The Impact of Using Multi-Dimensional and Combinatory Vague Terms on the Possibility of Formulating Sorites Paradoxes

JAN ŠŤEPÁNEK

Faculty of Arts. Masaryk University.
Arna Nováka 1. 602 00 Brno. Czech Republic
honza.stepanek@mail.muni.cz

ABSTRACT: We cannot definitely determine precise boundaries of application of vague terms like “tall”. Since it is only a height of a person that determines whether that person is tall or not, we can count “tall” as an example of a linear vague term. That means that all objects in a range of significance of “tall” can be linearly ordered. Linear vague terms can be used to formulate three basic versions of the sorites paradox – the conditional sorites, the mathematical induction sorites, and the line-drawing sorites. In this paper I would like to explore a possibility of formulating sorites paradoxes with so called multi-dimensional and combinatory vague terms – terms for which it is impossible to create a linear ordering of all objects in their range of significance. Therefore, I will show which adjustments must be made and which simplifications we must accede to in order to formulate any version of the sorites paradox with multi-dimensional or combinatory vague terms. I will also show that only the conditional version of the sorites paradox can be construed with all three kinds of vague terms.


1 I would like to thank the anonymous reviewer for his inspiring insights and comments.
1. Linear vagueness

Vague terms such as “heap” or “tall” lack sharp boundaries of application. In other words, with vague terms we can clearly distinguish some cases in which the vague term either applies or does not apply to some object, and so called borderline cases — cases in which we are not sure whether the term applies to a given object or not. Although we know that a man measuring 215 centimetres is tall and a man measuring 130 centimetres is not, there is no precise height at which we could draw the line separating tall people from the rest of the population. So while there are many people that either clearly are or clearly are not tall, there are also many people that we are not sure which group they should belong to. This lack of sharp boundaries of application is what gives rise to sorites paradoxes.

The simplest type of vagueness is so called linear vagueness. The applicability of any linear vague term is determined by one and only one dimension of variation — for “tall” it is the height of a person, for “old” it is the age of a person, etc. This dimension can be expressed numerically, though for some vague terms this numerical value is going to be only arbitrary. For phenomenal vague terms like “sweet” (see Hyde 2008, 11-12), there is no objective way of assigning the numerical value to particular members of the range of significance of the term in question.

Linear vagueness is often associated with terms “heap” and “bald” which were used to formulate The (Paradox of the) Heap and The Bald Man paradox — most likely the first sorites paradoxes that were formulated. Neither of the aforementioned terms is, however, truly linear, so I will use another common example of linear vagueness — the term “tall”.

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2 I use the term “object” in a very broad sense, since sorites paradoxes can be formulated for vague terms a range of significance of which can consist of physical objects, set-theoretical objects, or propositions.

3 It won’t be based on any physical quantity which can be precisely measured (like wavelength or height).

4 If all conditions necessary for classifying some object as a heap — except for a precise number of grains needed — could be decidedly given, then “heap” would indeed be the linear vague term. Yet it is also necessary to take a structure of a heap into consideration, although some philosophers either overlook or disregard it. Since there is no possible way to define precisely structure of a heap, we cannot count “heap” as a linear vague term. With “bald” we also need to consider placement of hair — so even this term is not linearly vague. I would like to thank Václav Hynčica for sharing his insights and
Linear vagueness can be subsumed under so called *degree-vagueness*. Hyde (2008, 16) defines degree-vagueness as follows: “Degree-vagueness consists of those cases in which the vagueness stems from the lack of precise boundaries between application and non-application – or at least their apparent lack – along some dimension.” Degree-vagueness can be compared to a greyscale. There are clearly light shades representing objects to which the vague term is applicable and there are clearly dark shades representing objects to which that term is not applicable, yet there is no clear and precise boundary between the two. As its name suggests, degree vagueness is considered to be a matter of degree\(^{6}\) – the more centimetres you measure, the taller you are.

If we want to formulate any version of the sorites paradox, we need two things to begin with. The first is a vague term and the other is an ordering of objects in a range of significance of the vague term, for every vague term is soritical only relative to the ordering of objects in its range of significance. Such ordering must satisfy three conditions which were for the first time explicitly formulated by Barnes in his (1982, 30-32). Barnes states that a vague term \(F\) is soritical with respect to an ordered sequence of objects \(\langle a_1, a_2, ..., a_n \rangle\) iff following three conditions are satisfied:

1. The term \(F\) is, to all appearances, TRUE of \(a_1\).
2. The term \(F\) is, to all appearances, FALSE of \(a_n\).
3. All adjacent objects \(a_i\) and \(a_{i+1}\) are, to all appearances, indistinguishable in all respects relevant to \(F\).

The third condition is tantamount to what Wright (1975) coined with the term *tolerance*. \(F\) is said to be tolerant iff small changes in aspects relevant to \(F\) do not seem to make a difference to applicability of \(F\).\(^{7}\)

\(^{5}\) Predicate “tall” is not exactly ideal too for it is context sensitive. Nevertheless, its context sensitivity has no impact whatsoever on its linear nature.

\(^{6}\) This is the main intuition behind infinitely many truth-valued approach to sorites paradoxes. According to Kolář (1998, 22): “Vague predicate, such as ‘small’, ‘bald’, or ‘red’, denotes property that objects can possess in ‘a different degree’.” This definition is not a good one since it only considers degree vagueness and omits combinatory vague terms such as “chair” or “religion”.

\(^{7}\) Kolář in his (1998) argues that the concept of tolerance is incoherent and argues for applying many-valued truth-functional approach to sorites paradoxes and vague terms.
Objects in the range of significance of the linear vague term are ordered with respect to the numerical value of the dimension of variation – men can be ordered with respect to their height in centimetres and millimetres or with respect to their age in minutes and seconds. It is easy to order objects in ranges of significance of such vague terms as “tall”, “old”, or “bald”, because dimensions of variation of these terms are founded upon either physical quantities or on a quantity of some objects.

If all three conditions specified above are satisfied, three basic versions of the sorites paradox can be construed. First, and probably the most common, version is the conditional sorites:

\[ \begin{align*}
F_{a_1} \\
F_{a_1} \rightarrow F_{a_2} \\
F_{a_2} \rightarrow F_{a_3} \\
\vdots \\
F_{a_{n-1}} \rightarrow F_{a_n} \\
F_{a_n}
\end{align*} \]

Consider the following argument: A man whose height is 130 centimetres is not tall. If the man whose height is 130 centimetres is not tall then a man whose height is 130 centimetres and 1 millimetre is not tall. If the man whose height is 130 centimetres and 1 millimetre is not tall then a man whose height is 130 centimetres and 2 millimetres is not tall and so on. This will, however, lead us to a conclusion that a man of a towering stature whose height is 215 centimetres is not tall. This is evidently absurd since that man is clearly tall. So where should we draw the line between tall people and those who are not tall?

The conditional sorites is based on a concatenation of many instances of modus ponens. Although this version can also be presented as a conjunction of many individual arguments (each being one instance of modus ponens), the polysyllogistic version depicted above is more common. In each step of this paradox, with each conditional premise or with each instance of modus ponens, we are getting closer to an unacceptable conclusion. It should be noted that not every argument that has the form of the sorites polysyllogism is paradoxical. The paradox only arises when a vague word and the proper ordering of objects is used.

Second version is called the mathematical induction sorites:
This version uses the mathematical induction in order to get to the paradoxical conclusion that the soritical term in question either can or cannot be applied to all objects in its range of significance. Second premise of this version of the sorites paradox is based on condition (3). Since every two adjacent objects are indistinguishable with respect to their features relevant to \( F \), \( F \) can be applied either to both of them or to neither of them – and therefore either to all or to none of the objects in the range of significance of the term \( F \).

The last version is called the line-drawing sorites:

\[
Fa_1 \\
\forall n (Fa_n \rightarrow Fa_{n+1}) \\
\forall n (Fa_n)
\]

This version of sorites paradoxes is based on denying the conclusion of the mathematical induction sorites. However, denying the conclusion of the mathematical induction sorites entails that there must be a sharp boundary of application of the given vague term, so called cut-off point. Consequently, there is the sharp boundary dividing all men into bald men and hirsute men – this entails that either losing or growing a single hair makes the difference between these two groups. To elaborate further, if we have two men that are located along the boundary of baldness and they differ only by one hair, then although those two men are indistinguishable for us, one of them is bald and the other one is not.

Since every object in the range of significance of any given linear vague term only contains objects that can be ordered on the basis of one single dimension, it is possible to create an “ultimate” sorites paradox. To formulate such sorites paradox, all objects in the range of significance of the vague term are used. The range of significance of some linear vague terms can, at least potentially, contain infinitely many objects, yet this does not rule out the possibility of formulating the “ultimate” sorites paradox.

The last feature all sorites paradoxes have in common is their reversibility. For every sorites paradox that proceeds by addition, a paradox proceeding by subtraction can be construed. We can use the negated version of some vague terms, e.g. “heap” and “not heap”; with other terms, we can use
their opposites, e.g. “bald” and “hirsute”. Whichever vague term we use, we can always turn the paradox over.8

2. Multi-dimensional vagueness

It is the case with many vague terms that “several different dimensions of variation are involved in determining their applicability” (cf. Keefe 2000, 11). These vague terms are labelled as so called multi-dimensional vague terms. While linear vague terms had only one quantifiable dimension of variation, multi-dimensional vague terms have at least two such dimensions. While even these vague terms can be subsumed under degree-vagueness, there are alterations to be made before any form of the sorites paradox can be formulated. In this section I will show that some philosophers make these alterations without even realising it.

One of the best examples of the multi-dimensional vague term was presented by mistake by Burks in his (1946, 482). It was Burks who was the first one to introduce the distinction between linear and multi-dimensional vagueness, citing colours such as “blue” or “green” as examples of linear vague terms. He, however, only took hue into consideration – with respect to “blue”, for example, he only considered the ordering of different hues ranging from green to blue. Yet for any colour there are three dimensions of variation that need to be taken into account – hue, brightness, and saturation.9 Although each of these dimensions independently allows formulation of the linear ordering of some members of the range of significance of the multi-dimensional vague term, there is no possible way to create the ordering of all members of the range of significance (e.g. all different shades of blue differing in hue, brightness, and saturation) that would satisfy all three conditions mentioned in the previous section of this paper. Formulation of the “ultimate” sorites paradox is therefore completely out of the question. Yet this in no way means that the sorites paradox cannot be formulated using multi-dimensional vague terms since there are at least

8 With regard to above stated conditions this means that $a_i$ from condition (1) need not have the smallest numerical value, $a_n$ from condition (2) need not have the highest numerical value, and $F$ can be a negative term such as “not bald”.

9 The first one to notice this mistake was, according to Hyde (2008, 17), Bertil Rolf in his (1981).
two ways of creating the sorites paradox using multi-dimensional vague terms.

It seems that to formulate virtually any variant of the sorites paradox we need to assent to some simplifications. In a majority of linear sorites paradoxes we limit the number of objects constituting the soritical ordering to a subset of all objects in the range of significance of the vague term. With the term “tall” I only considered men measuring between 130 and 215 centimetres, thus omitting all who do not fall within this range. Nevertheless, it is not enough to simply limit the range of significance of the multi-dimensional vague term.

Let us consider the term “big”. If we say that someone is “a big guy” we mean that he is both tall and massive. This means that there are two dimensions\(^{10}\) of variation that determine applicability of “big”. We can limit both of these dimensions, yet it would not make the formulation of the sorites paradox possible. We need to take a more severe action.

With multi-dimensional vague terms, there is a chance to omit all but one dimension. So with colour, for example, there is an option to only consider one dimension – either hue, or brightness, or saturation. As you can see, this is exactly what Burks did when he wrote about colours. As long as there are only minute differences between adjacent objects in the ordering of objects so that all three conditions mentioned earlier are satisfied, we can formulate any version of the sorites paradox that we could formulate with linear vague terms.

Aforementioned reduction of dimensions, however, means that we simply swap the multi-dimensional vague term for the linear one. If we apply such a reduction to “big”, we end up formulating the sorites paradox either for “tall” or for “massive”. This of course means that the sorites paradox formulated this way does not really make use of the multi-dimensional vague term.

Another way to formulate the sorites paradox using multi-dimensional vague terms is to handpick objects along all dimensions to formulate the soritical ordering. This way we take into consideration all of the dimensions and we handpick objects that differ slightly along all dimensions, yet each two adjacent objects are indistinguishable. This method is not exactly new, since it can be traced back to Carneades who employed it to formulate

\(^{10}\) This also means that we reduce “massive” to one easily numerically expressible dimension, e.g. linear ordering of people according to their volume in cubic centimetres or according to their weight in grams. This, of course, is a considerable simplification.
the sorites paradox for the term “god”. I consider this method crucial for formulating sorites paradoxes since, as I will show later, this method can be used to formulate the conditional sorites paradox for every vague term there is, not just for multi-dimensional vague terms.

To formulate the sorites paradox for “big” we can handpick people to create an ordering by gradually adding both height and weight. We omit many objects in the range of significance of “big”, yet we consider both its dimensions. This means that we must be careful not to pick objects that would be considered indistinguishable along one dimension but distinguishable along the other one. If we handpick objects carefully, all three abovementioned conditions are satisfied to classify the term as soritical. This way we are able to formulate the conditional sorites for any multi-dimensional vague term.

The same method described here can be employed when formulating the sorites paradox for any linear vague term. It is one of two simplifications that we accede to when we formulate the conditional sorites. The first simplification is already mentioned limitation of objects in the range of significance of the vague term (e.g. we only consider men that are between 130 and 215 centimetres tall). The second simplification is our omitting some objects from the range of significance of “tall” by only considering men that differ in their height by exactly 1 millimetre. It is much easier to handpick objects in the case of linear vague terms since the possibility of numerical ordering of objects makes our job easier. Our method of picking these objects is always driven by a need to satisfy the same three rules mentioned in the previous section of this paper.

Even if our handpicked ordering satisfies all three abovementioned conditions, we cannot use it to construct the mathematical induction sorites and the line-drawing sorites. The reason is that both these versions depend on the possibility of formulating the total linear ordering of objects in the range of significance of the vague term. With some modifications, however, we can form a version of the sorites paradox using the multi-dimensional vague term that on the first glimpse looks similar to the traditional mathematical induction variant. The modified sorites paradox for “big” would look roughly like this:

A man who measures 130 centimetres and weighs 40 kilograms is not big. The difference in both weight and height between any two adjacent men is so minute that both of them have to be judged identically as either big or not big. Since the first man in ordering is not big, no man in ordering is big – no matter his weight or height.
I need to emphasize that this version only resembles the mathematical induction sorites since this version is not based on the mathematical induction. This version of the sorites paradox is similar to arguments presented by Colyvan – Weber (2010). They are based on so called *Leibniz continuity condition* (see Priest 2006, 165-171). The problem of both my and Colyvans’ and Webers’ versions of the sorites paradox is that Leibniz continuity condition is not a generally valid mathematical schema (cf. Colyvan – Weber 2010, 315-316).

We can do similar modification for the line-drawing sorites which would be based on counter-intuitiveness of the conclusion that there must be objects which are in all relevant aspects indistinguishable, yet which differ as to applicability of the vague term used. This version of the sorites paradox is not exactly true to the linear version of the line-drawing sorites as well. While the original line-drawing sorites is based on the counter-intuitiveness of ascribing different statuses to two directly neighbouring objects, in the modified version there can be no such objects because there can be no total linear ordering of objects in the range of significance of the vague term used.

Consequently, the conditional sorites is the only version of the sorites paradox that can be construed with no formal alterations even when using multi-dimensional vague terms and it thus retains its strength. It doesn’t matter whether we omit one dimension or whether we handpick objects comprising the range of significance of the multi-dimensional vague term, because the conditional sorites is exactly the same as it would be if we had used the linear vague term.

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### 3. Combinatory vagueness

*Combinatory vagueness* 11 has one thing in common with multi-dimensional vagueness – with combinatory vague terms there are also multiple

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11 Bueno – Colyvan (2012) use the term “non-numerical vagueness” because with terms in question there is no natural numerical ordering. I consider this quite misleading since there are even linear or multi-dimensional vague terms like “looks red” or “sweet” for which there can be linear ordering, yet no natural numerical values can be assigned to individual components of such orderings – only arbitrary values based on a placement of a component in the ordering.
factors determining whether the vague term in question can or cannot be applied to some object. The difference between these two kinds of vagueness is the lack of quantifiable dimensions in the case of purely combinatory vague terms. As a result of this there is no dimension that would allow any linear ordering of at least some objects in the range of significance of the combinatory vague term. This is what distinguishes combinatory vagueness from degree vagueness.

For combinatory vagueness the problem does not consist in the inability to draw the sharp boundary along one or more dimensions, but in the inability to pinpoint the exact conditions which need to be satisfied in order to apply the combinatory vague term in question.

“Religion” is considered to be the example of the combinatory vague term (see Bueno – Colyvan 2012). What are conditions that need to be satisfied in order to classify some practice as a religion? If we have a set of conditions, which of them are necessary? Is there any combination of conditions that is, if satisfied, sufficient to count something as a religion? These and other similar questions contain the crux of combinatory vagueness.

There are, of course, many vague terms that combine both degree-vagueness and combinatory vagueness. I consider “heap” to be one of such terms, since we need to take a structure of any given arrangement of grains of sand into consideration apart from a number of grains of sand used.

There are many conditions which must be satisfied in order to call some object a combat knife, yet we can never be sure which of these conditions are necessary or sufficient – or even what these conditions are. Conditions such as “the material of the blade” do not allow the possibility of formulating any numerical linear ordering of objects in the range of significance of the term “combat knife” at all, yet the length of the blade allows us to linearly order at least some objects along this dimension. This also means that this kind of vague terms allows formulating both the mathematical induction and the line-drawing sorites, since its linear dimension allows the linear ordering. Even though we are able to formulate all three variants of the sorites paradox that can be formulated using linear vague terms, we can never use all objects in the range of significance of the com-

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12 This is probably the reason why some authors, like Keefe (2000), only distinguish between linear and multi-dimensional vagueness.
binatory vague term. This reduction to one quantifiable dimension would, once more, mean that we just replaced one vague term with another.

The question at hand is: what of combinatory vague terms lacking any dimension allowing linear ordering of objects in their range of significance? Although there can be no numerical linear ordering of the objects in the range of significance of the combinatory vague term based on some quantifiable dimension there is no reason whatsoever to abandon the strategy of handpicking objects to form the ordering. The situation was quite easy with multi-dimensional vague terms since naturally ordered dimensions simplified handpicking. With combinatory vague terms, however, we have to employ our ingenuity.

Whenever we want to formulate any version of the sorites paradox, we need to have the ordering of at least some objects belonging to the range of significance of the vague term. This ordering must fulfil all three conditions (1) to (3). Although it is not appropriate to say that, for example, Christianity is more of a religion than Buddhism, it is completely justifiable to say that some practices definitely count as religions, some are definitely not religious, and some of them we just cannot determinately assign as definitely religious or definitely non-religious. Bueno – Colyvan (2012, 30) offer a rough drawing of how should a handpicked ordering of objects look for the term “religion” – starting with Christianity and ending with schoolyard play. Although “religion” is a good example of the combinatory vague term, it is not the ideal term to illustrate the combinatory sorites paradox with, since it is not exactly easy to imagine what should be filled in between Buddhism and Brazilian soccer.

I consider much easier to imagine the ordering relative to which the term “sport” will be soritical. FIFA World Cup football is most certainly sport, kicking stones on a way home from work is not. Yet it is possible to imagine an ordering connecting these two examples, that would start with FIFA World Cup, UEFA Champions League, English Premier League, and would continue with friendly match between students and teachers at school, football match 7-a-side during PE, and so on all the way to kicking stones on a way home. In each step through the ordering we move slightly closer to kicking stones which is not a sport, yet no two adjacent steps are so different that one would be called sport and the adjacent would not. This way, we can formulate the conditional sorites for combinatory vague terms.

Although we can formulate the conditional version of the sorites paradox for terms like “religion” or “sport”, there is simply no possible way to
formulate the mathematical induction version since objects in their range of significance cannot be numerically ordered. We can, however, formulate the modified line-drawing sorites the same way as we did with multi-dimensional vague terms, yet this will also be a very simplified version.\(^{13}\)

Finally, there is no way for us to formulate the “ultimate” sorites paradox using all objects in the range of significance of any combinatory vague term. The reason is the same as with multi-dimensional vague terms – with every combinatory vague term we can construe multiple soritical orderings that cannot be combined into one ordering containing all the objects in the range of significance of the term in question which would satisfy all three conditions Barnes (1982) described.

4. Conclusion

There are many vague terms and even more sorites paradoxes, for with every vague term we can formulate multiple versions of the sorites paradox. We can also multiply the number of sorites paradoxes construed since for every sorites paradox that proceeds by addition we can formulate the sorites paradox proceeding by subtraction and vice versa, and we can formulate the paradox with the negated vague term. It does not matter whether we start at one end of the given ordering or the other as long as all three conditions described by Barnes (1982) are satisfied. This is true of all sorites paradoxes.

With linear vague terms, we are able to formulate all three basic versions of the sorites paradox – the conditional sorites, the mathematical induction sorites, and the line-drawing sorites. Theoretically it is possible to formulate one “ultimate” sorites paradox using all objects in the range of significance of any linear vague term. In practice, however, we predominantly use only a subset of all objects in the range of significance of the vague term, since many linear vague terms have infinitely many objects in their range of significance. The important thing is that it is possible to formulate such “ultimate” paradox for linear vague terms.

\(^{13}\) Conclusion of such paradox would look something like this: “There are at least two practices that are indistinguishable from each other, yet one of them is a religion and the other one is not.”
With multi-dimensional and combinatory vague terms, on the other hand, there is no way for us to formulate this “ultimate” sorites paradox since we cannot formulate one total linear ordering of all objects in their ranges of significance. We can, however, create many different sorites paradoxes based on handpicked soritical orderings with both multi-dimensional and combinatory vague terms. Though we can only formulate the conditional sorites this way.

Moreover it is impossible to formulate both the mathematical induction sorites and the line-drawing sorites when using multi-dimensional or combinatory vague terms. We either have to reduce these vague terms to linear vague terms – thus replacing one term with another – or we have to formulate versions that lack generality of linear versions of both the mathematical induction sorites and the line-drawing sorites.

Handpicking objects forming ordering relative to which the term is soritical is not limited only to multi-dimensional and combinatory vague terms. Even when we formulate the sorites paradox using linear vague terms we often handpick ordering of objects. Though in these cases our job is simplified by the existence of quantifiable dimension of variation. The advantage of handpicking is that it enables us to formulate the conditional sorites paradox without superfluous stretching the number of conditional premises of the paradox.

The most important thing about handpicking is that it enables us to formulate the conditional sorites paradox for any vague term whatsoever, while the other two forms of the sorites paradox can be properly construed only with linear vague terms but they cannot be construed with multi-dimensional and combinatory vague terms. The conditional sorites thus remains the only version of the sorites paradox that can be formulated with any vague term. In this sense, the conditional sorites is the most general version of the sorites paradox.

References


