

ON THE DEFINITIONS OF BASIC KINDS OF PROPERTIES

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In the recent Festschrift dedicated to Pavel Cmorej, a rigorous classification concerning various kinds of properties modeled within the intensional framework was published (see DBKP¹). However, one subtle error, not found before the publication, infiltrated into several passages of DBKP. Now the author suggests a way to rectify the definitions affected by it. The error is rather 'technical' and philosophically quite uninteresting – it 'only' turns few intuitively correct and philosophically welcomed definitions of kinds of properties into incorrect ones. Of course, there is a condition under which the straightforward simple classification of properties is valid (see below). In this rectification, I do not expose the apparatus, shortcut conventions, or preliminary definitions; for this purpose please see DBKP. Properties (and other attributes) were modeled as *intensions*, i.e. functions from possible worlds to classes of (n -tuples of) individuals. Within this functional framework, 'classical' classes are construed as total characteristic functions. But in DBKP, also partial characteristic functions, partial classes serving as extensions of properties, were allowed.² This leads to inaccuracies in DBKP. Generally, the partiality always causes troubles.

A property is called *trivial* provided it is a total function having a constant course of values-extensions (or it is a partial function undefined for all arguments); otherwise, the property is *non-trivial*. A property is *purely essential* provided all objects possessing it instantiate this property in every possible world. Given this fact, one might conclude that purely essential properties are trivial, i.e. having only one and the same (non-empty) extension invariantly across the logical space (i.e. in all worlds) – however, this is not correct. They are various non-empty

¹ Raclavský, J. (2007): Defining Basic Kinds of Properties. In: Marvan, T. – Zouhar, M. (eds.): *The World of Language and the World beyond Language (A Festschrift for Pavel Cmorej)*. Bratislava: Veda, 69 – 107.

² I did not originally handle them; they were accepted only after I decided to turn the study into an exhaustive and general one.

partial classes (and one total class) as characteristic functions which have the same cardinality and assign T to the same (and no other) objects: say individuals I_1-I_k . Assume that there is a property F which is purely essential for I_1-I_k but randomly changes such characteristic functions as its mutually distinct extensions. According to the above definition, F is non-trivial, but it is still purely essential. On the other hand, *partly essential* properties, say G , are such that there is at least one individual, I_m – distinct from I_1-I_k (individuals for which G is essential) – that can possess G (in at least one possible world); no doubt, these properties are non-trivial. It seems then that the border line between purely essential and partly essential properties, diagrammed in the Rose of Kinds of Properties (DBKP 103; cf. also below) as ‘south borderline’, is a bit fuzzy. It is not exactly the case: the axis ‘north-south’ always divides properties into non-trivial and trivial ones. However, the symmetrical division of properties (according to certain criteria) into quadrants disappears: purely essential properties cover more than one quadrant because they inhabit the ‘south-east’ and also a part of the ‘south-west’ (they reach the bold-dotted line; cf. the modified Rose below). There are two other important kinds of properties which, jointly with purely essential and partly essential properties, complete the quadruplet of kinds of properties: properties *purely empirical* – non-trivial properties which are not essential, properties *trivially void* – trivial properties which are not essential.

The existence of partial classes as extensions of properties makes 12 definitions (from more than 120) inaccurate or even (though rarely) invalid. Before repairing them, notice that purely essential properties split into the two following kinds ([ExtensionOf_w f] constructs the same as f_w):³

being a *trivially purely essential* property f (in w) =_{df} being a property such that there exists an individual x which is in the extension of f in every possible world w' and there does not exist a possible world w'' in which the extension of f is not identical with the extension of f in w

[TriviallyPurelyEssential_{|w|} f] \equiv (o ξ)^o

$$[[\exists \lambda x [\forall \lambda w' [f_w' x]]] \wedge \neg [\exists \lambda w'' [f_w'' \neq f_w]]]$$

being a *nontrivially purely essential* property f (in w) =_{df} being a property such that there exists an individual x which is in the extension of f

³ Of course, intensions (or other functions) not having (o ξ)-objects as values (btw., DBKP suggests classification of all intensions) cannot be classified in this way (moreover, the disputed bug does not arise for the case of the respective definitions of their kinds).

in every possible world w' ; and in every possible world w'' , the cardinality of the extension of f in w'' is identical with the cardinality of the extension of f in w ; and there exists a possible world w''' in which the extension of f is not identical with the extension of f in w ; and for every individual x' , if x is in the extension of f in w , then there does not exist a possible world w'''' such that there exists an individual x'' which is in the extension of f in w'''' and is not in the extension of f in w

$$\begin{aligned} [\text{NonTriviallyPurelyEssential}_{|w|} f] \equiv^{(\text{op})\omega} [& [\exists.\lambda x [\forall.\lambda w' [f_{w'} x]]] \wedge \\ & [\forall.\lambda w'' [[\text{Card } f_{w''}] = [\text{Card } f_w]]] \wedge [\exists.\lambda w''' [f_{w'''} \neq f_w]]] \\ & \wedge [\forall.\lambda x' [[f_w x'] \rightarrow \neg[\exists.\lambda w'''' [\exists.\lambda x'' [[f_{w''''} x''] \wedge \neg[f_w x']]]]]] \end{aligned}$$

Now the reader is asked to accept the following rectifications of DBKP. The first two definitions of purely essential properties in the section „Purely essential properties” (p. 95)⁴ define in fact trivial properties, thus insert properly ‘trivially’/‘Trivially’ into them (the next definitions in the section correctly define purely essential properties; the definitions of trivially/nontrivially purely essential properties may be added into this section). Similarly, there should occur ‘TriviallyPurelyEssential’ instead of mere ‘PurelyEssential’ in the definiens of „trivial” (p. 103). The first two definitions of partly essential properties (p. 96) should be completed by inserting ‘and it is not nontrivially purely essential’/‘ $\wedge \neg[\text{NonTriviallyPurelyEssential}_w f]$ ’ in the definiens. The first two definitions of purely empirical properties (p. 102) need analogous correction.⁵ Moreover, the same rectification applies on the right side of the sixth formula concerned with „purely empirical”, and, similarly, with „partly essential” (both on p. 104). Then ‘ $\vee [\text{NonTriviallyPurelyEssential}_w f]$ ’ should be inserted in the right side of the fifth formula on p. 103, and also in the consequent of the first formula concerned with „purely essential” on p. 104. In order to make the definition of trivial singular properties (p. 87) accurate enough, add to it (it is not mere ‘insertion into’): ‘and f is trivial’, ‘ $\wedge [\text{Trivial}_w f]$ ’ (and outermost brackets surrounding the whole definiens).⁶

Explaining the classification of properties, a philosopher like me frequently needs to state the definitions in an easily comprehensible (i.e. not

⁴ One of them is repeated on p. 104.

⁵ In the case of these two we have also another, easier, possibility: using ‘not essential’ / ‘ $\neg[\text{Essential}_w f]$ ’ instead of ‘not partly essential’ / ‘ $\neg[\text{PartlyEssential}_w f]$ ’.

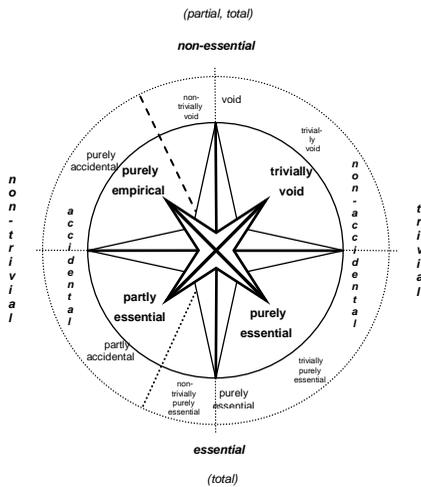
⁶ Without this correction they would specify properties instantiable only by (each of them ‘particularly’) one and the same individual; however, some of them are non-trivial.

technically complicated) manner. For example, to define them in terms of ‘quadrants’ and ‘semicircles’ diagrammed in the Rose. Before doing it, the following restrictive condition has to be explicitly formulated:

assuming only total (not partial!) classes as extensions of properties, ...

Once partial classes as extensions of properties – which cause the asymmetry in the ‘south’ – are excluded, it is correct to state, e.g., that partly essential properties are those which are non-trivial and not purely empirical; or that purely essential properties are those which are trivial and are not trivially void.⁷ (Unfortunately, the asymmetry in the ‘north’, near the dashed line, can be correctly ignored under much stronger condition: properties randomly changing empty class(es) with no value at all, i.e. nontrivially void properties, are also excluded from our considerations. As soon as it is assumed, purely empirical properties are just purely accidental properties.)

The Rose of Kinds of Properties⁸



⁷ The restrictive condition enables us to simplify also the wording of one of the most important contribution of DBKP – the very complicated definition of mutually complementary properties (e.g., „being a smoker” / „being a non-smoker”); i.e. the preliminary formulation on p. 83 would be sufficient.

⁸ If no restriction is assumed, accidental properties inhabit the (left) sector between the dashed line and the bold-dotted line. (Independently on restrictions, partly essential properties are the same as partly accidental properties.)