The Role of Priors in a Probabilistic Account of “Best Explanation”

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ABSTRACT: In this paper, I argue that the notion of “best explanation”, as it appears in the Inference to the Best Explanation (IBE), can be defined in terms of explanatory power (EP) (i.e. the best explanation among a set of possible explanations is the one having the highest EP), if we employ a probabilistic measure of EP, which takes into account both the likelihoods and the prior probabilities of the compared explanatory hypotheses. Although the association between the EP of a hypothesis and its likelihood is largely uncontroversial, most of those working on EP do not see an association between EP and the prior probability of an explanatory hypothesis. I provide three examples (two toy examples and one from real scientific practice), in order to show that the role of priors in decisions about the best explanatory hypothesis deserves a serious consideration. I also show that such an explication of “best explanation” allows us to compare IBE and Bayesian confirmation theory (BCT) in terms of the probabilities they assign to two competing hypotheses, and thus to elicit the conditions under which both IBE and BCT lead to the same conclusion and are in this sense compatible.

KEYWORDS: Bayesianism – Bayesian confirmation theory (BCT) – Inference to the Best Explanation (IBE) – explanatory power – prior probabilities.

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1. Introduction

The present state of the literature on Inference to the Best Explanation (IBE) reveals two problems of considerable importance. On one hand, there is no clear explication of the term “best explanation”. The main idea driving the need for such an explication is that, in order to “infer to the best explanation” out of a set of competing explanations, we need a clear way of comparing and/or grading the explanations within that set. However, one of the best-known accounts of a mechanism of comparing explanations, i.e. Lipton’s (2004), uses the term “loveliest explanation”, where “loveliest” is an umbrella term for a set of undefined number of explanatory virtues, such as simplicity, unification or scope, most of which also lack clear formal explications (cf. Norton 2016). Another example of the same problem is the classical (or textbook) form of IBE, which is often expressed by the following rule:

Given evidence \( E \) and candidate explanations \( H_1, \ldots, H_n \) of \( E \), infer the truth of that \( H_i \) which best explains \( E \). (Douven 2011)

The above rule fails to answer the crucial question at the heart of IBE – how do we find which is the best explanation of the evidence, out of a set of competing explanatory hypotheses. Its failure in that respect has in fact prompted Schurz to claim that IBE “is epistemically rather uninformative” (Schurz 2008, 204).

On the other hand, there is the issue of IBE’s compatibility with Bayesian Confirmation Theory (BCT). Incompatibilist philosophers of confirmation claim that IBE and BCT are two irreconcilable methods of confirmation, of which only one is rational. Here belong arguments such as van Fraassen’s, who claims that any probabilistic formulation of IBE should either: be equivalent to Bayes’ rule, and is thus redundant; or, if it is not equivalent to Bayes’ rule, it has to provide a satisfactory answer to the Dutch book argument (cf. van Fraassen 1989). Another argument for IBE – BCT incompatibilism is the claim that “confirmation is logically independent of explanation” (Salmon 2001, 88), and so explanatory considerations, such as those driving IBE, should not enter into a method of confirmation, such as BCT. In the end, most incompatibilists’ accounts are skeptical towards the role of IBE as a genuine rule of non-deductive inference (see also Iranzo 2008; Norton 2016).
However, both incompatibilists and compatibilists tend to view the issue of IBE – BCT compatibility as a two-sided matter. Either these approaches to confirmation are deemed completely incompatible – as one is rational and the other is not, or they are deemed completely compatible and assumed to work in tandem. The problem with such views is that there are different possible explications of IBE, and different models of BCT. Some of these may turn out to be compatible, while others may not. What is more, an IBE explication may be compatible with a specific Bayesian model of confirmation only under certain conditions.

Ultimately, the question whether IBE and BCT are compatible or not will depend upon a future investigation into these conditions; and in order to enable such an investigation, IBE and BCT should first be translated into the same language. The key to such a translation is to find an adequate probabilistic explication of the term “the best explanation”. There have been several attempts in the literature to give such explications of “the best explanation”, in the form of measures of explanatory power (EP). However, no direct measure of EP that I know of accounts for the prior probabilities of the explanatory hypotheses. As I show in the next part, insensitivity to priors may lead to very counterintuitive conclusions in certain cases. Therefore, it seems that an adequate probabilistic explication of “the best explanation” should take into account not only likelihoods, but priors as well. Interpreting IBE probabilistically in this way has three distinct advantages. First, it provides a mechanism for actually finding the best explanation. Second, it can account for cases in science, which can be accounted for by neither classical IBE, nor explications of “the best explanation” insensitive to priors. Third, it enables comparisons between IBE and specific Bayesian models of confirmation, in order to investigate the conditions under which these two approaches to confirmation may turn out to be compatible.

2. A probabilistic measure of EP should account for priors

A viable approach to solving the first problem outlined in the introduction – IBE’s lack of mechanism of comparing competing explanations – is

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2 For lists of such measures see e.g., Schupbach (2011) and Glymour (2015).
to seek a probabilistic explication of the key term “best explanation”. In other words, we may strive to provide a probabilistic mechanism to compare competing hypotheses in terms of their EP. The question “which one is the best explanation”, would then receive the answer “the one that has the highest explanatory power according to such-and-such probabilistic measure”. There are a few direct measures of EP in the literature, such as the one by Schupbach & Sprenger (2011):

\[
Ep_1(E, H) = \frac{P(H|E) - P(H|\neg E)}{P(H|E) + P(H|\neg E)}
\]

Crupi & Tentori (2012) have proposed another measure:

\[
Ep_2(E, H) = \begin{cases} 
\frac{P(E|H) - P(E)}{1 - P(E)} & \text{if } P(E|H) \geq P(E) \\
\frac{P(E|H) - P(E)}{P(E)} & \text{if } P(E|H) < P(E)
\end{cases}
\]

In addition, there are several more direct measures of EP, which have been created from different measures of confirmation (see Schupbach 2011).³

So far, all proposed direct measures of EP share a common characteristic – they are not influenced by prior probabilities. For example, the measure of Schupbach & Sprenger (2011) is, at first glance, dependent on posterior probabilities and thus on the priors which form them, yet calculations reveal that this is not the case and the priors actually cancel each other out.

However, there are examples, which seem to show that prior probabilities play a major role in our decisions about the best explanation. Consider the following simple case:

³ There are also some probabilistic measures of unification or coherence, which have been proposed as indirect measures of EP, e.g., Myrvold (2003), Fitelson (2003), Glass (2007), Wheeler (2009). These fall outside the scope of the current argument, which focuses on direct measures of EP.
Waking up in the morning you see that the grass is wet ($E$). You form two hypotheses explaining this fact:

$H_1$: “It rained tonight.”

$H_2$: “The gardener watered the lawn earlier in the morning.”

These hypotheses have the same likelihoods, i.e. $P(E|H_1) = P(E|H_2)$, because both $H_1$ and $H_2$ entail $E$ (the wet lawn). Suppose, however, that you made this observation in July and you live in a place where rains in July are extremely rare. Based on this background knowledge, you assign a very low prior probability to $H_1$. What is more, intuitively, the gardener watering the lawn is a far better explanation of the wet grass, than the extremely improbable rain in July. In order to capture that intuition, a measure of EP should account for prior knowledge. In other words, it should reach the intuitive conclusion, that even though the likelihoods of the two hypotheses are the same, the one with the higher prior better explains the evidence you have observed. The likelihood-only based measures cannot reach that result, and are forced to conclude that $H_1$ and $H_2$ are of equal EP, which is highly unintuitive in that case.

The above example is quite simplistic, so let us push the argument for the importance of priors in EP with a second, more complex example:

Patient X (aged 45) has paresis ($E$). This could be the result of various medical conditions, but for the sake of simplicity, we will take into account only two:

$H_1$: “X had untreated syphilis.”

$H_2$: “X had a stroke.”

By previously consulting X’s medical record, as well as various other sources of medical information, his physician brings into the case the following information:

i) X was diagnosed with syphilis, but not treated for it: $P(H_1) = 0.9$;

ii) About 25% of those who have untreated syphilis get paresis in later age: $P(E|H_1) = 0.25$;

iii) About 80% of stroke survivors get some kind of paresis: $P(E|H_2) = 0.8$;
iv) The physician does not know whether X had a stroke, but she knows that about 0.2% – 0.4% of the population of his age are at a high risk of stroke, i.e. $P(H_2) \approx 0.004$.

Although the likelihood of the stroke hypothesis is greater than the likelihood of the paresis one (i.e. $P(E|H_2) > P(E|H_1)$), most physicians, given the information in i) – iv) would assign higher EP to $H_1$. In other words, most physicians would explain the paresis with the untreated syphilis. What this example aims to illustrate again is that priors may play an important role in some decisions about the best explanation, so much so, that they may overturn a large difference in likelihoods. A probabilistic measure of EP, which is not sensitive to priors, and depends solely on likelihoods, will not be able to account for such cases, i.e., if we applied such a measure to these kind of cases, we would be led to counterintuitive results.

In summary, an adequate probabilistic interpretation of IBE should aim at a probabilistic explication of the key term “best explanation”, thus giving IBE the means to answer the question “how do we find the best amongst competing explanations”. This is a clear advantage over classical IBE, which is silent on this crucial question. The explication of the “best explanation” would be in the form of a probabilistic measure of EP; however, the measure should be influenced by the prior probabilities of the evaluated hypotheses, in contrast to the direct measures of EP proposed in the literature. If the measure does not account for priors, it runs into two kinds of problems. On the one hand, it will provide counterintuitive results in certain cases, as illustrated by the above toy-examples. On the other hand, it will fail to account for real cases in science, as will be shown in the next part.

Providing and defending the prescribed new measure of EP are aims for future research, which fall outside the scope of the current paper. The focus here is on arguing for the important role of priors in a probabilistic explication of the “best explanation”. We will now turn towards a third example for their importance, this time from real scientific practice.
3. The role of priors in scientific practice:  
the case of Planet Nine

In January 2016, two Caltech astronomers – Konstantin Batygin and Michael Brown – inferred the existence of a still unobserved planet in the outer Solar System (cf. Batygin & Brown 2016). This conjecture was made in order to provide the best explanation of certain peculiarities in the otherwise stable orbits of a set of trans-Neptunian objects. It was observed that six objects in the scattered disk of the Kuiper Belt (Kuiper Belt Objects or KBOs), which had perihelia greater than the orbit of Neptune, and semi-major axes greater than 150 AU \((a > 150 \text{ AU})\), exhibited a strange clustering of their arguments of perihelion \((\omega \sim 0)\). In other words, the perihelion of every one of these objects lied on the ecliptic, and their ascending nodes coincided with their perihelia, i.e. they all shared the same orbital direction – from south to north. Batygin and Brown calculated that orbits with \(a > 50 \text{ AU}\), clustered this tight, occur only 0.007% of the time, which means a probability of only 0.00007 that the clustering is due to chance. They stated that, considering the age of our Solar System, such groupings are expected to randomize, unless held together by some physical mechanism.

At that point in 2016, the above peculiarities in the six KBOs’ orbits have already been noted, and there were three contending explanatory hypotheses. The first one was proposed by Trujillo & Sheppard (2014). They concluded that the observed perihelia, which librated around \(\omega = 0\), might be held by a massive body on an outer orbit, about five times the mass of Earth. The second explanatory hypothesis was that the observed phenomenon was due to a self-gravitational instability of the scattered disk population of the Kuiper Belt (see Madigan & McCourt 2015). The third one was Batygin and Brown’s own model, predicting the existence of an unobserved planed. Batygin and Brown also systematically criticized the other two explanations.

As for the first one, the mechanism employed to explain the data in Trujillo & Sheppard (2014) had certain assumptions that would require the existence of several massive bodies, on orbits exactly tailored in order to explain the peculiarities in the orbits of the six KBOs. Furthermore, the same mechanism could not explain by itself why we observe objects clustered at \(\omega \sim 0\), but there is no such observed clustering in \(\omega \sim 180\) (see
Batygin & Brown 2016). This explanation required the assumption that our Solar System had a strong stellar encounter in the past – an assumption that does not fit with the rest of our knowledge about the Solar System. All of these assumptions reduce the quality of Trujillo & Shepard’s (2014) explanation of the KBOs’ orbits, making it more ad hoc.

As for the second explanation by Madigan and McCourt (2015), which employed a so called “inclination instability”, it assumed the scattered disk of the Kuiper Belt was once much more massive than current estimations, and stayed that way for a prolonged period of time, or it could not have provided enough gravity for the proposed mechanism of instability. Not only do we lack sufficient evidence for such an assumption, but also it is highly unlikely for theoretical reasons. Most of the mass that the disk might have contained in the past was most probably ejected from the Solar System due to interactions with the gas and ice giants. As Batygin & Brown (2016) noted, such interactions usually end up in hyperbolic trajectories for the less massive objects.

The best available explanation of the clustering of the six KBOs is that there is a massive body of about or above ten Earth masses, with a semimajor axis $a \sim 700$ AU and an estimated perihelion of about 200 AU, and an aphelion of about 1200 AU, which has eluded observation so far (Batygin & Brown 2016). It is speculated that this “perturber” of the orbits of the KBOs would likely be an ice giant, formed by an ejected giant planet core during the early phases of development of our Solar System. It probably has an orbital period in the range of 10 to 20 thousand years, and most of the time it is too far from Earth to be observed without very high-resolution equipment, which would explain why it has not yet been discovered. If its existence is confirmed by observation, the planet will receive an official name, but in the meantime, it has been called “Planet Nine”. Batygin & Brown (2016) point out that their explanation of the clustering of the six KBOs by the existence of Planet Nine also has implications about other features of the Kuiper Belt, which not only increase the scope of their explanation, but also provide “a direct avenue for falsification of our hypothesis” (Batygin & Brown 2016, 2).

In summary, we have three competing explanatory hypotheses, all of which entail the evidence, i.e., the observed clustering of KBOs. These are: Trujillo & Shepherd’s (2014) hypothesis, whose model requires the existence of several undiscovered massive bodies; Madigan & McCourt’s
hypothesis, which presupposes that the scattered disk of the Kuiper Belt was much more massive and for a longer period of time, than current estimations indicate; and Batygin & Brown’s (2016) hypothesis, which presupposes the existence of Planet Nine. When interpreting IBE probabilistically, if we explicate the “best explanation” through any of the measures of EP sensitive only to the likelihoods, we would be forced to the conclusion that the above three hypotheses are equally good explanations of the observed evidence. This conclusion, however, will go against the expert opinion of those astrophysicists who believe that Batygin and Brown’s hypothesis is the best available explanation (e.g. see opinions by Rodney Gomes, quoted in Lovett 2012, and by Alessandro Morbidelli, quoted in Achenbach & Feltman 2016). If we include in our explication of the “best explanation” the differences in prior probabilities of the competing hypotheses, then this controversy is resolved. Trujillo and Shepherd’s hypothesis requires the existence of several massive bodies, whereas Batygin and Brown’s hypothesis requires just one. According to the rules of classical probability, the probability for the existence of a single massive body would always be higher than the combined probabilities for the existence of several massive bodies. Madigan and McCourt’s hypothesis requires that the Kuiper Belt was once much more massive, and for a longer period, than current estimations indicate. Furthermore, it is unlikely for theoretical reasons – most of that mass would have been quickly ejected out of the Solar System due to planetary encounters. Ceteris paribus, a hypothesis, which is not in agreement with current estimations, and is unlikely from a theoretical point of view on top of that, cannot receive a higher prior than a hypothesis that does not run into such problems. Based on these considerations, Batygin and Brown’s hypothesis seems to have the highest prior probability of the three competing explanations thus far. If we include that consideration in our decision about which one of them is the best explanation of the evidence, we would reach a conclusion in accordance with the opinions of the experts. In other words, if our decision about the “best explanation” takes the prior probabilities of the assessed hypotheses in consideration, then it could adequately account for scientific cases, such as the one with Planet Nine.
4. A method for exploring the conditions of IBE – BCT compatibility

Another advantage of the described probabilistic explication of “best explanation” is that it makes investigating the conditions under which IBE and BCT are compatible relatively straightforward. As was already mentioned in the Introduction, the problem of IBE – BCT compatibility depends on the particular explication of IBE and the particular Bayesian model of confirmation we want to compare. Furthermore, an IBE explication and a Bayesian model may turn out to be compatible only if certain conditions hold. How do we know which conditions should hold? We may find that out through a method of comparing inequalities, and we will consider an example demonstrating how the method works.

For the sake of argument, let us take as a probabilistic explication of “best explanation” the following simple measure of EP:

\[ E_E(E,H) = P(E|H) \times P(H) \]

In other words, let us assume that the best explanation, out of a set of competing explanatory hypotheses, is the one that has the highest value of \( E_E \). We interpret IBE to mean that this hypothesis is also the most confirmed one.

This specific measure of EP was chosen because it cannot have values above 1 or below 0, which would be hard to interpret and would violate the axioms of classical probability. It also accounts for prior probabilities, as has already been argued. Nevertheless, it is introduced for the purposes of this example and should not be taken as a proposal to measure EP in real life.

Using the above measure, we will define the EP of two competing explanatory hypotheses \( H_1 \) and \( H_2 \), both of which explain some empirical evidence \( E \):

\[ E_E(E,H_1) = P(E|H_1) \times P(H_1) \]

\[ E_E(E,H_2) = P(E|H_2) \times P(H_2) \]

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4 By “compatibility”, I will understand the ability to provide equal results, when applied to the same case. Although there could be other meanings of the term, these will not be addressed here.
We would like to know under which conditions our interpretation of IBE may turn out to be compatible, in the sense of providing compatible results, with the measure of confirmation proposed by Eells (1982) and defended by Jeffrey (1992):

\[ C(E, H) = P(H|E) - P(H) \]

In other words, according to Eells and Jeffrey, confirmation is an increasing function of the difference between posterior and prior probabilities. We will define the measures of confirmation of our two hypotheses as:

\[ C(E, H_1) = P(H_1|E) - P(H_1) \]
\[ C(E, H_2) = P(H_2|E) - P(H_2) \]

We start comparing the two methods by exploring the scenario in which \( H_1 \) is better confirmed by \( E \) than \( H_2 \). According to our interpretation of IBE, \( H_1 \) is better confirmed than \( H_2 \) if the following condition is satisfied:

\[ E(E, H_1) > E(E, H_2) \]
\[ P(E|H_1) \times P(H_1) > P(E|H_2) \times P(H_2) \]

And according to our chosen Bayesian measure of confirmation, \( H_1 \) is better confirmed than \( H_2 \) if:

\[ C(E, H_1) > C(E, H_2) \]
\[ P(H_1|E) - P(H_1) > P(H_2|E) - P(H_2) \]

We may immediately notice that, as both \( H_1 \) and \( H_2 \) explain the same evidence, we may transform (8) by dividing both sides of the inequality by \( P(E) \), assuming \( P(E) > 0 \):

\[ \frac{P(E|H_1) \times P(H_1)}{P(E)} > \frac{P(E|H_2) \times P(H_1)}{P(E)} \]

\[ P(H_1|E) > P(H_2|E) \]

We may transform (10) into:

\[ P(H_1|E) - P(H_2|E) > P(H_1) - P(H_2) \]
After which we may transform (12) into:

\[ (14) \quad P(H_1|E) - P(H_2|E) > 0 \]

Now, if we assume that:

\[ (15) \quad P(H_1) - P(H_2) \geq 0 \]

From (13) and (15) we can infer:

\[ (16) \quad P(H_1|E) - P(H_2|E) > 0 \]

As (14) and (16) are obviously equivalent, we may argue that (13) and (14) are also equivalent, given that (15) is satisfied. In other words, both IBE and BCT would conclude that \( H_1 \) is better confirmed by \( E \) than \( H_2 \) if \( P(H_1) \geq P(H_2) \), and that when IBE and BCT lead to different predictions, this is due to a violation of condition (15).

Now that we know the above condition, one way to proceed would be to rationalize why it should hold. However, if the result is deemed indefensible or strongly counterintuitive, another way to proceed is to seek a different measure of explanatory power, or a different Bayesian measure of confirmation, and employ the method again to find if they are compatible and under what conditions.

The bottom line is that investigating the conditions of IBE – BCT compatibility, by employing the method presented above, lends itself naturally to an interpretation of IBE, which uses a probabilistic explication of “best explanation” (as outlined in section 2). This is an advantage of the probabilistic interpretation of IBE over classical IBE, as it allows us to explore the issue of IBE – BCT compatibility in much higher detail (i.e., on a model-by-model basis), rather than just announcing them completely compatible or incompatible.

5. Conclusions

A probabilistic interpretation of IBE should aim at providing an adequate probabilistic explication of the term “best explanation” – in this way it would be able to complete IBE with a formal mechanism for finding the best out of a set of explanations. An adequate probabilistic explication of
the “best explanation” should be a measure of explanatory power influenced by the prior probabilities of the explanatory hypotheses. There are cases, which show that priors have a key role in decisions about the best explanation, and that measures, which are sensitive only to the likelihoods of the assessed hypotheses, would lead to counterintuitive conclusions, when applied to these cases.

Such a probabilistic interpretation of IBE has three distinct advantages. The first one is the above-mentioned formal mechanism for finding the best explanation, whereas classical IBE lacks such a mechanism. The second advantage is that it can account for cases in science as the one with Planet Nine. Interpretations of IBE, which use probabilistic measures of explanatory power that are not influenced by priors, fail to account for such cases. They would consider all competing hypotheses as equally good explanations of the evidence, against the experts’ better judgment, whereas a priors-sensitive measure would be able to explain why one of the hypotheses is considered a superior explanation.

The third advantage is that such a probabilistic interpretation of IBE allows for investigation of the particular conditions, under which specific explications of IBE and specific Bayesian models of confirmation give compatible results. The issue of IBE – BCT compatibility is more nuanced than outright compatibility or incompatibility: there are different explications of IBE and different models of BCT. Some of these may turn out to be compatible, but only under certain conditions. In order to resolve the issue, these conditions will have to be explored in future research, which may be done in a straightforward way by employing the method presented in the previous part.

There are also several other topics, which remain open for further research. The main one is to provide a probabilistic measure of EP that accounts for priors and test it against the already proposed measures of EP. There is also a decision to be made whether competing hypotheses should have EP above a certain threshold, in order to be considered “good enough” in Lipton’s term, and avoid van Fraassen’s (1989) “argument from a bad lot”. Last but not least, introducing priors in EP may give rise to objections against making EP “subjective”, similar to the objections against BCT. These objections will have to be addressed, and one way to do it is to argue that priors may be formed by considerations about simplicity, unification, scope and other explanatory virtues.
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References


