Papers

Lowe on Locke's and Frege's Conception of Number

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Abstract: In his last book about Locke's philosophy, E. J. Lowe claims that Frege's arguments against the Lockean conception of number are not compelling, while at the same time he painstakingly defines the Lockean conception Lowe himself espouses. The aim of this paper is to show that the textual evidence considered by Lowe may be interpreted in another direction. This alternative *reading* of Frege's arguments throws light on Frege's and Lowe's different *agendas*. Moreover, in this paper, the problem of singular sentences of number is presented, and Frege's and Lowe's views are confronted with it.

Keywords: concept of number, Frege, Lowe, the Lockean, singular and general sentences of number, existence, identity, number zero, number one.

In a recent publication, E. J. Lowe takes on Frege and Locke, facing off their respective theses on the concept of number and positioning himself somewhere within the Lockean camp.² For my purposes here, let us employ the expression 'the Lockean conception of number' to denote any view close to Locke's (as Lowe himself puts it).

Locke accepts *grosso modo* that number is a primary quality of bodies. Additional primary qualities include bulk, figure, situation and motion:

The *Qualities* then that are in *Bodies* rightly considered, are of *Three sorts*:

- ¹ I would like to thank to Raul Zuñiga and Jon Umerez for their comments.
- My concern is specifically with the arguments on pages 82 87 of Lowe's excellent book on Locke (Lowe 2005).

First, The Bulk, Figure, Number, Situation, and Motion, or Rest of their solid Parts;...

The First of these, as has been said, I think, may be properly called *real Original*, or *primary Qualities*, because they are in the things themselves, whether they are perceived or no...³

A more Fregean reading of this thesis might state that 'in Locke's view, *number is a* (primary) *property of objects.*'

Frege deals with his concept of number in *Grundlagen*.⁴ The first half of the book (§5-§54) criticizes several conceptions of number. One such conception criticized by him holds that number is a property of objects. This is, from now on, *the Lockean view*. In the second half of the book (from §55 on), Frege presents his own conception: generally speaking, a sentence of number, or cardinality sentence, expresses that number is applied or ascribed to a property, or (as Frege puts it) 'to a concept.' And yet, according to Frege, numbers are objects (§57). Clearly, Frege's position and the Lockean view are incompatible.

Lowe charges that Frege's arguments against the Lockean view are not compelling, while at the same time he painstakingly defines the Lockean conception Lowe espouses. Moreover, Frege's conception of number is sometimes criticized by Lowe, mainly in what concerns the case of number *zero*.

However, in this paper, I argue that the textual evidence considered by Lowe may be interpreted in another direction. What Lowe interprets as a *reductio ad absurdum* against the Lockean view, is rather an argument (or part of an argument) in favor of a particular way of construing sentences of number. This alternative *reading* of Frege's (main) argument throws light on Frege's and Lowe's different *agendas*. For instance, Lowe's discussion of number zero clearly shows that we are facing two very different ways of understanding ontological issues. Moreover, in this paper, the *problem of singular sentences of number* is presented, and Frege's and Lowe's views are confronted with it.

³ Locke (1689, II, VIII, 23). About the idea of number, ibid., II, XVI.

⁴ Frege (1884).

1 The Supposed Reductio ad Absurdum

Lowe claims that one of Frege's arguments against the Lockean view constitutes something of a *reductio ad absurdum*.⁵ According to him, one ought to interpret Frege's argument against the Lockean view in this way:

Let us suppose that numbers are properties of objects. If so, then different numbers may be assigned to the same object. However, to say that different numbers may be assigned to the same object is incoherent. Consequently, the Lockean view leads us to an incoherent conclusion. Thus, this view is false, and numbers are not properties of objects.

Lowe suggests that the Lockean conception of number *need not imply* any such incoherence. To avoid this implication Lowe distinguishes between two different relations, namely *consisting of* and *identity*. For instance, Frege says that "while looking at one and the same external phenomenon, I can say with equal truth 'It is a copse' and 'It is five trees' " (§46). So, the incoherence arises when we *identify* one copse with five trees, and we assign the properties of being one and of being five to the same thing. Nevertheless, according to Lowe, no defender of the Lockean view need arrive at this incoherence: she can assert that one copse *consists of* five trees (without being identical with five trees), and in this way the incoherence disappears. Thus, Frege's objection to the Lockean view is moot, or so claims Lowe.

It is not my goal here to analyze this interesting distinction between two relations, namely *identity* and *consisting-of*. Perhaps Lowe is right, and said distinction proves enough for the Lockean scholar to avoid the incoherence put forward in Frege's argument at least *as this argument is described by Lowe*. My objection to Lowe is that Frege did not put forward any such *reductio ad absurdum*. The textual evidence proposed by Lowe may be interpreted in another direction.

Texts quoted by Lowe refer to those situations in which we apparently have 'one external phenomenon', and we can *describe* it in several ways:

I can say with equal truth both 'It is a copse' and 'It is five trees,' or both 'Here are four companies' and 'Here are 500 men.' (§46)

⁵ The core of this argument appears in Lowe (2005, 84).

Or,

If I place a pile of playing cards in (someone's) hands with the words: Find the Number of these, this does not tell him whether I wish to know the number of cards, or of complete packs of cards, or even say of honour cards at skat. (§22)

Or,

... I am able to think of the Iliad either as one poem, or as 24 Books, or as some large Number of verses. (§22)

Or finally,

... For example, a bundle of straw can be separated into parts by cutting all the straws in half, or by splitting it up into single straws, or by dividing it into two bundles. (§23)

Lowe is right in his assertion: Frege recognizes the incoherence of stating that any object could be one thing (one pack of cards; one copse) and, at the same time, more than one thing (54 cards; 5 trees).⁶ Nevertheless, by using these examples, Frege did not intend to *directly* attack the Lockean theory of number. What he suggests is a more general issue concerning sentences of number.

His argument can not be *reductio ad absurdum* of the Lockean view. This is because for Frege there was no such absurdity: our aforementioned incoherence is not a real one. Frege's words are quite clear about this. Let us examine how he punctuated the lines quoted just above:

If I place a pile of playing cards in (someone's) hands with the words: Find the Number of these, this does not tell him whether I wish to know the number of cards, or of complete packs of cards, or even say of honour cards at skat. To have given him the pile in his hands is not yet to have given him completely the object he is to investigate; I must add some further word – cards, or packs, or points. (§22)

Furthermore,

... an object to which I can ascribe different numbers with equal right is not what really has a number. (§22)

True, it *may sometimes seem* that Frege availed himself of this supposed incoherence or absurdity to attack the Lockean conception of number.⁷ Nevertheless, a much better explanation fits this textual evidence.

Frege considers these examples as a proof of vagueness and confusion in our everyday discourse: The object to investigate or the domain of investigation simply have not been precisely determined. This is not problematic just for Lockean scholarship; this problem affects everyday discourse in general. Vagueness is implicit in everyday discourse in the use of terms such as 'pile', 'conglomeration', 'multitude', 'set', and 'plurality'.8 When we use these terms we speak informally: It seems that the same conglomeration (multitude, set, or plurality) is one copse and, at the same time, five trees. However, by introducing a concept (in Frege's sense of concept), the domain of investigation is defined. Frege's point is that the introduction of concepts fine-tunes our discourse *clearly* and distinctly, and the logical form of sentences of number also becomes clear and distinct. At the common discourse level, we may be confusedly referring to a given conglomeration but, at the logical level, the message is clear: a sentence of number expresses that a number is applied to a concept. I would compare Frege's project here to Russell's one concerning definite descriptions: we pass from grammatical form of sentences of number to their logical form, and logical form is transparent concerning truth-conditions of sentences.

Thus the textual evidence quoted by Lowe may be interpreted in the way just mentioned: Sometimes, by means of sentences of number, *it appears* that we apply different numbers to the same object, i.e., *this* is one copse, *this* is five trees. Nonetheless, this is a *false impression*. By fixing the domain of investigation, we see which one the logical form of sentences of number is: 'there is one copse', 'there are five trees', or, alternatively, 'one object falls under the concept *copse'*, 'five objects fall under the concept *tree*'. The logical form shows that in sentences of number some number is applied to some concept: number *n* is applied to concept C. Such a sentence of number is true if and only if *n* objects fall under the concept C. By determining the concept (*copse* or *tree*), we abso-

⁷ See Frege (1884, §48).

⁸ Ibid., §23 (about the vagueness of 'agglomeration'), §45 (about the vagueness of 'set', 'plurality' and 'multitude').

lutely determine 'its' number (one or five). (Frege gives us other reasons in favor of this logical form, but Lowe does not deal with them.)

Once Frege has laid down the logical form of sentences of number, he can conclude that the Lockean conception of number is wrong, since sentences of number (in their logical form) express that numbers are applied to concepts (or properties) — rather than to objects: logical form is transparent. So, Frege's attack against the Lockean may be understood as a *side effect* of his logical analysis of sentences of number.

Now then, Frege provides (other) arguments which directly assailed the Lockean view in §21-25. In Locke's view, as we have seen, numbers (as primary Qualities) are in the things themselves, whether they are perceived or not. In §21-25, Frege asks, "Is number a property of external things?" Frege compares number with other properties as colour and solidity, and he puts forward several problems concerning this comparison. On the one hand, in §21, he says, "It is not in totally different senses that we speak of a tree as having 1000 leaves and as having green leaves? The green colour we ascribe to each single leaf, but not the number 1000." On the other hand, he adds, "I can point to the patch of each individual colour without saying a word, but I cannot in the same way point to the individual numbers... an important difference between colour and Number, that a colour such as blue belongs to a surface independently of any choice of ours." In §24, curiously, Frege mentions Locke's words (Essay, II, XVI, 1): "Number applies itself to men, angels, actions, thoughts - everything that either doth exist or can be imagined", and agrees with him. However, Frege uses this thesis again to attack the conception of number as a property of external things:

It would indeed be remarkable if a property abstracted from external things could be transferred without any change of sense to events, to ideas and to concepts. The effect would be just like speaking of fusible events, or blue ideas, or salty concepts or tough judgments.

It seems that Frege identifies some tension within Locke's view. However, since Lowe does not examine these arguments, I prefer not to deal with them here.

What about Lowe's crucial distinction between *consisting of* and identity? On the one hand, Lowe tries to block the (supposed) *reductio ad absurdum* by means of such distinction. But, as we have seen,

Frege's main goal is to establish the logical form of sentences of number, not just to attack the Lockean by means of *a reductio ad absurdum*. On the other hand, as far as I see, Frege could state that 'there are 5 trees' means that five is applied to the concept *tree*, that 'there is one copse' means that one is applied to the concept *copse*, and that copses consist of trees. In this sense, Lowe's distinction could be absorbed into Frege's view without putting forward any problem to such view.

Until now I have tried to show that, against Lowe's interpretation, there is other way to interpret textual evidence. By the way, we have seen which the logical form of sentences of number is in Frege's view. What about the Lockean?

2 The Lockean Conception of Number. Arguments around Number *One*

Lowe states:

Now, if we are to take seriously Locke's suggestion that number is a property of things, then, clearly, the only number that can be assigned to a *single* thing... is *one*. But this means, of course, that numbers other than one can only be assigned to *more than one* thing – that is, to *pluralities* of things.⁹

This last paragraph constitutes a more detailed account of Lowe's *Lockean conception*, expressed as:

- (i) Number one is the only number that can be assigned to single objects.
- (ii) Other numbers are assigned to pluralities.

Alas, a heterogeneous theory: there is a difference between number one and the rest of the numbers. Frege, as we have seen, does not make such a difference: every sentence of number expresses that a number is applied to a property.¹⁰

- ⁹ Lowe (2005, 85 86).
- Frege (1884, §51). Frege says that number 1 is a number in the same sense than 2 and 3 are numbers. Frege looks for a homogeneous theory, because sentences of number 1 (and 0) behave as other sentences of number. However, as Frege saw, singular sentences of number 0 and of number 1 put forward problems to Frege's

Oddly enough, Frege might have accepted (ii). However, allow me to flesh out some of the nuances of this statement. With respect to (ii), Lowe states:

In the sentence 'The planets are nine', the subject term, 'the planets', refers plurally to Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto, and the predicate attributes the property of being nine to that plurality. 11

Hence, this plurality is the object that Lowe needs for cases of numbers other than one.

For Frege, the sentences of number 'The planets are nine' and 'There are nine planets', both exhibit the logical form 'The number nine is assigned to the concept *planet*.' Moreover, he would have accepted that both sentences have the following *derived* form: 'The number nine is assigned to the extension of the concept *planet*'. In such a case, Frege might have agreed with Lowe's statement on numbers other than one (and zero).

As Blanchette puts it:

Though Frege's view is opposed to the attribution of cardinality to individuals or to collections, it is not opposed to the attribution of cardinality to sets, in the modern sense... The cardinality of sets is just the cardinality of every concept whose extension is that set.¹²

So, Lowe and Frege might agree concerning (ii), if 'plurality' is interpreted as 'set' (in the modern sense proposed by Blanchette). However, I have some doubts about this alleged agreement. It seems to me that from Lowe's view, when we affirm 'the planets are nine' we affirm that the number nine applies to the plurality {Mercury, Venus, Earth, Mars...}. Does this mean that the sentences 'the planets are nine' and 'Mercury, Venus, Earth... are nine' affirm the same thing? If so, as Lowe seems to state, this is a mistake, because such sentences affirm different things (the Fregean, as we will see, can make sense of this distinction). We will come back to this issue later.

project because they apparently behave in other way. In this sense, heterogeneous theories may be on the right track. Later, I deal with the (general) problem of singular sentences of number.

- ¹¹ Lowe (2005, 86).
- ¹² Blanchette (1999, 208).

Nevertheless, it is quite clear that Lowe and Frege disagree with respect to the number one. According to Lowe, one is the only number that can be applied to *single things*. I could agree that this is true. But, once again, let me flesh it out a bit more.

Number one may be applied to both properties and single things, as in the following sample sentences,

- (1) There is just one cat in this room.
- (2) Lewis Carroll is one.

Concerning (1), Frege's theory says: number one is assigned to the concept *cat in this room*. I will not venture to speculate here on how Lowe might respond in this case. May number one only be assigned to single things? Or to pluralities which contain just one element, as well? I suppose Lowe would argue that number one may be applied to this kind of plurality. So, concerning sentences like (1), let us suppose, Frege and Lowe might agree (taking into account Blanchette's warning, of course).

Let us move on to (2). For Lowe, the number one is assigned to the *single object* Lewis Carroll. In this case, it seems that there is no property (or concept) at all. Hence, Lowe's view would seem more adequate than Frege's view. Which is Frege's answer? Frege's theory should pronounce itself on (2), but it does not. Why?

From my point of view, the problem is more general: Fregean and Lockean theories ought to have dealt in general with 'singular sentences of number' such as 'a₁, a₂, ..., a_n are m' (where $m \ge 1$, $n \ge 1$ and 'a₁', ..., 'a_n' are singular terms; I will deal with those cases in which m = 0, later on in this paper). They do not, however.

- (2) is a case of singular sentence of number. Here are two more cases:
 - (3) Lewis Carroll and Charles Dodgson are two.
 - (4) Lewis Carroll and Charles Dodgson are one.

Frege's theory certainly suffers from a lacuna: what do singular sentences of number mean? Which are their truth-condions? He does not systematically deal with these questions. And the Lockean?

With this in mind, let us see what Lowe says concerning number one. Lowe takes issue with one argument of Frege's, as being allegedly critical of defenders of number one as a property of single objects. Frege argues against the Lockean conception of number one, on the basis that such a conception forces us to accept an all-embracing property, namely the property of being one: each and every single object is one. Because all sentences similar to (2) are apparently true, the property of being one becomes an all-embracing property. In Frege's view, this type of property, precisely because it is all-embracing, is under suspicion. Hence, the Lockean conception proves problematic. These are Frege's own words about all-embracing properties:

It is not easy to imagine how language could have come to invent a word for a property which could not be of the slightest use for adding to the description of any object whatsoever. (§29)

Lowe's surprise at Frege's rejection (of the all-embracing property of being one) stems from the inconsistency, since Frege would have readily acquiesced to the all-embracing property of self-identity, for instance.

On this, I am in complete agreement with Lowe. His surprise at such inconsistency on Frege's part is justifiable, and I accept that Frege's argument is not a *good* argument. Does this mean that the Lockean is the winner? I do not think so.

Let us consider the Fregean and the Lockean as they deal with singular sentences of number. Frege has doubts concerning singular sentences of number. Frege's (inconsistent) rejection of the all-embracing property of being one might show his doubts. Allow me to speculate that Frege never intended for cardinality sentences such as (2) to be objects of study where a conception of number is concerned. As a matter of fact, he scarcely alludes to them.¹³ This is true. Do consider now my observation on his omission: for instance, the singular sentence 'a is one' could be interpreted as 'a is identical with a'. This might be an interpretation compatible with Frege's ideas. Thus, this sentence should not be considered a standard cardinality sentence, since in its logical form there is no explicit number: in fact, it is an identity-sentence. As such, Fregean theory, untarnished and homogeneous, states: a standard cardinality

One exception to the rule is Frege (1884, §29). True, Frege's own discussion here proves unconvincing. I deal with this subject in Arrieta-Urtizberea (2007).

sentence says that a number is assigned to a concept. This is just one example which gives us an idea of one of the possible Fregean strategies to deal with singular sentences of number.¹⁴

For the Fregean the main goal is to establish the logical form of these singular sentences which (independently of Frege's doubts) appear to be completely meaningful and informative. As far as singular sentences of cardinality are related to identity-sentences, we could speak about the *puzzles of singular sentences of number*. These puzzles, of course, would be related to well known (Fregean) puzzles concerning identity-sentences. For instance, do 'a and b are one' and 'a and a are one' express the same proposition?

What about the Lockean? Let us take sentences (3) and (4). In a Lockean view, what do these sentences mean? What are their truth-conditions? Does (3) mean that number 2 applies to the plurality {Lewis Carroll, Charles Dodgson}. What kind of plurality is {Lewis Carroll, Charles Dodgson}? From a Lockean view, what does 'Neptune and Vulcan are one' mean? The Lockean ignores the issue of the meaning of (singular) sentences of cardinality.

Frege (or the Fregean) before dealing with ontological issues (What is number zero? What is number one?...) tries to solve the problem of the meaning of sentences of number. On the contrary, the Lockean agenda seems to be a different one. The discussion around number zero confirms this hypothesis.

3 Arguments around Number Zero

What about number zero? What about existence and non-existence sentences, such as 'Lewis Carroll exists' or 'Lewis Carroll does not ex-

In my view, singular sentences of number are Boolean combinations of (singular) identity-sentences. For instance, (3) means 'Lewis Carroll is not identical to Charles Dodgson'. (4) means 'Lewis Carroll is identical to Charles Dodgson'. Therefore, singular sentences of number are not standard cardinality sentences. I propose this alternative to extend Frege's view in Arrieta-Urtizberea (2007). Blanchette extends Frege's view keeping the compatibility with Frege's ideas. The singular sentence 'a₁, a₂, ..., a_n are m' should be interpreted as a (non-singular) sentence about the concept *identical with a₁ or ... or with a_n* (Blanchette 1999, 221 – 2). In any case, the concept of identity plays an important role in the building of these extensions of Frege's conception.

ist'?¹⁵ Frege's theory of number *should pronounce itself* on this subject, especially when questions of existence force us to deal directly with issues concerning the number zero.

In this respect existence is analogous to number. Affirmation of existence is in fact nothing but denial of number nought. (§53)

As far as *general* non-existence sentences are concerned (i.e., 'there is no cat in this room'), Lowe holds that *prima facie* Frege has the upper hand on the Lockean conception. That is because the Lockean needs some object (a single thing or plurality), a holder of the property of being zero. But there is no such object. On the contrary, Frege would say that in the case of general non-existence sentences, the number zero is assigned to the concept C (*cat in this room*). Recognizing *empty concepts* does not represent a significant problem, at least for Frege.

In light of this seemingly (or *prima facie*) technical advantage in Frege's favor, Lowe's charges are a rather curious novelty. Lowe now considers Frege's thesis that 'zero' *denotes a number* (or an empty set) 'at best a bad joke to be told.' Lowe attacks Frege's thesis that zero is an object. In so doing, Lowe attacks Frege's realism concerning number zero. Number zero cannot be conjured into existence by mere stipulation. This is an ontological mistake. Lowe seems convinced that number zero actually poses no problem for Lockean scholarship because there is no such property as being zero. In this way, the Lockean dispenses with the problem of number zero.

Let us see which Frege's strategy is. First, Frege deals with the meaning of sentences of number zero. Although Frege's answer to *general sentences* of non-existence is that number zero is assigned to a concept, he leaves the problem of singular non-existence sentences ('Lewis Carroll does not exist') up in the air. I hypothesize that, as is the case with other singular sentences of number, these existence sen-

While there is no mention of singular existence sentences (except God-sentences) in *Grundlagen*, Frege, in other works, offers us some insights on such sentences. It is very clear how hesitant was Frege with respect to this kind of sentences. One time he says that these sentences are vacuous and evident; the next he says that they do not make sense; and even sometimes he transforms them into general existence sentences. I deal with this subject in Arrieta-Urtizberea (2007).

¹⁶ Lowe (2005, 87).

tences are not 'standard' cardinality sentences. In which case, said sentences do not pose problems for Frege's view of sentences of number. Second, after fixing the meaning of these sentences, we can deal with the ontological question, what is number zero? And Frege's answer roughly is: number zero (like other numbers) is a set of concepts. All this is what Frege does. Of course, if Frege's view about sentences of number zero implies Platonism concerning number zero (or Realism about the empty set), and such kind of Platonism is false or problematic, then Frege's view about sentences of cardinality zero is false or problematic. Anyway, I do not intend to muddle this discussion by delving into the drawbacks of Fregean Realism-Platonism's understanding of numbers and sets. Nor will I examine number zero and empty sets. Such a discussion, in my estimation, demands a much more in-depth analysis than Lowe does in this book. Realism-Platonism has still important followers.

It is difficult to see how Lowe's statement that there is no such property as *being zero* helps us to deal with the issue of the meaning of singular (and general) non-existence sentences. Again, it seems that Lowe ignores the issue, or, at least, he is not interested in it.

4 Conclusion

We have seen that Frege is mainly concerned with the logical form of sentences of number: ontological issues, let us say, come after. Paradigmatically, in The *Grundlagen*, Frege puts language at the core of philosophical research: the inquiry into the concept of number takes into account how number expressions appear in the language. On the one hand, language *deals with* any number (zero and one included) in a similar way. On the other hand, there are general and singular cardinality sentences. Any theory about the concept of number must fit into these *linguistic facts*. As the discussion concerning number zero makes clear, Lowe is mainly or directly interested in ontological issues. He proposes a heterogeneous theory. From his point of view, there is a difference between number one and the rest of the numbers. Moreover, number zero does not exist. This theory does not fit well into *linguistic facts*, probably because Lowe has a different strategy and conception with respect to ontology. Probably for this same reason, Lowe's counter-arguments

have no bearing upon the core of Frege's thesis and argumentation about the logical form and semantics of sentences of number.

Concerning the distinction between general and singular sentences, Frege's conception ought to respond to a series of questions related fundamentally to singular cardinality sentences, but he does not do it. However, as I have argued, this problem might be settled within a Fregean framework. It is not clear, instead, how the Lockean conception (as Lowe puts it) could deal with these singular sentences, and with general and singular sentences of non-existence. It seems that the Lockean ignores these issues. As we have seen, concerning number zero, Lowe's discussion is purely ontological: no *linguistic facts* are considered. He affirms that *zero does not exist*: most ordinary folk would consider to think of "zero" as denoting a number at best a bad joke. ¹⁷ In Frege's *onto-semantics*, number zero and the concept of (non-) existence are strongly related, as language itself shows: zero, as any other number, lives normally within language.

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