Jan Dejnožka: *The Concept of Relevance and the Logic Diagram Tradition*
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Consider the relevance criterion (R) If A follows from (a set of) assumptions X, then A has *something in common* with the assumptions. The criterion seems to be plausible enough, yet one can argue that our good old classical logic is at odds with it. The argument usually builds on the notorious "limit cases" of classical consequence. For example, one might point out that any A classically follows from any B & $\sim B$ but it is far from obvious that any contradiction should have something in common with *any* proposition.

Relevance logics are motivated by the attempt to formalize "following from" in the sense of (R) as an implication *connective*. In fact, relevance logicians have come up with a more precise version of the relevance criterion, namely the *variable sharing* requirement (PR) If $F \rightarrow G$ is a theorem, then the formulas F, G share at least one propositional variable. Theorems of the form $F \rightarrow G$ can be read as "G follows from F" (F can be a conjunction) and the shared variable can be seen as securing a "link" or "meaning connection" between the assumption(s) F and the conclusion G.

Indeed, the "flagship" relevance logics such as **R** and **E** comply with (PR). Consequently, not every formula of the form (FQ) "ex falso quodlibet" (F & $\sim F) \rightarrow G$ is a theorem (simply put, the schema is not a theorem). But (PR) has more implications, some of which are considered to be controversial. For example, **R** and **E** cannot prove the (DS) "disjunctive syllogism schema" (F & $(\sim F \vee G)) \rightarrow G$ (note that it could be used to derive (FQ)). However, (DS) seems to be quite plausible, mainly because of the resemblance it bears to the *rules* of disjunctive syllogism (If F and $\sim F \vee G$ are both provable, then so is G) and "material" Modus Ponens (remember that $\sim F \vee G$ simply *is* the material implication $F \supset G$). From the viewpoint of a relevance logician, schemas such as (FQ) or (DS) are not "*relevantly valid*" – if one wishes to comply with (R) and, hence, the relevantist's reading (PR), then one has to reject these schemas as general logical laws.

Dejnožka bucks this point of view as he argues in the present book that there is a sense of "having something in common" in which classical logic is *not* at odds with (R) and even that this sense is more natural and rooted deeper in the logical tradition than (PR). The crucial assumption of his main argument links (R) with *logic diagrams* and states that *diagramming shows containment* (DC) "if in the very act of diagramming all the premises of an argument we also diagram its conclusion, then the premises contain its conclusion" (p. xiii). The remaining two assumptions are (AR) "If the premises of an argument

contain its conclusion, then the argument is relevantly valid" and (AD) "Many arguments that violate the [...] variable sharing requirement can be and have been so diagrammed" (both quotes p. xiii).[1] Consequently, many schemas on the relevance logician's blacklist, such as (FQ) and (DS), *are* in fact relevantly valid: "classical logic is relevant after all, even if that does not accord with the relevantists' conception of what relevance is" (p. 6).

The first three chapters of the book discuss and defend the assumptions of the main argument. The assumption (AR) is taken "to be non-controversial" (p. 1) and the first chapter is devoted to explaining "containment" and "relevant validity" in more detail. In short, Dejnožka understands the "containment" of (AR) as *truth-ground containment*, "the truth grounds of [the premises] contain the truth grounds of [the conclusion]" (p. 5) and regards this kind of containment relation as "the most basic kind of logical relevance" (p. 5). Importantly, the *truth grounds* of a proposition can be thought of as situations in which the proposition holds. The notion of truth ground containment is traced back to Wittgenstein's *Tractatus*.

Chapter 2 defends the crucial assumption (DC), according to which "showing containment by a diagram is a *sufficient condition* of relevant containment" (p. 7). In accord with (AR), relevant containment is construed as truth ground containment or containment of truth grounds. Hence, in effect, (DC) means that diagrams show containment of truth grounds (or lack thereof). It should be noted that "diagram" is used in a deliberately vague fashion and the reader learns in Chapter 3 that diagrams can be "truth tables, truth trees, and literally geometrical figures such as circles or squares" (p. 13). In addition, (AD) simply points out that classical implicational tautologies such as (FQ) and (DS) can be shown to be valid by the abovementioned kinds of diagrams: for example, truth tables show that every truth ground of the antecedent is also a truth ground of the consequent.

The main argument of the book is interesting for suggesting that truth ground containment, i.e. the classical notion of consequence, embodies a meaningful notion of a connection between the assumptions and the conclusion of a valid argument. However, this seems to be plain enough, and I am not sure if any relevance logician would insist that classical consequence invokes *no* connection *whatsoever*. It does, but the connection is too weak – *almost* too weak to be called a connection at all. Dejnožka holds that "the relevantists overlook the best and, so to speak, most visible concept of their own tradition: diagram

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[1]   The labels (DC), (AR) and (AD) are mine.

containment" (p. 2). A rejoinder to this might be that they *do not* overlook the concept – they simply consider it too weak.

In any case, the argument is not meant to be an attack on relevance logics: "This book has been written toward reunion in logic, or at least toward peaceful co-existence, in a Carnapian spirit of mutual tolerance" (p. 102). A noble goal, even if some would tend to consider it as stating the obvious.

Chapter 4 goes back in time and claims that Russell was an *express relevantist*, in that "he does expressly endorse Wittgenstein's view that deductive inference is a matter of containment and of following from" (p. 32). Chapter 5 discusses several "antecedents" of the view that "modern classical validity is the primary form of relevant entailment and there are secondary, more restrictive forms" (p. 45). In particular, the author distinguishes *extensional* relevance (truth-ground containment) and *intensional* relevance in the style of Anderson and Belnap (variable sharing) and provides examples from the literature where these have been discussed side by side. Dejnožka points out that, in addition to him, only David Lewis is willing to call classical entailment "relevant" but he submits the belief that the view is "implicit in every modern classical logician who uses truth tables, truth trees, or Venn diagrams to show validity" (p. 33).

Chapter 6 discusses ten objections to the presented view. I will not go into details of every objection since some of them are somewhat random. But there is an interesting discussion of the relevantist claim that "modern classical logicians are in the business of formally analyzing ordinary 'if-then' [and that] the supposed modern classical analysis [is] perverse" (p. 56). Indeed, this is the usual starting point of the expositions of relevance logics. Dejnožka offers four replies to this objection against classical logic. First, a "tu quoque" rejoinder is offered. Dejnožka points out that relevance logicians are themselves at odds with the ordinary usage of language, in that they willingly disregard the distinction of the object-language "If-then" and the metalanguage "implies" (see the "Grammatical propaedeutic" of Anderson and Belnap's *Entailment*, vol. 1): "[t]hey say that violating ordinary usage is perfectly all right when it comes to their own logic, yet they criticize modern classical logicians for their 'perversity' in violating ordinary usage" (p. 59). However, one might insist that the "If-then" / "implies" distinction is much more subtle that the material conditional / "If-then" distinction and, hence, disregarding the latter distinction is much more "perverse" that disregarding the former one.

Second, Dejnožka claims that classical logic and formulas such as (FQ) and (DS) are themselves supported by strong intuitions. However, this view is shared by relevance logicians (at least in the case of (DS)), and much work is devoted to advocating their status (see, for example, Restall 2000, 344-346). It

is perhaps important to note as well that, albeit (DS) is not a theorem of the "flagship" logics **R** and **E**, the *rule* is admissible in both logics.

Third, Dejnožka argues that relevance logicians do not seem to "understand the modern classical project" (p. 60), in that the modern classical logicians did not intend to faithfully represent "If-then": "In fact, it is disingenuous to pretend that modern classical logicians are secretly insincere or do not know exactly what they are doing when they use technical notions like material implication" (p. 61). This is an interesting point. But one might want to replace classical logic because of its failure to meet a specific standard even if it was not the intention of classical logicians to meet the standard. One can simply identify a specific problem of classical logic and try to come up with a logic that does not suffer form the problem while *disregarding* the original intentions of classical logicians.

Fourth, Dejnožka assumes that relevance logicians "apparently believe that modern classical logicians would actually use the falsehood (or impossibility) of a premiss to materially (or strictly) imply the truth of a conclusion" (p. 61). He insists that this assumption is plainly false and cites Russell who himself claims that material implications are practically useless since "they can only be *known* when it is already known either that their hypothesis is false or that their conclusion is true; and in neither of these cases do they serve to make us know the conclusion, since in the first case the conclusion need not be true, and in the second it is known already" (p. 62, Dejnožka cites the first volume of *Principia Mathematica*, pp. 20-21).

It is interesting to note that this reasoning assumes that a disjunction can be known only if at least one of the disjuncts is known – which is a slightly controversial assumption.

Chapter 7 states the plain thesis that classical validity is extensionally equivalent with truth ground containment and, hence, "since virtually all modern classical logicians from Wittgenstein on would accept at least [this thesis], virtually all are implicitly extensional relevantists" (p. 69). Moreover, "the insight that modern classical validity is visibly containment-relevant is the sort of insight that was right in front of us all the time" (p. 69). The chapter also discusses several "extensions" of the classical notion of validity, or "relevance filters," that would yield a stronger notion of consequence. For example, one could adopt the requirement that in order for A to follow* from assumptions X, the set of assumptions has to be satisfiable or, as Dejnožka puts it, the assumptions have to be "compossible" (p. 73). However, some of the constraints are not explained clearly enough, to wit the "deducibility constraint that the (formal) validity of an inference be showable *by some formal means*" (p. 76).

The book concludes with Chapter 8, where some of the main theses are reiterated. Dejnožka insists once again that relevance logicians "make it so clear that Russell is their chief nemesis *because* he has no concept of relevant containment validity, and yet it is so clear that Russell *does* have just such a concept" (p. 94). I do think that this is not quite right: I believe that relevance logicians would submit only that Russell has no *satisfactory* concept of relevant containment validity.

In conclusion, Dejnožka stresses that "[t]he thesis that a conclusion necessarily follows if and only if it is in *some* sense contained in the premiss(es) seems to be a basic synthetic a priori intuition many on both sides would accept. The thesis that truth-ground containment is *the* basic sense, or at least *a* basic sense, of relevant containment is a second basic synthetic a priori intuition which I accept, I think many modern classical logicians would accept, and I hope relevantists will at least consider. If I am right that these two theses are basic, then perhaps no positive argument for them should be expected, since that would seem to require more basic premisses" (pp. 98-99).

However, Dejnožka does offer a positive argument for the reader's consideration: "My theory is actually a synthesis of both sides. It cancels the claim of each side to have the sole truth, reveals each side to be a limited viewpoint, and preserves and transcends the merits of each side in a comprehensive unified theory in which, broadly speaking, one side is the genus and the other side is its chief species" (p. 99).

I do think that the book's main claims hold, but this is largely due to the fact that they are not overly ambitious. Yes, classical validity can be seen as invoking a (rather weak) notion of containment – containment of truth grounds. Yes, this notion can be found in the writings of outstanding modern classical logicians such as Wittgenstein or Russell. Yes, the relevantist's notion of relevance, embodied in (PR), can be seen as a species of a broader genus. Perhaps the greatest merit of the present book is that it emphasizes these points explicitly. I do not think that relevance logicians will find much to argue against (but perhaps this does not apply to Chapter 6 as I have indicated above). Hence, the "reunion in logic" is a goal that can be established, if it has not been established already some time ago.

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References

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