differing meanings. To wit, the proposition that "if you don't stand for anything, then you don't stand for anything" means something entirely different than the proposition that "two plus two equals four." This problem, the problem of hyperintensionality[12] is outside of the scope of this essay – but it definitely motivates the position about the meaning of propositions I will argue for.

Finally, as I have indicated earlier, the meaning/content of fully unknowable proposition is also problematic. What is the relevance of differentiating two propositions if both are unknowable to us? Alternatively, our

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9 Given that our preferred modal system is S5, the difference between being true and being factual plays any role only if the proposition is *fully unknowable*. If a proposition is known in at least one possible world, then the proposition is true in all those possible worlds in which it is factual. As the issue of fully unknowable propositions will be considered soon in some length, the distinction will be downplayed here.

10 This is the result of our dependence on Kripke's take on the ontological status of possible worlds – or at least of the way I interpret his claim.

11 To substantiate this point, a Quinean argument for the primacy of theory over the individual sentences may be handy here. And surely, advocates of coherence theories of truth and/or of knowledge (e.g. Davidson 1986), can also help here. However, this issue has little significance for our project and so it will not be pursued on these pages.

12 On this problem, see e.g. Jago (2014).

MAR approach is built – at least partially – on the idea that meaning should not outstrip knowability. In light of this consideration, we may ask: what kind of theory of meaning can accommodate to this anti-realist expectation?

4. On the meaning of unknowable propositions

Let us consider a fully unknowable, contingent proposition, p. This proposition, like any other, can be individuated by the set of possible worlds in which p is factual. But what content/meaning does p have? As mentioned earlier, identifying meaning with the set of possible worlds appeals to our intuition that the meaning of propositions can be grasped by the situations in which they are true, and the situations in which they are false. Ideally, we could assemble a list of propositions, $p_1, \dots, p_n$ such that $(p_1 \;\&\; p_2 \;\&\; \dots\; p_n) \leftrightarrow p$. This list of propositions, p's truth conditions, would then explicate the meaning of p. Given that this biconditional, $(p_1 \;\&\; p_2 \;\&\; \dots\; p_n) \leftrightarrow p$, fixes the *meaning* of p, it should hold not only in the actual world, but in every possible world as well.

One may find it more natural to identify a given proposition not with the *conjunction*, but rather the *disjunction* of a set of propositions. As I see it, both options are viable, but they are motivated by very different considerations. The latter option envisions the identification of a proposition, p, with listing all the possible scenarios in which p is factual. Accordingly, each disjunct is a detailed description of a possible world (or perhaps a narrowly defined situation, i.e. a "small" set of possible worlds). The former option, more fitting for an anti-realist approach, looks for defining criteria, such that each criterion is necessary, and they are jointly sufficient as well. Actually, we have already utilized this approach earlier, when we defined or identified the concept, or rather the *meaning*, of truth with two individually necessary and jointly sufficient conditions: factuality and knowability. Similarly, the traditional justified true belief approach to knowledge that we listed among our assumptions in the previous section also follows this pattern.[13]

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[13] Propositions can, and perhaps should, be identified by sets of propositions. There are two different strategies to identify sets: either by listing their elements, or by giving

In practice, we would probably settle for less: we would describe a scenario (i.e. a collection of worlds) in which p is true and another scenario (i.e. another collection of worlds) in which p is false. These scenarios can be described by sets of propositions, $q_1, q_2, \ldots, q_l$ and $r_1, r_2, \ldots, r_m$ and then we would claim that $(q_1 \& q_2 \& \ldots \& q_l) \rightarrow p$ and $(r_1 \& r_2 \& \ldots \& r_m) \rightarrow \neg p$. These two sets represent the truth and falsity conditions of p – hereafter the T&F conditions. Intuitively, the problem is that if p and ¬p are unknowable, then so are the T&F conditions that meant to explicate them. But how can we grasp the meaning of a proposition if it is couched in descriptions that are themselves unknowable? Of course, the meaning of the propositions constituting the lists can be further explained by further sets of lists, but the same must hold true: some of those propositions on those lists must also be unknowable, and so on.[14]

To support our intuition, I will briefly argue first that if the T&F conditions are known, then p cannot be unknowable. Then we will consider in what ways these conditions themselves can be unknowable. The two conditionals, presenting a conceptual analysis for the meaning of p, are *analytic* as they explicate meanings and they are obviously *known* in our world. There are two ways these conditionals can fail to transfer knowledge from T&F conditions to p:

(i) in all the possible worlds where the world-describing T&F conditions are actually known, the conditionals themselves are not known, or

(ii) although the conditionals themselves are known, but epistemic closure does not hold. Neither of these options are reasonable, though.

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rules. The former corresponds to the *disjunctive* approach, while the latter to our preferred approach of using *conjunctions*.

[14] Let me acknowledge two possible, highly connected, objections – at least in a footnote – which will not be discussed here. First, one may object that there are no unknowable facts, i.e. all facts are within our epistemic reach. Second, that even if there are unknowable facts, there are no unknowable propositions to express them. As I do not think that these objections are reasonable, they will not be discussed here.

Considering the former option, we may ask: is there any reason to assume that in all the possible worlds where the T&F conditions, (q_1, q_2, …, q_l) and (r_1, r_2, …, r_m), are fully known, the analytic conditionals, (q_1 & q_2 & … & q_l) → p and (r_1 & r_2 & … & r_m) → ¬p, known in our own actual world, would not, or rather could not, be known (at any time, by any epistemic agent)? Since these conditionals are true in any world, it is either the belief or the justification/evidence condition that can prevent putative knowers from knowing them. If so, then there must be some *inherent, structural* difference between our world in which these knowledge conditions are met and the worlds in which the T&F conditions are known to account for the difference in knowing the analytic conditionals. But, as far as I can see, there is no such inherent difference.

Turning our attention now to the latter option, these conditionals can fail to transmit knowability only if epistemic closure itself is challenged: these challenges operate with a familiar line of reasoning, summarized in a conditional, against a non-standard, unexpected circumstance (Dretske's zebra-looking mules, etc.). But our conceptual analysis does not fit into this pattern – it outlines the very circumstance in which the analyzed concept must be true. If both the T&F conditions are knowable and the analytic conditionals are known, then transmitting knowability is unavoidable.[15]

Accordingly, if p is unknowable, then the set of T&F conditions must also be unknowable. There are 3 different ways these T&F conditions can be unknowable:

(i) The set of T&F conditions is inconsistent, and so the conditionals are vacuously true and p and ¬p may be propositions out of our epistemic reach.

(ii) Even if the T&F set is consistent, some of the elements of the sets can be unknowable themselves, and that accounts for the unknowability of p.

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[15] This claim may come with a caveat. It may be objected that even if both K(q_1 & q_2 … & q_l) and K((q_1 & q_2 … & q_l) → p) hold, epistemic agents may never actually attain the knowledge of q. That's certainly *possible*, i.e. there will be possible worlds in which p will not be actually known. But if closure holds, then attaining the knowledge of p is also *possible*, i.e. there will be possible worlds in which p is known. And that is enough for our purposes.

(iii) Even if the set of T&F conditions is consistent, and all the propositions in this set are knowable *in themselves*, their conjunctions may still not be knowable, i.e. the situation they describe are not fully knowable. NC, the sentence at the heart of the Church-Fitch hypothesis is an example of such conjunctions where both sentences can be knowable separately, but the conjunction itself is arguably unknowable. Importantly, this option undermines the compositionality of meaning – even if two propositions, p and q, both have meanings, p&q may not be knowable and thus the conjunction is meaningless.[16]

To sum it up, unknowable propositions cannot be explicated/illuminated/interpreted in terms of knowable propositions. They are meaningless, as they are beyond our epistemic reach. As these propositions may be individuated by a corresponding set of possible worlds, we can further conclude that meanings cannot be identified with the sets of possible worlds in which the proposition is factual, even in case of contingent propositions.

5. A solution to the problem

The insights of the previous section suggest that we should refine our intuition about the individuation and meaning of propositions by modifying our previous question in the following way: in what *knowable* circumstances would a proposition be *known* to be true, and in what *knowable* circumstances would a proposition be *known* to be false? Accordingly, we

[16] Jago writes: "Take our example from above, 'it is both snowing and not snowing here right now'. This sentence is perfectly meaningful, for both of its conjuncts are meaningful, and a sentence 'A ∧ B' is meaningful whenever its conjuncts 'A' and 'B' are individually meaningful" (Jago 2014, 7). Jago's claim about the compositionality of meaning comes without any argument or support. However, just because one understands the meaning of the proposition "it's snowing here right now," claiming that the proposition that "it is both snowing and not snowing here right now" has any meaning is far from obvious. Personally, I cannot imagine what would anyone aimed to express by that proposition. Furthermore, it is unclear how the meaning of this proposition is different from "Boston is in Massachusetts, but Boston is not in Massachusetts."

identify the *meaning* of a proposition not with the set of worlds in which the proposition is factual, but rather with the set of worlds in which the proposition is *known*. Informally, this approach emphasizes the relevance of context – the meaning of a proposition, p, is captured by considering *what else should be known* to understand p.[17]

In essence, our MAR approach suggests *two different identity relations* on the set of propositions. In one way, propositions can be individuated by the sets of worlds in which they are factual. In another way, the content/meaning of propositions can be identified with set of possible worlds in which the proposition is known. These two different identity relations correspond to our two different concepts of truth; the metaphysical concept (our concept of factuality) and the epistemically constrained MAR concept (our concept of truth).

To be more precise, it is not propositions, but rather pairs of propositions to which meaning is attributed. In the traditional account, individuating p with the corresponding set of worlds also individuates ¬p, as its corresponding set is the complement set. Accordingly, the meaning of the pair of propositions, p and ¬p (or rather, Tp and Fp) should be identified with the set of worlds in which p is known and with the set of worlds in which ¬p is known. Quite obviously, this approach solves our problem. If p and ¬p are both unknown in every possible world (i.e. p is a fully unknowable proposition), then the corresponding sets are empty and so no meaning is associated with p (and ¬p). It also explains in what sense truth is more than mere factuality – being true is being meaningfully factual.

6. The Liar Paradox

Let us turn our attention now to the Liar Paradox. Consider first the following sentence:

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[17] This point suggests how this approach can solve the problem of hyperintensionality. What defines the meaning of a necessary statement is the set of worlds in which that statement is known. Focusing on those worlds would reveal what should have to be known previously to be able to know that p. This approach is rather similar to the intuitionistic ideal of stages, or possible development, of knowledge (Beall 2003, 96-97).

(f) This sentence is false.

Traditionally, this sentence can be formalized as

(1) $f \leftrightarrow \neg f$,

and then it is easy to realize that no truth value can be attributed to f. This point, on its own, invites us to consider some version of a 3-valued (or many-valued) logic. The trap can be easily avoided if a 3rd value is attributed to both f and ¬f. Still, some explanation is required about the meaning of the 3rd truth value, i.e. what it means for a proposition to be neither true nor false.

Consider now the following, "strengthened" version of the paradox:

(n) This sentence is not true.

It is often claimed that the previous approach, based on a 3rd truth value, is inefficient here. If n is neither true, nor false, then obviously n is not true, so the sentence that "n is not true" *is* true and so we are back at the paradox. We may anticipate at this point that this conclusion is just too fast; all we should be able to conclude from this reasoning is that "n is not true" *is factual.*

Let us now switch from the traditional approach to our MAR approach. The previously introduced sentence, f, can be written as

(2) $f \leftrightarrow Ff$,

Where F is our *falsity* operator. Since (2) fixes the meaning of the sentence referenced as f, it is an analytic statement and so

(3) $\Box(f \leftrightarrow Ff)$.

Given that Ff is defined as $\neg f \;\&\; \Diamond K\neg f$, what (3) really amounts to is

(4) $\Box(f \leftrightarrow (\neg f \;\&\; \Diamond K\neg f))$.

As f and $\neg f \& \Diamond K \neg f$ cannot be simultaneously factual, (4) can only be true if both are nonfactual. Accordingly, f must be non-factual, and so ¬f must be factual, but then $\Diamond K\neg f$ cannot be factual. Putting all these considerations together, 4 implies that

(5) $\Box(\neg f \,\&\, \neg\Diamond K\neg f)$.

Furthermore, according to 5, $\neg f$ is factual in all possible worlds and thus f is not knowable either:

(6) $\Box(\neg f \,\&\, \neg\Diamond Kf \,\&\, \neg\Diamond K\neg f)$.

Less formally, the crucial sentence in the liar paradox is, on the one hand, nonfactual, but, on the other hand, it is also *fully unknowable*, and so *meaningless*.[18]

Similarly, the formal version of the “strengthened” liar sentence is

(7) $\Box(n \leftrightarrow \neg Tn)$,

and then, by using the definition of T:

(8) $\Box(n \leftrightarrow \neg(n \,\&\, \Diamond Kn))$.

Following a reasoning similar to our previous one shows that (8) implies that

(9) $\Box(n \,\&\, \neg\Diamond Kn \,\&\, \neg\Diamond K\neg n)$.

(9) shows that the crucial sentence of the *strengthened liar* is factual, but, just like the *liar*, it is also fully unknowable,[19] and so meaningless.

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18 Even if we know f's (non-) factuality (that it holds in none of the possible worlds), f itself is unknown to us. This point brings to the front one of the basic tenets of antirealism: knowledge of meaning is knowledge of truth conditions. Knowing the factivity of a proposition is not the same as knowing its meaning, or, simply put, knowing the proposition itself.

19 Here is the reason: assume that the sentence within the scope of the necessity operator, $n \,\&\, \neg\Diamond Kn \,\&\, \neg\Diamond K\neg n$, is knowable. Then there is a world, w, in which $K(n \,\&\, \neg\Diamond Kn \,\&\, \neg\Diamond K\neg n)$ is factual/true. Assuming that K is closed under conjunction-elimination, both Kn and $K\neg\Diamond Kn$ would hold in that world. Given that K is factive, $\neg\Diamond Kn$ (i.e. $\Box\neg Kn$)

Our outcomes, (6) and (9), show that the MAR definitions of truth and falsity render definite factuality and truth values to the *liar* and *strengthened liar* sentences. The former is unknowably nonfactual, while the latter is unknowably factual; as such, neither of them is either true or false.

Our language is limited by the logical/epistemic norm that sentences should be meaningful. Any assertion in the form that "p is factual" should be interpreted as p is *meant to be meaningfully factual*, which, according to our interpretation means that p is meant to be true. If so, then the distinctions between untrue and false and, similarly, between true and unfalse disappear when we consider assertions. False and untrue sentences differ in their knowability and no one should aim to assert a proposition that's unknowable, and thus, according to our reasoning, meaningless. The difference between these sentences surfaces only as an explanatory device: they explain one's mistake to assert something that should not or could not be asserted. Arguably, this is an important advantage of this approach: the distinction between truth and factuality is almost imperceptible – in most of our everyday (and perhaps even in our theoretical) dealings truth and factuality are the same. The consequence of this insight is that the two liar paradoxes collapse into one; if there is no real, *meaningful* difference between the assertions that "this sentence is false" and that "this sentence is untrue" then these assertions express the same proposition that can be expressed, using the combination of our previous formulations, (6) and (9), as follows:

(10) $\Box((f \,\&\, \neg\Diamond Kf \,\&\, \neg\Diamond K\neg f) \vee (\neg f \,\&\, \neg\Diamond Kf \,\&\, \neg\Diamond K\neg f))$.

In other words, the proposition that expresses the liar-sentence is the staple meaningless sentence which is either untruly factual *or* unfalsely nonfactual.

Furthermore, our insight about the normative character of assertions prevents the emergence of an "iterated" liar paradox. In light of our previous insights about the *normative* standards governing assertions, the sentence that

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would also hold in that world, and so would $\neg Kn$ then. Given that both Kn and $\neg Kn$ would both hold in w, $n \,\&\, \neg\Diamond Kn \,\&\, \neg\Diamond K\neg n$ is not knowable.

(g) this sentence is nonfactual,

should be interpreted as “this sentence is meaningfully nonfactual”, i.e. it is false. But then we are back at the original liar paradox.

7. Concluding remarks

The underlying, fundamental assumption of this paper is that the concept of knowledge plays a central role in our concepts of truth and meaning. It is the possibility of knowing that makes truth (understood as being different than mere factuality) and meaning possible for us, epistemic agents. I did not argue for this fundamental assumption in this essay. Rather, I argued that such a moderate anti-realist approach to the concepts of truth and meaning offers a way of solving or dissolving a number of semantic paradoxes and other challenges. I demonstrated that our MAR approach to truth and meaning, offers an interpretation of the *Liar* and *Strengthened Liar* sentences. This interpretation renders *definite* truth and factuality values to these sentences and shows that the difference between their truth and factuality values emerge from their *meaninglessness*. Even if these sentences do not seem to be meaningless, they are meaningless as there is no possible situation in which these sentences (or their negations) could be known. This should not surprise us: if someone tells us that the sentence she is uttering is false (or it is not true), then we would not know what she meant by it, what kind of knowable fact she tried to impart to us.

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