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Logic and Rational Requirements

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ABSTRACT: In this paper, I discuss the relation between logic and rationality. I develop (formally and conceptually) a rational requirement which can respond to the classic objections by Harman (1986). On the one hand, the requirement pays attention to the *relevance* of the premises and the conclusion, which is formally expressed by the notion of weak relative closure. The requirement also takes care of the *complexity* of the inferences. This notion of complexity is formally represented by a partially ordered scale of the difficulty of inferences, which is weaker than the notion of complexity as number of steps.

KEYWORDS: Logic and rationality – normativity of logic – rational requirements.

1. Introduction

In this paper, I discuss the relation between logic and rationality. The notion of rationality is too complex, so I will just focus on some aspects which are relevant to the discussion. I am interested, following Brome (1999) and MacFarlane (2004), on developing a specific *rational requirement* for logic. Rational requirements are statements which express what rationality asks from us with respect to a certain epistemic or practical issue.

Many authors, such as Broome (1999), Kolodny (2005), Way (2010), and Shpall (2013), discussed which was the best way of expressing rational requirements. One of the main issues that this discussion introduced is the difference between a wide scope and a narrow scope for the rationality operator ("rationality requires that …").[1] Taking R as a rationality operator, the narrow scope principles have the form $A \rightarrow R\mathrm{B}$, while the wide scope principles have the form $R(A \rightarrow B)$. For example, many authors have discussed the following rational requirements (where "WS" means Wide Scope and "NS" means Narrow Scope):

(NS Evidence) If you believe that there is conclusive evidence that *p*, then rationality requires you to believe *p*.

(WS Evidence) Rationality requires that (you do not believe that there is conclusive evidence that *p*, or you believe *p*)—cf. Kolodny (2005, 521).

(NS Enkrasia) If you believe that you ought to do *F*, and you believe that you can do *F*, then rationality requires you to intend to do *F*.

(WS Enkrasia) Rationality requires that (either you don't believe that you ought to do *F*, or you don't believe that you can do *F*, or you intend to do *F*)—cf. Broome (2014, 171).

Here I will not focus on the precise formulation of each non-logical rational requirement. I will take for granted that some of these pragmatic or epistemic requirements are indeed true. In this paper, I will focus on the following two possible requirements of logical rationality. The first one has narrow scope, while the second one has wide scope:

(NS Validity) If $\Gamma \vDash \mathrm{A}$, then if you believe Γ, rationality requires you to believe A.

(WS Validity) If $\Gamma \vDash \mathrm{A}$, then rationality requires that (you do not believe some sentence of Γ, or you believe A).[2]

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1 Strictly speaking, most authors in this discussion use deontic operators such as "should …" or "has reasons to …". Following Broome (1999; 2014), I prefer to use rationality operator, and then discuss whether the requirements can be read as duties or reasons. On the other hand, a rationality *predicate* could also be used instead of an operator, but it does not give more clarity to the discussion (for example, we would need to add sentence names, etc.).

2 As two anonymous referees observed, some authors such as MacFarlane have argued for this negative requirement: if Γ implies A, then rationality forbids you to believe

The aim of this paper is to develop a new requirement for logical rationality, which will be based on WS Validity. But before going into this, it is convenient to say some words about another discussion, which is sometimes taken as more fundamental: should we be rational; or in other words, is rationality normative?

There are different arguments in favor and against the idea that rationality is necessarily normative. In general, the arguments in favor can be Kantian or utilitarian. Kantians consider that rationality is a fundamental aspect of the human being, and as such, it is certainly normative (cf. Southwood 2008). Utilitarians claim that following rational requirements leads us to taking better decisions or believing true propositions. For example, Joyce (1998) appeals to accuracy arguments to justify the rational requirement of epistemic coherence.[3]

Arguments against the normativity of rationality usually point out two things. First, that rationality can lead us to taking wrong decisions or believing false propositions (see Kolodny 2005). Second, that in cases in which rationality takes us "closer" to the right action or the true belief, it is superfluous, since it can be subsumed under other requirements. For example, the rational requirement of epistemic coherence can be subsumed under the evidential norm of believing what the evidence suggests (cf. Kolodny 2008).

Γ and $\neg A$. Under some plausible assumptions (such as Explosion), WS Validity gives a similar result: it forbids you to believe $\Gamma \cup \{\neg A\}$ and disbelieve something else (i.e. there is no rational way of believing $\Gamma \cup \{\neg A\}$ without going fully trivial). If we add a requirement of non-triviality, then WS Validity implies MacFarlane's negative requirement.

Now, why is WS Validity better than MacFarlane's negative requirement? Because the negative requirement is just equivalent to a consistency requirement. But there is something more to tell about logical rationality: if someone believes that Canada is a country, and that every country has a capital city, but does not yet believe that Canada has a capital city, there is *something wrong* with the belief set of this person. Ignoring the obvious consequences of your beliefs is something to be criticized. WS Validity (unlike MacFarlane's negative requirement) can point out this kind of mistake.

[3] Joyce (1998) shows that an incoherent probability distribution (i.e. one which does not correspond to the probability calculus) is necessarily "dominated" by a coherent distribution. This means that the coherent one will be closer to the truth in every possible world.

The word "rationality", and similarly the word "normativity", have been used to name different things. This is why the discussion on the normativity of rationality is often confusing. To be clear, I can specify what I mean when I say "normativity of rationality":

(Normativity of rationality)
If rationality requires you to do *F*, then you *ought* to do *F*.[4]

In this paper, I will not take a stance on the normativity of rationality. I will be interested, mainly, in formulating a requirement of logical rationality. Moreover, given my suspension of judgment on the normativity of rationality, I will offer logical rational requirements that are *compatible* with the possible normativity of rationality. In other words, I will provide some requirements such that, if rationality requires you to do *F*, assuming that you *ought* to do *F* does not lead to inconsistency.[5]

2. Scope and normativity

In this section I will explore the problem of the scope of logical rational requirements. In particular, I will mention the Bootstrapping objection (which affects the narrow scope requirements) and discuss which kind of normative force corresponds to a logical rational requirement.

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[4] A referee observed that this definition does not clarify the concept of normativity. Admittedly, it is not a proper analysis of normativity, but rather a semantic clarification. Normativity is understood in many different ways across the literature; for example, it can be applied to meaning or content (cf. Boghossian 2003). Even the normativity of rationality can be understood in more inflationary ways (see Southwood 2008). The identification between "normativity" and "ought" is usual but not trivial; thus, the semantic clarification could be useful for some readers.

[5] This methodology was also adopted by Broome (2014, chap. 11). A referee observed that this definition makes the requirements incompatible with the *non-normativity* of rationality. This is true in one respect: the requirements cannot logically imply the non-normativity of rationality. If this were the case, they would be *a priori* incompatible with the normativity of rationality. According to the methodological principle I adopted, rationality could be normative or non-normative, but this should not be a logical consequence of rational requirements.

2.1. Bootstrapping

The Bootstrapping objection has frequently been raised against the normativity of rationality, although it affects mainly the narrow scope formulations. This problem can be expressed in this way, schematically:

> (Bootstrapping)
> Suppose that the requirement *r* of rationality has narrow scope, i.e. it has the form "If you have the attitude X, then rationality requires you to have the attitude Y". Suppose that you ought not to have the attitude Y. Now, in case you have the attitude X, rationality requires you to have the attitude Y anyway. If rationality is normative, then in this case you *ought* to have the attitude Y, which by hypothesis we assumed you ought *not* to have.

Until now, the setup was rather abstract. But we can illustrate the problem with some clear examples:

> (Bootstrapping for NS Enkrasia)
> For unjustified reasons, you believe that you ought to kill your son (and you believe you can do it). Therefore, if rationality is normative and NS Enkrasia holds, you ought to intend to kill your son. But obviously you ought not to do it.
>
> (Bootstrapping for NS Evidence)
> For unjustified reasons, you believe that there is conclusive evidence that the world is squared. Therefore, if NS Evidence holds and rationality is normative, you ought to believe that the world is squared. But obviously you ought not to believe it.
>
> (Bootstrapping for NS Validity)
> For unjustified reasons, you believe *p*. Given that logic is reflexive (i.e. *p* implies *p*), if NS Validity is normative, you ought to believe *p*. The same can be reproduced for any of your beliefs: you ought to believe everything you believe. But this is absurd.

Most authors in this discussion consider that Boostrapping rules out the normativity of narrow scope requirements.[6] Fortunately, the *wide scope* version

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[6] For reasons of space, I will not discuss the positions which defend the narrow scope requirements against the Bootstrapping objection. See Schroeder (2009).

of those requirements is immune to the Bootstrapping objection. In those cases, rationality gives the option of having or lacking some attitudes, but it does not require adopting a specific attitude. For example, WS Enkrasia requires you not to believe that you ought to do *F*, or not to believe that you can do *F*, or to intend to do *F*.

The same holds for WS Validity: it just requires that, if Γ implies A, you do not believe some sentences of Γ, or you believe A. Moreover, WS Validity can respond to one objection by Harman (1986, 11). Harman observed that, even though A and A → B imply B, sometimes we believe A and A → B but we ought not to believe B (for example, when B is false). WS Validity is not affected by this problem, for it does not require believing B in this case, but it gives the option of revising the belief in the premises.

Therefore, it is promising to adopt the Wide Scope rational requirements, which may provide *duties*. In other words, there might be duties of complying with disjunctive requirements such as WS Validity, WS Enkrasia, and WS Evidence, among others. In what follows, we will focus on these wide scoped rational requirements, for they are compatible with the normativity of rationality.

2.2. Strict normativity?

In the last paragraph, we argued that rationality could be normative. But can this normativity be *strict*? I will claim that no specific requirement (including the logical requirements) can be strictly normative. Reisner (2011) developed some mental experiments in order to prove this point. This is the clearest one:

> (Reisner case)
> Suppose that a millionaire makes the following bet with you: he gives you billions of dollars in case you believe *p* and you don't believe (*p* or *q*). With that money, you could and would feed all the hungry people in the world.

According to Reisner (and I share his intuitions), in this case you ought *all things considered* to adopt a belief set which includes *p* but not (*p* or *q*). As Reisner (2011, 41) claims, "it would be quite hard to explain how it is that saving all the starving people in the world does not have deontic or normative priority over violating a principle of rationality". Therefore, you ought to violate WS Validity.

This shows that the normativity of WS Validity cannot be strict, but weak or defeasible. The duty of being logically rational (if there is such a duty) can be defeated by a different duty. MacFarlane (2004) was the first one to hold this idea. He exemplified this problem with the Preface Paradox (cf. Makinson 1965), in which a person can have an inconsistent but rational belief set. According to MacFarlane, this is a case in which a logical rational requirement conflicts with a more global epistemic requirement, and the last one dominates. Here I will adopt the same approach with respect to these conflicting cases: these cases do not show the absence of normative force in logical rational requirements, but their defeasible nature. It is worth remarking that, even admitting the defeasible normativity of logical rationality, it is still better to adopt a Wide Scope requirement than a Narrow Scope one. Given the bootstrapping problem, a Narrow Scope requirement would be defeated in every case in which I have a false belief; on the contrary, a Wide Scope requirement only fails in very specific cases such as the Preface Paradox or the imaginary Reisner cases.

Finally, even though I reject the possibility of a rational requirement with strict normativity, I admit the possibility that rationality, taken as a global property, could be strictly normative. If rationality is taken as a property which emerges from the fulfillment of different requirements (epistemic or practical), the strict normativity of this "global" rationality cannot be so easily ruled out.

3. Relevance

Until now, I have argued for a wide scope requirement, and I claimed it could possess a defeasible normativity. But the wide scope requirement I advocated for, WS Validity, is still affected by many problems.

The first one was described by Harman (1986, 12). Harman observes that we may intuitively ignore some *irrelevant* consequences of our beliefs. According to WS Validity, it is irrational to believe "it rains" and not to believe "it rains or $2 + 2 = 6$", "it rains or it is Tuesday", "it rains or $2 + 3 = 4$", and many other completely irrelevant sentences. But, according to Harman, this attitude is rational, since believing *all* the consequences of your beliefs would make you lose time, energy and mental space in many strange, trivial or irrelevant beliefs.

My way of solving the problem of irrelevant consequences is to add a clause to WS Validity, which specifies that the premises and the conclusion must be contextually relevant. As we will see, my specification is similar (but different) to the proposals by Broome (2014, 157) and Steinberger (2015, 25).

Broome specifies that the rational requirement holds whenever the agent "cares about the conclusion" (Broome 2014, 157). A paraphrase of his position is the following:[7]

(WS Validity – Broome)
If Γ implies A, and you care whether A, then rationality requires you not to believe some sentence of Γ, or to believe A.

This principle holds in most cases, but it is affected by some problems. One of them is the exaggerated subjectivity of the notion of *care*. Suppose that I hear the fire alarms, and I know that if the fire alarms sound, then the house is burning. But still, my belief set does not include "the house is burning", since I don't care about this in this particular moment (suppose I am writing a difficult article on logic, and all my attention is focused on that). Intuitively, my attitude is irrational, but Broome's notion takes it as rational, for in that case I don't *care* about the conclusion.

Steinberger (2015, 25) solves this point, for he changes the specification and he introduces the idea of "having reasons to consider the conclusion". He suggests the following requirement:[8]

(WS Validity – Steinberger)
If $\Gamma \vDash A$, and you *consider* or *have reasons to consider* A, then rationality requires you not to believe some sentence of Γ, or to believe A.

Here, he makes room for a disjunctive notion between a subjective aspect (to actually consider) and an objective aspect (to have reasons to consider). In the

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[7] Broome applies this idea to WS Modus Ponens, not to WS Validity. However, his considerations about relevance do not depend on that.

[8] This is not exactly Steinberger's formulation. His requirement also includes, as we will see, the fact that the agent believes that the inference is valid. However, for the purpose of this section, I ignore that aspect of the requirement (I leave it for the next section).

previous example, the agent does not consider the conclusion, but clearly has reasons to consider it.

Anyway, Steinberger's proposal is still affected by a problem, which is the emphasis on the *conclusion*. This is unreasonable. Suppose that my belief set includes a remote and complicated inconsistent set. For example, I believe in the axioms of naïve set theory, which I learned at primary school. But in the context, the discussion is focused on something completely unrelated, say, the size of the countries. In that context, I consider the proposition "Spain is larger than France", though I reject it. To put it simpler, my belief set is *Naïve set theory* ∪ {Spain is *not* larger than France}. According to Steinberger's notion, logical rationality does not permit me to be in that state, given that my remote inconsistent beliefs also imply "Spain is larger than France" (by Explosion).[9] In other words, given that I believe the axioms of Naïve set theory (which is inconsistent), and I consider "Spain is larger than France", I must also believe that sentence. This is not completely unjustified (after all, logical rationality does not permit me to have inconsistent beliefs), but it is clearly inadequate if the *relevance* of logical requirements is taken into account. For the inconsistent set I remotely believe is absolutely irrelevant in the context.

My position makes a modification to solve this problem, where both premises and conclusion must be relevant in the context. In other words, I will adopt Steinberger's notion of relevance, but also extended for the premises:

(WS Validity + Relevance)
If $\Gamma \vDash A$, and Γ and A are relevant in the context, then rationality requires you not to believe some sentence of Γ, or to believe A.

Following Steinberger, I will define "relevance in the context" in the following disjunctive way:

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9 As a referee observed, it is possible to avoid this problem by rejecting Explosion and adopting a paraconsistent logic. However, moving towards a paraconsistent logic such as *LP* or *FDE* has very strong consequences: it means rejecting the rational force of very intuitive rules such as disjunctive syllogism or modus ponens. I also think that my description of the situation is more accurate; the problem is not the background logic but the excessive demands of the ideal rational requirements.

(Relevance)
In the context *c*, the sentence *p* is relevant for agent *i* iff *i* considers or has reasons to consider *p*.[10]

In the fire alarms example, my beliefs are {the fire alarms are sounding; if the fire alarms sound, the house is burning}, and I have reasons to consider the belief {the house is burning}. Then, according to my criterion, the set of relevant propositions in the context is the union of these sets, say: {the fire alarms are sounding; if the fire alarms sound, the house is burning; the house is burning}. In this case I am violating WS Validity+Relevance: my belief set is not closed relatively to the set of relevant propositions.

Instead, in the case of naïve set theory as irrelevant belief, my belief set is *Naïve set theory* ∪ {France is not larger than Spain}, and it complies with WS Validity+Relevance. For, even though "France is larger than Spain" is relevant and can be deduced from my belief set, it cannot be deduced from my set of *relevant* beliefs. It is worth mentioning that the axioms of naïve set theory are irrelevant in this context for, even though I believe them, I am not considering them and I don't have any reason to do it.

3.1. Relevance: a formal approach

In what follows, I will formally develop the notion of relevance that I introduced in the last paragraph. In order to do it, I will use the concept of a consequence operator, which is widely used in non-classical logics and belief revision theories.[11]

In the literature on belief revision, it is usual to presuppose that the belief set is closed under logical consequence. Formally, there is an operator *Cn* which takes a set of sentences and gives as output the set of its logical consequences. In other words, $Cn(X) = \{A \mid X \vDash A\}$. A consequence operator is *Tarskian* iff it satisfies these three conditions:

(Inclusion) If $a \in X$, then $a \in Cn(X)$

10 The reader can notice that I introduced the notion of *relevance* in the requirement, and then I defined it. Strictly speaking, I could have introduced the defined notion from the beginning. I presented the requirement in this way for simplicity.

11 See Hansson (1999) for a complete introduction to belief revision theories, and Wójcicki (1988) for a classic monograph on consequence operators.

(Idempotence) $Cn(X) = Cn(Cn(X))$
(Monotony) If $X \subseteq Y$, then $Cn(X) \subseteq Cn(Y)$

The most popular logics (classical, intuitionistic, relevant, etc.) can be represented with a Tarskian operator, for they are structural (i.e. they satisfy monotony, reflexivity and cut). Belief revision theories usually take the belief set X to be closed under consequence, i.e. $Cn(X) = X$.

However, we have seen that it is exaggerated to ask a real individual to have a closed belief set. A non-closed belief set can be adequate, when the consequences of the beliefs which are not included in the set, or the sentences which work as premises, are irrelevant. Now I will try to give a formal characterization of these conceptual aspects of relevance.

Relative closure

In order to formally characterize the notion of relevance, I have to start from the definition of a *context*. As I said before, the evaluation of a belief set takes place in a context. The set Δ of relevant propositions is the set of propositions which the agent considers or has reasons to consider in a particular context.

By now, the only restriction on Δ is the following:

(Closure under negation)
If $A \in \Delta$, then $\neg A \in \Delta$

This cannot be so problematic. If a sentence is relevant in a context, its negation must also be relevant.[12] In general we will make a simplification to avoid Δ being necessarily infinite: we will allow the cancellation of double negations. So, if A and $\neg A$ belong to Δ, then $\neg\neg A$ may not be in Δ.[13] We will use the symbol $\pm$ to simplify, where $\pm\Gamma = \Gamma \cup \{\neg\gamma \mid \gamma \in \Gamma\}$.

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[12] This condition is similar to the closure under negation in judgement aggregation. See List (2012) for an introduction to this area of research.

[13] This property is adequate in cases where A is equivalent to $\neg\neg A$, such as classical logic, K3, LP, FDE, etc. It may be not entirely adequate for intuitionistic logicians, for they do not regard A and $\neg\neg A$ as equivalent. However, this simplification may be dropped for philosophical reasons and all the essential features of the proposal would remain the same.

Another restriction over the contexts that could be adopted is taking every set Δ to be closed under subformulas:

(Closure under subformulas)
If A ∈ Δ, and B is a subformula of A, then B ∈ Δ.

This is fairly intuitive too. If "it rains and it is Wednesday" is relevant, then "it rains" and "it is Wednesday" are relevant. The same should apply to the other connectives.

I will use a notion from belief revision theory (see Hansson 1999, 32), which is the concept of relative closure. A set Γ is closed relative to Δ iff Γ contains all the consequences of Γ that also belong to Δ. Formally:

(Relative closure)
A set X is closed relative to a set Δ iff $Cn(X) \cap \Delta \subseteq X$.

For example, the set $\{p\}$ is not logically closed relative to $\{p, q, q \rightarrow p\}$, for $\{p\}$ does not include the sentence $q \rightarrow p$, which can be inferred from $\{p\}$. Instead, the set $\{p\}$ is closed relative to $\{p, q, q \vee r\}$, for $\{p\}$ includes all the consequences of $\{p\}$ that are included in $\{p, q, q \vee r\}$ (i.e it includes itself).

Let's see how this concept can be applied to more concrete cases. Suppose that the context in question is a football match, and the agent has the following belief set:

Γ = {Messi will score a goal;
If Messi scores a goal, Barcelona will win}

In the context *c* of a football match (and of course, with a considerable amount of simplification), suppose that the set of relevant propositions is the following:

Δ = ±{Messi will score a goal;
If Messi scores a goal, Barcelona will win;
Barcelona will win;
Neymar is playing with number 5}

Here, the belief set Γ has a clear shortcoming: it includes a set of beliefs which are relevant in the context, but it does not include one relevant consequence of these beliefs ("Barcelona will win"). Formally we can say that the set Γ of the example is not closed relative to Δ.

Let's compare Γ with the following set:

Γ* = {Messi will score a goal;
If Messi scores a goal, Barcelona will win;
Barcelona will win}

This new set, unlike Γ, is closed relative to Δ. The same would happen to the following set:

Γ** = {Messi will score a goal}

Even though Γ** includes fewer elements than Γ, it is closed relative to Δ. This shows that the way of reaching a closed belief set is not necessarily to accumulate beliefs, but also to abandon beliefs when it is necessary.[14]

Weak relative closure

The notion of relative closure is much more realistic than the ideal notion of closure. However, it still has a shortcoming (that we mentioned in the previous part, from a conceptual point of view). Suppose that our belief set is inconsistent with respect to a completely irrelevant topic. Just to follow with the previous example, take the set of relevant propositions as Δ, but now the belief set is:

Γ′ = *Naïve set Theory* ∪ {Messi will score a goal}

Intuitively, this set should be taken as contextually adequate. Even though it includes an inconsistent belief, the inconsistency is not relevant in the context (since we are not considering it, and we have no reasons to do it in the context). With respect to the relevant propositions, Γ′ is actually closed.

However, following the previous definition of a relatively closed set, the set Γ′ is not closed relative to Δ, for "Neymar plays with the number 5" (and any other sentence in Δ) can be inferred from Γ′ by Explosion; but Γ′ does not contain that sentence. A way of solving this problem is to adopt a weaker notion of relative closure:[15]

[14] The concept of relative closure, like the concept of closure, is synchronic. In other words, it does not guide the processes, but it evaluates states.

[15] It is worth mentioning that there are many ways of solving this problem. Restall & Slaney (1995) use a background paraconsistent logic, so that an inconsistent belief does not make the set trivial. However, this proposal pays high costs: the paraconsistent logic

(Weak relative closure)
A set X is weakly closed relative to a set Δ iff $Cn(X \cap \Delta) \cap \Delta \subseteq X$.

Indeed, the set Γ' is not closed relative to Δ, but it is weakly closed relative to Δ. For even though Γ' does not include all the relevant consequences of its members, it does contain all the relevant consequences of its *relevant* members. This notion of closure is stronger than the previous one, and allows us to formally define a rational requirement which pays attention to the contextual relevance. Indeed, we can translate WS Validity+Relevance as the following requirement:

(WS Validity+Relevance – Formal)
Rationality requires your belief set to be contextually adequate; i.e. when the contextually relevant propositions are Δ, your belief set X must be such that $Cn(X \cap \Delta) \cap \Delta \subseteq X$.

This requirement asks the set to include the relevant consequences of those propositions that were initially relevant. This avoids the intermission of irrelevant beliefs that might imply other relevant propositions. In a nutshell, I argued that introducing the notion of weak relative closure in the logical rational requirement can respond adequately to Harman's objection.

4. Excessive demands

4.1. The objection and the first answers

The second important objection against WS Validity was also expressed by Harman (1986, 17):

(Excessive demands)
It is rational to ignore the least obvious consequences of our beliefs. For example, one can believe the Peano axioms and not believe some of its consequences, without being irrational.

This objection has received many answers.

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they use (*FDE*) does not admit Modus Ponens, so the demands over the agents are considerably low.

The first answer, and probably the least interesting one, just denies the problem. According to this view, if we assign a belief set to an agent, and we represent the set as a set of possible worlds, logical closure will follow. This answer was formulated by Stalnaker (1987), though it is not particularly strong. In theories of rationality, idealizations are frequent. However, this does not mean that we do idealize at this point when we attribute beliefs. If that were the case, we would not understand how can someone ignore the consequences of her beliefs.

The second answer to Excessive Demands admits that sometimes we do not comply with logical closure. But it claims that logical rationality is an *ideal* condition, and as such, there is always some level of irrationality if you believe the Peano axioms but you ignore some of their consequences. This is, for example, Broome's first position (1999), and one of the proposals of MacFarlane (2004).

It is convenient at this point to introduce the important distinction between *ideal rationality* and *applied rationality*.[16] *Ideal* rationality is a set of requirements that can be used as a point of reference, or regulative ideal, for dealing with our beliefs or evaluating the beliefs of the others. According to this kind of rationality, the objection of excessive demands does not apply, since even when no agent can comply perfectly with the closure requirement, one can evaluate how close is her belief-set to the ideal. Every agent should, in any case, take logical closure as a point of reference.

On the other hand, *applied* rationality is a set of requirements that we use ordinarily to evaluate real agents and classify them as rational or irrational. Undoubtedly, logical closure is too demanding in this respect, for we do not classify an agent as irrational when she ignores the last consequences of her beliefs. It shall be clear that, in this paper, I am looking for a requirement of *applied* rationality. Therefore, this second answer to the problem of excessive demands is not useful for my purpose.

4.2. Epistemic variations

A common response to the problem of excessive demands, which was anticipated by Harman (1986, 17) and suggested by Field (2009, 253) and Steinberger (2015, 25), holds that logical rational requirements apply just in cases

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16 See Smithies (2015) for a defense of the distinction between ideal and applied rationality, or between "ordinary standards" and "ideal standards" for rationality.

in which the agent *recognizes*[17] that the premises logically imply the conclusion. In other words, their proposal is to replace WS Validity by the following requirement (the considerations about relevance that were introduced in the previous section will be ignored by now):

(Recognized WS Validity)
If you *recognize* that $\Gamma \vDash A$, then rationality requires you not to believe some sentences of Γ or to believe A.

This epistemic variation of WS Validity has, nevertheless, a clear shortcoming. There are some *obvious* cases of validity, which must have rational force even when one does not recognize them. In other words, it seems that, even though not closing your belief set under *recognized* consequences is wrong, it is also wrong not to believe some simple consequences of your beliefs. In what follows, I will present some variations of the requirement that are immune to this objection.

4.3. Objective and inferential scales

Adopting epistemic variations is not the only way of restricting the range of application of rational requirements to a subset of valid inferences. It is possible to develop more objective restrictions, based on the level of difficulty. If this strategy is adopted, the subset of inferences with normative force will not be the *recognized* inferences, but the *simple* inferences, according to a certain scale. The idea is that the agent must have her belief set closed under some simple inferences, but not necessarily under more complex inferences.

A usual strategy for restricting the requirements to "simple" cases of validity is to consider that the complexity of an inference can be measured by the *number of steps* that you need to prove its validity (i.e. the *length* of its shortest proof). Some authors, such as Field (2009), D'Agostino & Floridi (2009) and Jago (2009) have proposed ideas of this kind.[18] In this case, rationality could

[17] It is hard to mention all the subtleties of each position. Strictly speaking, the word "recognize" comes from Harman. Field uses "realizes" and Steinberger "believes".

[18] Actually, Field is the only one who appeals to inferential measures for logical rational requirements. D'Agostino & Floridi, and also Jago, just try to establish a reasonable measure of the complexity of an inference.

require that the agents believe what can be derived from their beliefs in a certain (small) number of applications of rules:

> (WS Validity – Proof-length criterion)
> If $\Gamma \vDash A$ can be proved in at most *k* applications of rules, then rationality requires you not to believe some sentence of Γ, or to believe A.

The proof-length criterion is initially plausible (in fact, I will apply a similar one). However, it faces several objections. The first one was observed by Field (2009, 260): there is no way of complying with this requirement without having a logically closed belief set. For, imagine a set which is closed under *one* application of rules. Could it be non-closed under *two* applications? Certainly not; if the first thing happens, every number of steps can be reached one by one.

Anyway, we may focus on what does the requirement ask in each case. And effectively, with respect to certain initial non-closed set, this requirement can point out which beliefs are we to blame for not adopting. For example, if you believe just p and $p \rightarrow q$, you can be blamed for not believing q,[19] but you cannot be blamed for not believing $\neg\neg\neg(\neg q \vee \neg(r \vee q))$.

However, even though this variation is promising, there is another important and not so commonly observed problem: the number of applications of rules is not a correct measure of the complexity of an inference. In fact, suppose that an agent has the beliefs $p_1, \ldots, p_{120}$. Intuitively, it is easy for the agent to infer $p_1 \wedge \ldots \wedge p_{120}$. However, this involves 120 applications of rules. Now, suppose that the agent believes in the two axioms of Naïve set theory. It is possible to prove a contradiction from them in a few steps, but it is not an easy proof (the intelligence of Russell was needed to find the proof). According to the inferential approach, finding a contradiction in Naïve set theory is much easier than introducing 100 conjunctions. In this sense, the inferential criterion

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19 I am using an *explicit* notion of belief: a type of belief that, when you have it, you know that you have it. Using an implicit notion of belief may be useful for other discussions, but it would obscure this particular discussion. For, according to the usual concept of "implicit belief", we implicitly believe the obvious consequences of our beliefs; in this way, a considerable part of the problem of deductive closure would be automatically (and artificially) solved. I find much more illuminating to *explain* (rather than to rule out) the failure of deductive closure; that's why my notion of belief is explicit.

assigns difficulty to simple inferences, and takes some difficult inferences as easy. Therefore, an inferential criterion cannot establish a good measure of complexity.

It is tempting to adopt a more skeptical position in this debate, and to claim that the task of finding a scale of inferential difficulty is impossible. This reaction is somewhat justified. Harman (1986, 3) draws a distinction between inference and reasoning. Logical *inferences* are cases in which an agent arrives to a conclusion from certain premises, by using a set of rules. Instead, in a process of *reasoning* an agent arrives to a conclusion from certain premises by different informal methods, such as mental maps, rational intuition, suppositions, implicit "logical rules", etc. Logical inferences, given their precision, are measurable, and therefore can be ordered by complexity. But pieces of reasoning are not so precise.

Anyway, there is no strong reason to embrace skepticism at this point. Even though reasoning does not psychologically work as a logical apparatus, there are certain similarities. There are clear cases of simple or complex beliefs that are classified as such by both perspectives (logical and psychological). So, even if there are many functional differences, the level of "intuitive" difficulty of an inference hopefully may be formally captured, as well.

My proposal in the next paragraph will take some elements from the proof-length approach. I will develop a formal theory which can be used as a measure of complexity. Unlike the inferential approach, which establishes a total order, my proposal will establish a partial order, where some inferences are necessarily more difficult than others.

4.4. From recognized to recognizable

Conceptually, the restriction I will adopt has an element of subjectivity, but not as strong as in Recognized Validity. It is clear that, for each agent, some cases of logical consequence are *recognizable* and some are not. What is recognizable for each agent depends on her inferential capacity.[20] This allows to restrict the requirement in the following way:

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[20] Anyway, the two axioms of recognizability I introduce later are compatible with an objective interpretation of the criterion, where every agent could recognize the same set of inferences.

(Recogniz*able* WS Validity)
If you *could* recognize that $\Gamma \vDash A$, then rationality requires you not to believe some sentence of Γ or to believe A.

It is important to remark that my proposal is potential. This means that the agent is not limited by the inferences she actually recognizes, but by the inferences she *could* recognize. The modal element depends on the inferential capacity of the agent. In this sense, logic has a rational force over the agent no matter the actual logical knowledge she has; the only thing that matters is the knowledge she *could* have. Admittedly, potential notions such as "could know" are not completely clear. However, they describe the rational requirements in a much more accurate way. In ordinary talk, we usually appeal to *abilities*: for example, someone is responsible for not avoiding the death of another person whenever she had the *ability* or the *possibility* to save the other person. The same applies to logical rational requirements: if you are able to realize that A implies B, you can be criticized for believing A and not believing B; but you cannot be criticized for believing the Peano Axioms and ignoring whether the Goldbach conjecture is true or false. What does "ability" precisely mean is still an ongoing debate in philosophy, and is far from the scope of this paper. In what follows, I will provide some axioms which, at least for the cases of logical recognizability, help to make the notion more precise.

Now I will introduce some precisions on the notion of recognizability. In the last section I rejected the proof-length approach, for it doesn't give a correct analysis of the difficulty of inferences. My proposal is based on two conditions that every set of recognizable inferences should satisfy:

(Set of recognizable inferences)
Let R_i be the set of recognizable inferences for an agent *i*. R_i should satisfy the following two properties:

(Reflexivity) If $A \in \Gamma$, then $(\Gamma \vDash A) \in R_i$.
(Order) If every proof of $\Gamma \vDash A$ includes a proof of $\Gamma \vDash B$, then: $(\Gamma \vDash A) \in R_i$ only if $(\Gamma \vDash B) \in R_i$.

This definition establishes that if you can recognize that $p, q, r \vDash (p \wedge q) \wedge r$, then you can also recognize that $p, q, r \vDash (p \wedge q)$. For, in order to prove the first case of validity, you must be able to prove the second one. But it does not establish any previous relation between "incomparable" inferences: for example, between the inference from $p_1, p_2, \ldots, p_{100}$ to $p_1 \wedge p_2, \ldots \wedge p_{100}$ and the inference from Naïve Set theory to absurdity, even though the latter inference is shorter than the former. In this way, it avoids the objection I presented against the proof-length approach.

An anonymous referee observed that this criterion is still arbitrary, since there is no principled way of establishing the set of recognizable inferences for an agent. Admittedly, my criterion has some degree of arbitrariness; but it is still better than the proof-length perspective, for it does not suppose that every agent has a numerical limit *k* (a very unrealistic assumption). The set of recognizable inferences depends on each agent; the two axioms I provided give some restrictions on the structure of this set. Unlike the proof-length approach, my two axioms are intuitive and realistic. They are still too weak to determine *a priori* what an agent can recognize. But this is not necessarily a problem. The recognizability set can be thought as analogous to a possible world: there is no logical way of determining *a priori* what is true in a possible world, but there are some structural restrictions that every possible world satisfies.

In order to make Order more precise, it is necessary to specify some proof method. As I claimed above, no proof-theoretical apparatus "corresponds" perfectly to natural language reasoning. The discussion about which proof system is more similar to natural language reasoning is too complex to be covered here, so my proposal will take a simple proof method, with fairly intuitive rules. I will use a simple *tableaux* system,[21] although it is important to remark that the schematic definition of Proof-inclusion is suited for other calculi as well, such as *sequent calculus*. A *tableaux* is a tree which represents an argument *ad absurdum*. In the first step we enumerate the premises and deny the conclusion, i.e. we start the tree in the following way:

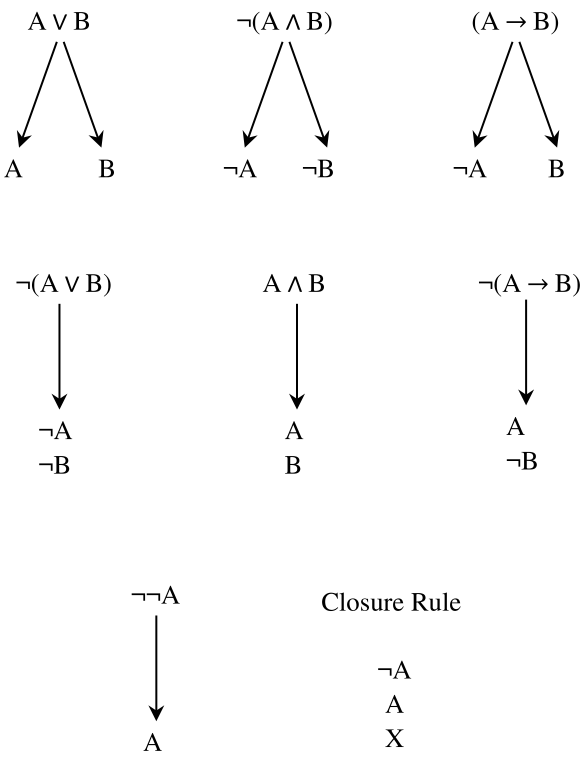
Premise 1
…
Premise *n*
¬Conclusion

Then, by valid transformation rules, we try to reach a contradiction. An application of a rule may extend a branch or open two new branches (for example, if A ∨ B is in the three, there will be a branch with A, and another branch with B). When a branch includes A and ¬A for one formula A, we say that the branch is closed, and we put the symbol "X". The formulae in

21 See D'Agostino et al. (1998) for a detailed presentation of *tableaux* systems.

each branch are *nodes*. The *length* of a proof is the number of nodes in it, without counting the nodes that express the premises and the initial hypothesis *ad absurdum*.[22]

The tableaux system for classical logic has the following rules. Each rule "decomposes" a formula, and reduces it to formulae with fewer symbols:

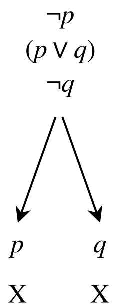


In propositional logic, one must decompose each formula which appears in a branch at most once. Put otherwise, once a rule *R* is applied for *A*, you

22 This measure of proofs is developed *ad hoc* for treating the problem we are discussing.

should not apply it again to *A*. The proof ends when we reach a contradiction in every branch; i.e. when we can close every branch. In this case, the inference is valid. Otherwise, the inference is invalid, and there are open branches in the *tableaux*. This system is complete and correct for classical logic (cf. Priest 2008, 16-17).

Let's see some examples of how this method is used. For example, a proof of $\neg p$, $p \vee q \vDash q$ can be performed this way:

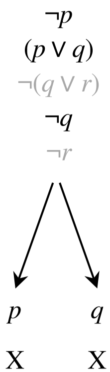


It is now possible to define more precisely what is for a proof to include another one, in order to determine more clearly how the axiom of Order is applied:

(Proof inclusion)
A proof of $\Gamma \vDash$ A *includes* a proof of $\Gamma \vDash$ B iff, by erasing nodes in the proof of $\Gamma \vDash$ A, you can obtain a proof of $\Gamma \vDash$ B.[23]

For example, we can think of the inference $\neg p$, $p \vee q \vDash q \vee r$, which might be obtained with the following *tableaux*:

[23] This notion of inclusion could be extended to a variety of proof systems. For example, in natural deduction, one proof includes another one iff one can obtain the latter by erasing steps of the former. However, introducing other notions of inclusion would also involve introducing other proof systems, and this task is far from the scope of this paper.



It is easy to observe that, if the grey nodes are erased, we get a proof of $\neg p, p \vee q \vDash q$. This implies that the second proof includes the first one.[24]

Now we can apply the notion of Order to the tableaux system:

(Order-Tableaux)[25]
If every *tableaux* proof of $\Gamma \vDash A$ includes a proof of $\Gamma \vDash B$, then: $(\Gamma \vDash A) \in R_i$ only if $(\Gamma \vDash B) \in R_i$.

The general idea is that, if you need to prove B in order to prove A, then if A is recognizable for an agent, B is recognizable for the agent too. The axioms

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[24] As I claimed before, the same considerations could be represented with a sequent calculus. As an example, see the following derivation:

$$\dfrac{\dfrac{\dfrac{\dfrac{p \Rightarrow p}{p \Rightarrow p,q} \quad \dfrac{q \Rightarrow q}{q \Rightarrow p,q}}{p\vee q \Rightarrow p,q}}{\neg p, p\vee q \Rightarrow q}}{\neg p, p\vee q \Rightarrow q \vee r}$$

[25] Notice the difference between Proof Inclusion and Order. Proof inclusion establishes a binary relation on proofs. Instead, Order asks for *every* proof of a certain valid argument to include a proof of another valid argument. In this sense, Order has a higher level of generality.

of Order and Reflexivity can be proved to be consistent. For example, the inferential criterion of the previous section, where R_i is the set of inference that are feasible in k steps,[26] satisfies both axioms:

Theorem
An inferential measure of an agent's recognizability (i.e. where R_i includes the valid arguments that can be proven in k or less steps) satisfies Reflexivity and Order.

Proof
Let us establish that $f(\Gamma \vDash \text{A})$, a measure of the length of inference, is equal to the number of nodes in the shortest proof of $\Gamma \vDash A$, without counting the premise nodes and the initial hypothesis *ad absurdum*. The set R_i of recognizable inferences for an agent i is the set of valid arguments that can be proved in k or fewer steps.
Reflexivity holds because $f(\Gamma \vDash \text{A}) = 0$ when $A \in \Gamma$, given that the *tableaux* is closed just after the enumeration of the premises and the hypothesis *ad absurdum* $\neg$A. Then, given that $k \geq 0$, $(\Gamma \vDash \text{A}) \in R_i$.
Order also holds. If every tableaux proof of $\Gamma \vDash A$ includes a proof of $\Gamma \vDash \text{B}$, then the shortest proofs of $\Gamma \vDash A$ include a proof of $\Gamma \vDash \text{B}$. The length of these proofs of $\Gamma \vDash \text{B}$ will be as much $m \leq n$, where n is the length of the shortest proofs of $\Gamma \vDash A$. So the shortest proofs of $\Gamma \vDash \text{B}$ have length $j \leq m \leq n$. Therefore, necessarily $f(\Gamma \vDash \text{B}) \leq f(\Gamma \vDash \text{A})$, i.e. the measure of difficulty of $\Gamma \vDash \text{B}$ is less or equal than the measure of $\Gamma \vDash \text{A}$. So, for an arbitrary k, if $(\Gamma \vDash \text{A}) \in R_i$, then $(\Gamma \vDash \text{B}) \in R_i$. □

It is worth remarking that, even though the inferential measure satisfies Order and Reflexivity, many other measures may satisfy it (including measures where a short proof is not necessarily easier than a large one).[27]

Following the concept of recognizability, it is possible to develop a consequence operator relativized to the recognizable inferences. In other words, if

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[26] Above we made an objective interpretation of the inferential criterion, according to which the limit k is identical for every agent. We could also make a subjective interpretation, where the limit k is different for each agent.

[27] The axiom of Order introduces a partial order between inferences that *share premises*. With respect to other valid arguments, there are no restrictions and they could be incomparable.

$\vDash_i$ means "recognizable for agent i": $Cn_i(X) = \{A \mid X \vDash_i A\}$. This consequence operator satisfies some structural properties.

First, Cn_i is reflexive. For an arbitrary i, the axiom of Reflexivity guarantees that $X \subseteq Cn_i(X)$. Now, Cn_i does not necessarily satisfy Monotony (if $X \subseteq Y$, then $Cn_i(X) \subseteq Cn_i(Y)$). In our definition of Order, the inferences have the same set of premises; therefore, inferences with different premises (even though one set of premises includes the other one) could be just incomparable. For the recognizability measure to satisfy Monotony, it is necessary to relax the axiom of Order for inferences with different sets of premises, and to admit *tableaux* where not every premise appears at the beginning. In this way, naturally every proof of $\Gamma \vDash A$ will include a proof of $\Gamma \cup \Delta \vDash A$. Otherwise, one could just add a third axiom to the concept of Recognizability:

(Monotony) If $(\Gamma \vDash A) \in R_i$, then $(\Gamma \cup \Delta \vDash A) \in R_i$

Monotony is also consistent with a purely inferential measure (since the premise nodes do not extend the length of the proof). But Monotony is still a controversial axiom. One might argue that adding premises to an inference makes the inference *less* recognizable, for now it is necessary to find which premises need to be used.

Finally, the operator Cn_i does not satisfy Transitivity (if $A \vDash_i B$ and $B \vDash_i C$, then $A \vDash_i C$). For example, you could recognize that $p \vDash p \vee q$ and $p \vee q \vDash r \to (p \vee q)$, without recognizing that $p \vDash r \to (p \vee q)$. The only thing that cannot happen, according to the axiom of Order, is that $p \vDash r \to (p \vee q)$ is recognizable and $p \vDash p \vee q$ is not, given that every proof of the first argument includes a proof of the second one.[28]

5. Relevance and difficulty

In this section, I will put together the considerations of the two previous sections, i.e. the concepts of relevance and complexity. The combination of WS Validity+Relevance and WS Validity+Recognizable is straightforward:

[28] Indeed, the failure of transitivity makes this notion functional. A transitive notion of recognizability would arguably become trivial.

(Logical rationality)
If $\Gamma \vDash A$, and both Γ and A are relevant; and you could recognize that Γ implies A; then rationality requires you not to believe some sentence of Γ or to believe A.

This requirement asks you to believe the *relevant* consequences of your *relevant* beliefs, whenever you *could recognize* that those beliefs imply those consequences. I can now develop a formal version of this requirement, using the two formal concepts of the previous sections: the set of relevant propositions Δ and the epistemically constrained operator Cn_i. The final requirement is the following:

(Logical Rationality – Formal)
Let i be your inferential capacity, and Δ the set of relevant propositions. Rationality requires your belief set X to be such that $Cn_i(X \cap \Delta) \cap \Delta \subseteq X$.

For example, let us assume that my belief set is $\Gamma = \{PA, \neg T\}$, where PA are the Peano axioms, and T is a formula that can be derived from PA but in a thousand of difficult steps. Suppose that I am seriously discussing T with a colleague; i.e. the set Δ is {PA, T, ¬T}. My belief set is inconsistent, but it is not irrational. Suppose that my inferential capacity is R_i, where $(PA \vDash \mathrm{T}) \notin R_i$. In this case, Γ is intuitively a rational belief set, in the applied sense of "rational". Even though Γ is not closed under logical consequence (it is indeed inconsistent), the proof from PA to T is too complex, so that the inference stays out of the set R_i. Therefore, I am not supposed to follow that inference, and my belief set Γ is logically rational.

6. Conclusion

In this paper, I proposed a specific rational requirement for characterizing logical rationality. First, I argued in favor of WS Validity, for it can avoid the Bootstrapping problem. Then I added a restriction to WS Validity in order to solve the problem of irrelevant consequences. The new version of the requirement, WS Validity+Relevance, restricts the requirement to the cases in which premises and conclusion are relevant (i.e. the agent considers or has reasons to consider it). This conceptual notion can be formally represented with the concept of weak relative closure.

After this, I introduced a second modification to address Excessive Demands. According to the new criterion, a valid argument has rational force over an individual whenever the individual *could* recognize its validity. Then WS Validity+Recognizable is obtained. The concept of recognizability can be formally characterized by two axioms. Reflexivity establishes that if $A \in \Gamma$, then $\Gamma \vDash A$ is recognizable; while Order establishes that, if in order to prove $\Gamma \vDash B$ you have to prove $\Gamma \vDash A$, then if you can recognize the first inference you can also recognize the second one.

Finally, I combined these two elements in the final requirement Logical Rationality: if you could recognize that Γ implies A, and both Γ and A are relevant in the context, rationality requires you not to believe some sentences of Γ or to believe A. This is, I think, a complete and formally precise logical rational requirement.

References

BOGHOSSIAN, P. (2003): The Normativity of Content. *Philosophical Issues* 13, No. 1, 31-45.

BROOME, J. (1999): Normative Requirements. *Ratio* 12, No. 4, 398-419.

BROOME, J. (2007): Wide or Narrow Scope? *Mind* 116, No. 462, 359-370.

BROOME, J. (2014): *Rationality Through Reasoning*. New Jersey: Wiley-Blackwell.

CARIANI, F. (2013): Ought and Resolution Semantics. *Noûs* 47, No. 3, 534-558.

D'AGOSTINO, M. & FLORIDI, L. (2009): The Enduring Scandal of Deduction. *Synthese* 167, No. 2, 271-315.

D'AGOSTINO, M., GABBAY, D., HANLE, R. & POSEGA, J. (eds.) (1998): *Handbook of Tableaux Methods*. Dordrecht: Springer.

FIELD, H. (2009): The Normative Role of Logic. *Proceedings of the Aristotelian Society* 82, 251-268.

HANSSON, S. (1999): *A Textbook on Belief Dynamics*. Dordrecht: Kluwer.

HARMAN, G. (1986): *Change in View: Principles of Reasoning*. Cambridge MA: MIT Press.

HINTIKKA, J. (1962): *Knowledge and Belief*. Ithaca: Cornell University Press.

HORTY, J. (2012): *Reasons as Defaults*. Oxford: Oxford University Press.

JAGO, M. (2009): Logical Information and Epistemic Space. *Synthese* 167, No. 2, 327-341.

JOYCE, J. (1998): A Non-Pragmatic Vindication of Probabilism. *Philosophy of Science* 65, No. 4, 575-603.

KOLODNY, N. (2005): Why Be Rational. *Mind* 114, No. 455, 509-562.

KOLODNY, N. (2008): Why Be Disposed to Be Coherent. *Ethics* 118, 437-463.

KORSGAARD, C. (1996): *The Sources of Normativity*. Cambridge: Cambridge University Press.

LIST, C. (2012): Judgment Aggregation: An Introductory Review. *Synthese* 187, No. 1, 179-207.

MACFARLANE, J. (2002): Kant, Frege and the Logic in Logicism. *The Philosophical Review* 111, No. 1, 25-65.

MACFARLANE, J. (2004): In What Sense (if Any) Is Logic Normative for Thought. Talk at APA, April 21, 2004; available at the author's website: http://johnmacfarlane.net/normativity_of_logic.pdf.

MAKINSON, D. (1965): The Paradox of the Preface. *Analysis* 25, No. 6, 205-207.

PRIEST, G. (2008): *An Introduction to Non-Classical Logics*. 2nd ed. Oxford: Oxford University Press.

RAZ, J. (2005): The Myth of Instrumental Rationality. *Journal of Ethics and Social Philosophy* 1, No. 1, 2-28.

REISNER, A. (2011): Is there Reason to Be Theoretically Rational? In: Reisner, A. & Steglich-Petersen, A. (eds.): *Reasons for Belief*. Cambridge: Cambridge University Press, 34-53.

RESTALL, G. & SLANEY, J. (1995): *Realistic Belief Revision*. Technical Report, Automated Reasoning Project, Australian National University.

SCHROEDER, M. (2004): The Scope of Instrumental Reason. *Philosophical Perspectives* 18, 337-364.

SCHROEDER, M. (2009): Means-End Coherence, Stringency and Subjective Reasons. *Philosophical Studies* 143, 223-248.

SHPALL, S. (2013): Wide and Narrow Scope. *Philosophical Studies* 163, No. 3, 717-736.

SMITHIES, D. (2015): Ideal Rationality and Logical Omniscience. *Synthese* 192, No. 9, 2769-2793.

SOUTHWOOD, N. (2008): Vindicating the Normativity of Rationality. *Ethics* 119, 9-30.

STEINBERGER, F. (2015): Three Ways in Which Logic May Be Normative. Manuscript; available at the author's Academia.edu site.

STEINBERGER, F. (2016): Explosion and the Normativity of Logic. *Mind*, forthcoming.

WAY, J. (2010): Defending the Wide-Scope Approach to Instrumental Reason. *Philosophical Studies* 147, No. 2, 213-233.

WÓJCICKI, R. (1988): *Theory of Logical Calculi. Basic Theory of Consequence Operations*. Dordrecht: Reidel.