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Quantifier Domain Restriction, Hidden Variables and Variadic Functions

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ABSTRACT: In this paper I discuss two objections raised against von Fintel's (1994) and Stanley and Szabó's (2000a) hidden variable approach to quantifier domain restriction (QDR). One of them concerns utterances of sentences involving quantifiers for which no contextual domain restriction is needed, and the other concerns multiple quantified contexts. I look at various ways in which the approaches could be amended to avoid these problems, and I argue that they fail. I conclude that we need a more flexible account of QDR, one that allows for the hidden variables in the LF responsible for QDR to vary in number. Recanati's (2002; 2004) approach to QDR, which makes use of the apparatus of "variadic functions", is flexible enough to account successfully for the two phenomena discussed. I end with a few comments on what I take to be the most promising way to construe variadic functions.

KEYWORDS: Quantifier domain restriction – hidden variables – Recanati – variadic functions.

1. The syntactic variable approach to Quantifier Domain Restriction

Consider the following sentence uttered by a student just before handing in her exam to the professor:

(1) Every mistake was corrected.

Assuming a simple unrestricted semantic value for the quantifier 'every' and the usual semantic values for the other expressions, the truth-conditions we obtain for the utterance of (1) are such that it is true iff *every mistake (in the world of the context) was corrected*. So the prediction is that the utterance of (1) is *false*, as there are many mistakes on many exams or in other places in the world that have not been corrected yet. This result seems incorrect; if, by hypothesis, every mistake *on this exam* had been corrected at the moment of the utterance, the utterance is intuitively *true*, not false. It is not made false by the existence of a mistake somewhere else in the world. Hence, the naïve semantic theory that yields the above truth-conditions has a problem. This is the problem of quantifier domain restriction (QDR): we need to find a mechanism to restrict the domain of quantification to a contextually salient subdomain (e.g. the set of all the mistakes on the student's exam), relative to which the semantic theory predicts intuitively correct truth-conditions.

One proposal to deal with QDR that has received much attention is the "syntactic variable approach", developed in Stanley and Szabó (2000a). It has been extensively discussed in the literature and received a good amount of criticism (e.g., Bach 2000; Recanati 2004; Collins 2007; Pupa and Troseth 2011). The proposal has a number of virtues, such as accounting for the phenomena of quantified contexts (cf. Stanley and Szabó 2000a, 250), accounting for cross-sentential anaphora (cf. Stanley and Szabó 2000a, 257), and accounting for the context-sensitivity of comparative adjectives (cf. Stanley 2002, 380), among others (see also Kratzer 2004).

In this paper I discuss two objections raised against this approach. I look at various ways in which the account could be amended to avoid these problems, and I argue that they fail. I start with the problem of the limiting case of QDR, i.e. the case of sentences involving quantifiers that do not require contextual domain restriction in order to get the correct truth-conditions. I subsequently discuss the problem of multiple quantified contexts, which are cases in which we need to postulate more than one bound variable in order to get the intuitively correct truth-conditions.

The syntactic variable approach is both a syntactic and a semantic approach, in the sense that it postulates syntactic constituents at the level of the LF of natural language sentences containing quantifiers. These constituents are not realized phonologically, that is, they are not present at PF (i.e. the superficial, or phonetic, form of natural language sentences). More specifically, Stanley and Szabó (2000a) postulate a complex aphonic expression, constituted by two variables: a variable 'f' of semantic type ⟨e,⟨e,t⟩⟩, and variable 'i' of semantic type ⟨e⟩. The value of both variables is provided by the context. The value of 'f' is a function that maps an object onto a set of individuals. It takes as argument the value of 'i', and maps it to a set of individuals that constitutes the restrictor of the domain of the quantifier.

Stanley and Szabó's (2000a, 251) implementation of this idea has both 'f' and 'i' "*co-habit* a node" with the CN that occurs in the quantifier phrase:

(2) $[_S\ [_{DP}\ [_{DET}$ Every$]\ [_{CN}$ mistake, f(i)$]]\ [_{VP}$ was corrected$]]$

The interpretation of the node in which 'f(i)' occurs is the intersection of the denotation of 'bottle' and the denotation of 'f(i)', after the context has supplied the values to the variables. If the context assigns to 'i' the exam the speaker has in mind when uttering the sentence, and to 'f' the extension of the relation of *being on* (relative to the world of evaluation), then the value of 'f(i)' will be *the class of entities that are on this exam.*[1] And this restricts the domain of objects we are quantifying over. According to Stanley and Szabó (2000a, 253), the semantic value of the node is given by the following meaning postulate (where 'c' above is an assignment determined by the context):

(3) $||\text{mistake, f(i)}||^c = ||\text{mistake}|| \cap \{x\text{: } x \in c(f)\ (c(i))\}$

The reason why the authors postulate a complex variable has to do with the phenomenon of quantified contexts (Stanley and Szabó 2000a, 250). Consider sentence (4):

(4) In most of John's classes, he fails exactly three Frenchmen.

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[1] With these assignments of semantic types to the variables, Stanley and Szabó (2000a) suggest a compositional combination of their semantic values. But it is not clear what rule of composition allows for these values to be computed. None of the ones in Heim and Kratzer (1998) does. The rules of composition, as introduced in Heim and Kratzer (1998, 105), take as input the semantic value of *different* nodes, so they do not apply to elements inside a simple node. This is a potential problem for the account, but it is not one that I address in this paper.

On one reading of (4), it is true iff *for most x such that x is a class of John's, he fails exactly three Frenchmen in x.* The authors maintain that the syntactic variable approach to QDR makes the correct predictions concerning this reading of (4). The first quantifier noun phrase (QNP) makes salient a certain set of individuals that it quantifies over, those that are *classes of John's* (in the educational sense). The QNP 'three Frenchmen' is implicitly completed to *three Frenchmen in a class of John's*. Therefore, it is not sufficient to posit in the LF of the second QNP a variable of type ⟨e⟩, for individuals that are classes of John's. The individual variable 'i' cannot do the job by itself. We also need to postulate a variable that gets in the context of utterance the value *being in* (relative to the world of evaluation). This is the variable 'f', of semantic type ⟨e,⟨e,t⟩⟩.

2. The problem of the limiting case and the default value solution

The first challenge to Stanley and Szabó's account I discuss here is the following: how does the theory account for those utterances of sentences where the QNP is *complete* and so no domain restriction is needed to predict correct truth-conditions? Consider an utterance of sentence (5):

(5) Every mistake on this exam was corrected.

Suppose the utterance is such that the QNP 'every mistake on this exam' is complete. That is, the speaker does not intend to convey the thought that every *formal* mistake on this exam was corrected, or that every *spelling* mistake was corrected, or any such proposition with an extra implicit completion, but simply that every mistake on this exam (say, the salient exam) was corrected. Now, the theory relies on the context to supply a value for 'f' and 'i', but it is not clear what these values could be in the case of (5). Apparently, the context does not supply any value at all to the variables.

Bach (2000) raises this issue as an objection to the Stanley and Szabó's proposal. He considers sentence (6):

(6) All men are mortal.

He writes: "Although this is a limiting case, the value of the domain variable must still be contextually provided. Otherwise, the sentence would not express a proposition at all" (Bach 2000, 274). Stanley and Szabó (2000b) do not address this objection in their reply to Bach's (2000) criticism. I know of no other place where they discuss this question.

A possible reply that might come to one's mind on behalf of the syntactic variable approach is that the value of 'f(i)' for the utterance of (5) is the same as for the utterance of (1) in the same context. That is, 'f(i)' stands for *the class of entities that are on this exam*, and that introduces a restriction without a difference. While this suggestion works for the case of sentence (5), it does not have a counterpart for the case of sentence (6), as here there are no corresponding plausible candidates for the values of 'f' and 'i'.

A general solution must provide default values for the variables in the case of complete quantifiers, which get us the intuitively correct truth-conditions for the utterance of the sentence. One could suggest, for instance, that the contextually determined assignment function assigns an *arbitrary* value to the variable 'i'. Indeed, there is no particular object that is salient, or in any other way relevant for the truth-conditions of (6) (in the case of (5), no other object apart from the room explicitly referred to). If the variable 'i' is to receive a value at all, even if the context does not pick out one, it can only be an arbitrary object from the domain D_e (relativized to the world of evaluation). The value of 'f' could be the extension (relative to the world of evaluation) of the property of being *either identical to or different from* an object. All individuals in the world of evaluation have the property of being *either identical to or different from* any arbitrary object. Therefore, the value of the CN would be the following:

(7) $||\text{mistake on this exam, f(i)}||^c = ||\text{mistake on this exam}||^c \cap \{x = c(i) \vee x \neq c(i))\}$

This way we get the desired outcome, that of having a restriction without a difference. However, while this proposal does provide the right truth-conditions for the utterances in question, it is artificial and it very much looks like an *ad-hoc* move. It is *ad hoc*, as the only reason to postulate these values is to obtain the intuitively correct results. It is artificial in the sense that it does not seem to be the case that either the speaker who utters (5) or (6), or the hearer, entertains a thought involving the property of *being self-identical*, or a singular thought *about* an arbitrary object. This problem is especially pressing if we consider a framework of *structured propositions*. On the other hand, if we take (7) to be the contribution of the expression to the *truth-conditions* of the utterance of (5) (and make no explicit claim about the structured proposition expressed), then this becomes an instance of the more general problem that truth-conditional semantics has with the fact that there are alternative but equivalent specifications of the truth-conditions of an utterance of a sentence. Thus, if a semantic theory assigns to an utterance of 'Snow is white' the truth-conditions: true iff *snow is white and* $2 + 2 = 4$, we might suspect that something has gone wrong.

There are other options of default values that one might take the variables responsible for QDR to have. Thus, one might take the value of 'f' to be the extension of 'in' relative to the world of evaluation, and the value of 'i' to be the world of the context. However, this option will not do, because it has an undesired result, as it leads to the QNP being rigidified. An utterance of (5) will have the following truth-conditions: true iff *every mistake on this exam in* c_w *was corrected* (where c_w is the world of the context). These are intuitively incorrect truth-conditions: if we evaluate the utterance relative to a world w other than the world of the context, the truth or falsity of the utterance depends intuitively on whether the mistakes *on this exam but in the world w considered* were corrected or not.

Now, it is true that in their original article Stanley and Szabó (2000a, 252) point out that the semantic types of the variables 'f' and 'i' are set to ⟨e,⟨e,t⟩⟩ and ⟨e⟩ only as a matter of convenience, and as a "simplifying assumption". Instead, "the domains contexts provide for quantifiers are better treated as intensional entities such as *properties*, represented as functions from worlds and times to sets" (Stanley and Szabó 2000a, 252). This might help avoid the problem of rigidifying the QNP, but a problem still remains. Suppose 'f' is an *intensional* variable of type ⟨⟨⟨s,i⟩,e⟩,⟨⟨s,i⟩,⟨e,t⟩⟩⟩, and 'i' of type ⟨⟨s,i⟩,e⟩ (where 's' stands for a possible world, and 'i' stands for a time). On this account, the value of the variable 'i' is not an individual, but what is sometimes called an *individual concept*. Furthermore, it might be suggested that the default value of 'i' for the limiting case (when no QDR is needed) could be a *non-rigid* individual concept that picks out *the relevant world*. The extension of 'i' is the relevant world of evaluation. The value of 'f' is the property of *being in*. But the problem now is that a possible world is not an individual, so it cannot be the extension of 'i', as defined here. A world is a semantic value of type ⟨s⟩ (see Fintel and Heim 2011, 10). In order for the suggested solution to work, the semantic type of the variable 'i' should be ⟨⟨s,i⟩,s⟩, but that would be of no help with the cases in which we do need a substantive domain restriction.

3. The problem of the limiting case and the ambiguity solution

The above discussion indicates that it would be preferable if the predicted truth-conditions of (5) and (6) did not contain a restriction without a difference. A suggestion along these lines would be to take the LFs of (5) and (6) to carry *no* hidden variables in those cases in which the QNP is (used as) complete. In the case of sentence (5), the semantic contribution of 'mistake' to truth-conditions would be the following:

(8) $||\text{mistake}||^{w,c} = \lambda x_{\langle e \rangle}.x$ is a mistake in w

In those cases in which QDR is required, (e.g., the utterance of sentence (1)) the LF of the sentence does contain the hidden variables, and the semantic value for the node $[_{CN}$ mistake, f(i)] is:

(9) $||\text{mistake, f(i)}||^{w,c} = \lambda x_{\langle e \rangle}.x$ is a mistake and is c(f) (c(i)) in w

So, on this proposal there are two different expressions in the LF that correspond to the superficial expression 'mistake'. The interpretation function assigns to each of them its semantic value. This means that 'mistake' turns out to be *ambiguous*, instantiating a kind of lexical ambiguity, given that 'mistake' sometimes expresses the concept *mistake*, but at other times it is a context-dependent expression, expressing the concept of *mistake standing in this relation to this object*.

Now, postulating ambiguities is generally not considered to be a great way to solve problems in philosophy. Methodological considerations concerning theoretical parsimony of the kind Grice (1978, 118-119) advances immediately come to mind. "It is very much the lazy man's approach in philosophy to posit ambiguities when in trouble", reads Kripke's (1977) insightful remark. Kripke suggests a policy of caution: "Do not posit an ambiguity unless you are really forced to, unless there are really compelling theoretical or intuitive grounds to suppose that an ambiguity really is present" (Kripke 1977, 268). Are there such grounds in this case?

A theoretical consideration in favor of the ambiguity solution is that it avoids the undesirable consequence that the "restriction without a difference" solution has. But there are no intuitive grounds for favoring the ambiguity solution. On the contrary, there are intuitive considerations *against* positing ambiguity in the CN: common nouns such as 'bottle' do not seem to be ambiguous in this way.[2,3] Now, one might find questionable the claim that intuitions about certain words being ambiguous or not are *bona fide* linguistic data for semantic theories. A semantic theory is not a study of the *intuitive* concept of meaning. Instead, it may postulate various theoretical notions of meaning or semantic value (e.g., intensions and extensions), even if these theoretical claims might be found unintuitive in some sense.

However, there are other good reasons to reject the ambiguity solution. As already mentioned, on the relevant reading of (4), the variable 'i' in 'three Frenchmen' is bound by the QNP 'most of John's classes'. But, as Breheny (2003, 63) points out, it is possible to find sentences with QNPs the domain restriction of which involves various quantificational dependencies. Consider sentence (10) (cf. Breheny 2003, 63):

(10) Some student thought no examiner would notice every mistake.

One reading of (10) is that some student x thought no examiner y would notice every mistake *made on a paper x turned in which y examines*. If we want to account for this reading in the way Stanley and Szabó do for the reading of (4) discussed above, then we need to postulate *two* complex variables of the form 'f(i)' that the QNP 'every mistake' contributes to the LF of (10). The lexical entry for 'mistake' in (9) above, which has one such complex variable, is not adequate for this case. That is, we need to introduce a new lexical entry for 'mistake', apart from the ones in (8) and (9), as follows:

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2 Pelletier (2003) uses similar appeals to intuitions against the Stanley and Szabó theory of QDR. The intuition is that a CN such as 'bottle' or 'student' is not context-dependent (or "contextually ambiguous", as he prefers to put it). He writes: "it seems simply unintuitive to claim that the interpretation of the same *noun* changes from context." (Pelletier 2003, 156-157)

3 An anonymous reviewer pointed out that a defender of such an approach could respond by arguing that this could be thought of as a case of *polysemy*. Polysemy is a particular form of ambiguity, in which there are "different senses of a lexical item that bear some intuitive relationship" (Jackendorff 2002, 339). Indeed, if the proposal discussed here has any plausibility then the different senses contemplated *should* be seen as instantiating polysemy, and not homonymy, as they are systematically related. However, I take the proposal to be still problematic, as there are no intuitive grounds for this claim.

(11) $||\text{mistake, f(i), g(j)}||^{w,c} = \lambda x_{\langle e \rangle}$.x is a mistake and is c(f) (c(i)) and is c(g) (c(j)) in w

This lexical entry still does not help us to account for cases which involve further dependences on quantified elements, such as in (12):

(12) Every year some student thought no examiner would notice every mistake.

On one reading of (12), it expresses the proposition that every year z some student x thought no examiner y would notice every mistake *made on a paper x turned in during z which y examines.* If Stanley and Szabó's example (4) shows that there is a complex variable in the LF of the sentence (given that it can be bound), then these readings of (11) and (12) show that there are two, and respectively three, complex variables in the LF of these sentences. With a little effort of imagination, we can build examples that involve even more dependencies of the restriction of the domain of quantification on previously introduced elements. This means that we need to postulate an indefinite number of lexical entries for CNs such as 'mistake' that differ from each other in the number of variables of the form 'f(i)' that they carry. While a language with such a lexicon is not necessarily unlearnable, as the lexical entries are introduced in a systematic way and following a pattern, this is clearly a very unattractive option.[4]

4. Other approaches to QDR that postulate hidden variables

To recap, Stanley and Szabó's syntactic variable approach to QDR gets into problems both when no QDR is required, and when the restriction needed requires that we postulate more than one complex hidden variable in the LF. In this section I argue that the two problems affect not only Stanley and Szabó's version of the hidden variable approach, but other versions as well. On von

[4] Stanley and Szabó (2000a, 232, n.16) discuss a possible ambiguity approach to QDR, but not the one considered here. On the approach they consider, a CN such as 'mistake' is multiply ambiguous, having one lexical meaning corresponding to each possible completion. They reject this option as "implausible".

Fintel's (1994, 30; 2014) proposal, the variables 'f(i)' cohabit the same node with the quantifier determiner.[5] The LF of (1) is (13), instead of (2):

(13) $[_S\ [_{DP}\ [_{DET}$ Every, f(i)] $[_{CN}$ mistake]] $[_{VP}$ was corrected]]

On the ambiguity solution to the limiting case of QDR, it is the quantifier determiner that is multiply ambiguous, having an indefinite number of lexical entries, starting with (14), (15), and so on:

(14) $||\text{every}||^c = \lambda g_{\langle e,t\rangle}.[\lambda h_{\langle e,t\rangle}.$ every x such that g(x) = 1 is such that h(x) = 1]

(15) $||\text{every}||^c = \lambda g_{\langle e,t\rangle}.[\lambda h_{\langle e,t\rangle}.$ every x such that g(x) = (c(f)(c(i)))(x) = 1 is such that h(x) = 1]

The proposal is as problematic as the similar one discussed above in relation to Stanley and Szabó's account of QDR. There are no strong intuitive of theoretical grounds for postulating a rampant ambiguity of the quantifier determiner.

Other versions of the hidden variable approach have been proposed: one of them takes 'f(i)' to occupy its own node. On this hypothesis, the LF of sentence (1) might look like this:

(16) $[_S\ [_{DP}\ [_{DET}$ Every] $[[_{CN}$ mistake] [f(i)]]] $[_{VP}$ was corrected]]

Stanley and Szabó (2000a, 255) reject this option, arguing that "one should not place such a burden on syntactic theory", but Stanley (2007, 248) explicitly embraces it.

This proposal obviously faces the same problem of the limiting case of QDR. The only significant difference with the previous cases discussed is that the ambiguity solution involves multiplying *the nodes* in which the variables occur, and that does not affect the semantic value of the CN 'mistake'. In this case the ambiguity solution does not boil down to a *lexical* ambiguity of the

5 Another difference with Stanley and Szabó's approach is that von Fintel (1994) does not commit himself to any syntactic claim. He writes that the question whether the variable is present in the syntactic representation of the sentence "is an important conceptual and empirical issue that we will not be able to do justice here" (von Fintel 1994, 33).

CN, but rather to something closer to a syntactic ambiguity. The superficial form of a sentence containing a QNP has various LFs that differ from each other in the number of nodes of the form ‘f(i)’ to be found in the vicinity of the CN, which in turn depends on how many quantified contexts are involved in a particular reading of that sentence. But the solution is equally unattractive as the previous ones unless we are given a plausible and not ad-hoc explanation of how the variables end up in the LF. However, the proposal, as presented above, fails to do so.

A final alternative I briefly mention here is due to Pelletier (2003), on which the complex variable ‘f(i)’ is placed in the NP node, but not in any of its daughters. That is, the variable does not occupy its own node, but it is also does not co-habit a terminal node with another expression. For that reason, Functional Application (cf. Heim and Kratzer 1998, 105) fails to deliver the right result in this case (as it only computes the values of the terminal nodes, ignoring any other expression that is not in the terminal node). Pelletier (2003, 152) introduces a different rule of composition (call it Modified Functional Application, or MFA), as follows (where ‘Det’ stands for a determiner and ‘N’ for a noun):

$$||\text{Det N}||^c = ||\text{Det}||^c\ (||\text{N}||^c \cap c(f)(c(i)))$$

This is different from standard Functional Application, which we might represent here as follows:

$$||\text{Det N}||^c = ||\text{Det}||^c\ (||\text{N}||^c)$$

The price to pay for achieving domain restriction is the need to introduce a new rule of composition. It might not be a price too high to pay, if the account proved satisfactory. But does it? Pelletier does not discuss the two problems mentioned above, but it is easy to see how his account can deal with the problem of the limiting case: whenever the domain is implicitly restricted Det and N combine by MFA, taking into consideration the values of the variables as well; whenever the NP is complete and no domain restriction is required to derive the correct truth-conditions, Det and N combine by standard FA, thus ignoring the values of the variables (in that case it simply does not matter what default values we assign to the variables). However, the account fails to deal satisfactorily with the phenomenon of multiple quantified contexts, which requires more than one variable in the LF. So, it turns out to be only in part better than the versions of the hidden variable approach to QDR previously discusses.

5. The variadic function approach

All the versions of the hidden variable approach discussed here face the problem of the limiting case (except Pelletier's) and the problem of multiple quantified contexts. The above discussion suggests that a "dynamic", more flexible, proposal is required, one that avoids the rampant multiplication of ambiguities, and at the same time provides the resources needed to account for the limiting case of QDR as well as for the cases of multiple quantified contexts. Fortunately, there are approaches that do allow for the needed flexibility (no variables, or more than one, up to as many as the restriction requires), as well as provide a systematic explanation of how the variables end up in the LF. In this section I briefly present the variadic function approach, and I argue that it offers a satisfactory solution to the two problems mentioned.

According to Recanati (2002, 319), a variadic function is a function from a predicate in natural language P*n* (a predicate with adicity *n*), to a predicate with a different adicity: P**n+1*, in the case of an *expansive* variadic function, and P**n-1*, in the case of a *recessive* variadic function. Thus, Recanati suggests that the prepositional phrase 'in Paris' in the sentence 'John eats in Paris' contributes a variadic function which maps the unary predicate eats (x), ascribed to John in the simple statement 'John eats', onto the binary predicate eats_in (x, l), which takes two arguments: an individual and a location. Following Recanati (2002, 321) the variadic function in the former case can be represented as follows:

$$V_{\text{location: Paris}} \text{ (eats (John)) = eats\_in (John, Paris)}$$

The general form of an expansive variadic function could be given as follows:

$$V(P(x_1,\dots x_n)) = P^* (x_1,\dots x_n, y)$$

A variadic function creates a new predicate from a pre-existing one by changing the adicity of the latter. But it does more than that: in some cases, it also changes the content of the function, so that P and P* need not have the same content. Such is the case of the above variadic function that takes eats (x) as input and gives eats_in (x, l) as output. Moreover, in the case of an expansive variadic function it also provides a value for the new variable that it introduces (unless the variable is bound). The role of the subscript ‘location: Paris’ in the above formula is precisely to indicate that the variadic function introduces a variable for location, and give to it the value Paris.[6]

Recanati introduces the apparatus of variadic functions in order to prove the invalidity of the Binding Argument, which Stanley proposes in Stanley (2000). According to Recanati, a key premise of this argument is the Binding Criterion, according to which “[a] contextual ingredient in the interpretation of a sentence S results from saturation if it can be ‘bound’, that is, if it can be made to vary with the values introduced by some operator prefixed to S” (Recanati 2004, 102). That is, if an ingredient dependent on the context is part of the truth-conditions of an utterance of a sentence S, and we can build a different sentence ΦS, such that the value of the contextual ingredient varies with the value of the operator Φ, then that contextual ingredient is a variable that Φ binds. If sentence S is (17), uttered with the intention to express the proposition that *John fails exactly three Frenchmen in his class*, then the contextual ingredient at issue is the nominal completion *in his class.*

(17) He fails exactly three Frenchmen.

The sentence ΦS might be (4) above, that is:

(4) In most of John’s classes, he fails exactly three Frenchmen.

Now the nominal completion of the quantifier ‘exactly three Frenchmen’ varies with the operator ‘In most of John’s classes’. In accordance with the Binding Criterion, a component of the truth-conditions that varies with a certain operator is bound by that operator, and in turn, binding requires that there be a bound variable. So, the nominal completion of the quantifier ‘exactly three Frenchmen’ in (4) results from binding a variable that is present in the QNP. Given that the presence of the variable does not depend on whether it is bound or not, a variable of the same form must be present in the LF of (17). This variable must be saturated in order for the sentence to be interpretable, which

6 For a more detailed presentation of variadic functions see Recanati (2002, 319-322) and Recanati (2004, 107-109).

means that the nominal completion *in his class* results from the saturation of this variable.

Recanati rejects this latter conclusion. He accepts that in (4) binding requires the presence of a variable that is bound, but not that this is evidence that the variable is present in (17) as well. The apparatus of variadic functions allows him to show how this is possible. Recanati (2004, 111) argues that not only prepositional phrases such as 'in Paris' introduce variadic functions, but also QNPs. According to Recanati (2004, 113), the quantified prepositional phrase 'In most of John's classes' contributes not only the quantifier 'for most x that are John's classes', it also contributes the variadic function 'in x'. To see how this result is obtained, let us first consider the LF of (17), before embedding it as in (4). This is (18):

(18) $[_S [_{NP}$ exactly three Frenchmen$] [\lambda_1 [_S$ he $[_{VP} [_V$ fails$] [_{NP} t_1]]]]]$

(18) results from Quantifier Raising the QNP 'exactly three Frenchmen' from its position at the superficial form to the first upper S node up on the tree. This solves the mismatch problem between the verb 'fails', of type ⟨e,⟨e,t⟩⟩, and the QNP 'exactly three Frenchmen', of type ⟨⟨e,t⟩,t⟩. According to Heim and Kratzer (1998, 193f), the movement leaves behind a variable of type ⟨e⟩, called a *trace*. The trace then combines with the transitive verb, the type of which is ⟨e,⟨e,t⟩⟩, thus solving the problem of the type mismatch. In order to get the right truth-conditions the trace must co-vary with the QNP. To achieve this, QR-ing the QNP also introduces a variable binder that will occupy the position in the sentence immediately after (i.e. below, in the phrase structure tree) the place where the QNP has landed, and which binds the trace. Binding requires that the binder be co-indexed with the variable it binds.

Now, the variadic function that 'In most of John's classes' introduces might be represented as follows:

$V_{\text{location: unspecified}}$ $([_S [_{NP}$ exactly three Frenchmen$] [\lambda_1 [_S$ he $[_{VP} [_V$ fails$] [_{NP} t_1]]]]]) = [_S [_{NP}$ exactly three Frenchmen *in x*$] [\lambda_1 [_S$ he $[_{VP} [_V$ fails$] [_{NP} t_1]]]]]$

The variadic function modifies the noun 'Frenchmen' to 'Frenchmen in x'. It increases its adicity: while 'Frenchmen' is of type ⟨e,t⟩, 'Frenchmen in' is of type ⟨e,⟨e,t⟩⟩.[7] If we QR 'most of John's classes' to the upper S node (an optional, not mandatory move, as the QNP does not produce a type mismatch), we obtain the LF in (19):

(19) $[_S [_{NP}$ most of John's classes$] [\lambda_2 [_S [_{PP}$ in $t_2] [_S [_{NP}$ exactly three Frenchmen in $x_2] [\lambda_1 [_S$ he $[_{VP} [_V$ fails$] [_{NP} t_1]]]]]]]]$

Moreover, if we co-index the binder λ_2 introduced by QR-ing 'most of John's classes' with the variable x that the variadic function introduces, as in (19), then x gets to be bound by λ_2. The calculation of the truth-conditions of (19), which I skip here in the interest of space, gives us the desired result. As a consequence, we see that the bound variable x_2 in (4) is the contribution of the variadic function that 'In most of John's classes' introduces, and is not part of the original sentence that 'In most of John's classes' takes as argument (i.e. sentence (17), the LF of which is (18)). Variables, Recanati argues, are not part of the contribution to the LF of the sentence of the CN in the QNP the domain of which they restrict, as in Stanley and Szabó's proposal, or of some other elements in the QNP. They are the contribution of another QNP higher in the sentence, the one that binds the contextual element that restricts the domain.

Having seen how variadic functions work, I turn now to a brief discussion of the nature of the processes that generate them. According to Recanati (2002, 322), variadic functions might be either the contribution of an adjunct in the sentence (such as 'in Paris' and 'In most of John's classes') or introduced "by purely contextual means". When the variadic function is not realized phonetically, as in simple cases of QDR, its presence in the truth-conditions of the sentence is optional. In that case it is the contribution to the truth-conditions of a purely pragmatic process of "free enrichment", a non-mandatory modification of the literal content of the quantifier. However, one might depart from Recanati's view on this point, and take on board variadic functions, but not the claim that they are pragmatic mechanisms. For instance, Marti (2006) uses the

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[7] This departs slightly from Recanati's (2004, 113) presentation of the output of the variadic function, where 'in x' is placed next to the S node, and takes as argument the S node. However, 'in x' has to modify the QNP 'exactly three Frenchmen', so it needs to make a direct contribution to the interpretation of this QNP. That is why I take 'in x' to modify the noun 'Frenchmen', and not the sentence node. I come back to this point in the next section.

apparatus of variadic functions, but does not take them to be the contribution to truth-conditions of a pragmatic process. Instead, Marti (2006, 141-142) suggests a purely semantic mechanism: free variables may be optionally generated in the syntax, and a variable thus generated might receive as value a contextually determined variadic function, which in turn modifies the adicity of the predicate it combines with.

As I argue below, both Recanati's and Marti's take on variadic functions have the advantage of avoiding the two problems that the hidden variable approaches to QDR discussed in the previous sections face. However, one might want to avoid any appeal to either free generation of variables or free processes such as pragmatic enrichment, which are always open to the objection of being too unconstrained and arguably ad hoc. For this reason, I prefer a semantic account, according to which quantifiers introduce a variadic function in the predicate they combine with, in virtue of their *lexical meaning*. The suggestion is that quantifiers are ambiguous, having one lexical meaning on which they introduce a variadic function, but also one on which they do not do so. Zeman (2015, 177) discusses this proposal briefly, in the context of a different, but relevantly similar debate, concerning the semantics of predicates of personal taste. He points out that worries that free pragmatic processes are too unconstrained are not sufficient reason to reject the apparatus of variadic functions *per se*, as "the variadic functions approach is in itself independent from Recanati's strong pragmatic commitments" (Zeman 2015, 178). Still, the proponent of the variadic function approach owes us an explanation of why a quantifier sometimes contributes a variadic operator and sometimes it does not. Zeman suggests that the quantifier might be responsible for introducing a variadic function in the logical form of the sentence whenever "*the truth-conditions of the uttered sentence require it*" (Zeman 2015, 178). One might further suggest, along these lines, that quantifiers have a lexical feature that allows them (without requiring that this be so on every use) to introduce a variadic function in the predicate they combine with. To avoid the charge of postulating a mechanism of ad-hoc generation of variadic functions, we could think of quantifier determiners as ambiguous: on one meaning they introduce a variadic function on the predicate they combine with, on the other they do not. For reasons that will become clear in the next section, I take the variadic function that quantifiers introduce to be able to take as argument *any* node within the predicate the quantifier combines with, and not necessarily the node of the predicate itself.

A worry might be raised at this point, as an anonymous reviewer notes. Doesn't the suggestion that quantifier determiners are ambiguous bring back all the difficulties that ambiguity approaches to QDR face, which were discussed in section 3? Indeed, some of the worries mentioned in section 3 might be reasonably raised here again. For one thing, quantifier determiners do not seem to be ambiguous. However, notice that if quantifier determiners are thought of as ambiguous, as I suggest here, this ambiguity is not multiplying beyond control. As we saw, in order to account for multiple quantified contexts, a defender of the hidden variable approach must postulate a rampant ambiguity in nouns (on Stanley and Szabó's account) or in the quantifier determiner (on von Fintel's account). The ambiguity postulated here is less problematic in this sense. So, while I acknowledge that the present proposal is not without difficulties, its comparative merits recommend it for serious consideration.

6. The two problems revisited

The virtue of the account of QDR proposed here is that it avoids the two problems discussed that the hidden variable approaches face, and it does so without postulating rampant ambiguities. Let us first look into the phenomenon of the limiting case of the QDR, i.e., when the intuitively correct truth-conditions do not require implicit completion of the nominal in the quantifier phrase. The suggestion that quantifiers are ambiguous, having one lexical meaning on which they introduce a variadic function, and one on which they do not do so, explains why in some cases the quantifier phrase is not completed in any way. The reason is that in these cases we make use of the meaning of the quantifier (for instance, 'Every mistake on this exam' in sentence (5)) that does not introduce a variadic function on the predicate it combines with ('was corrected', in the case of (5)).

Consider now the phenomenon of multiple quantified contexts. This is exemplified by sentences (10) and (12) introduced above. Consider again sentence (10):

(10) Some student thought no examiner would notice every mistake.

We can hear a reading of (10) on which it is true iff: some student x thought no examiner y would notice every mistake *made on a paper x turned in which* *y examines*. The restriction of ‘every mistake’ involves a double quantified context. In this case we deploy the meaning of the quantifier ‘some student’ that introduces a variadic function on ‘mistake’. We do the same for the quantifier ‘no examiner’. As a result, the second variadic function returns the noun ‘mistake made on a paper x turned in which y examines’, the type of which is ⟨e,⟨e,⟨e,t⟩⟩⟩. To see how this is achieved step-by-step, notice that (10) results from embedding (20) under ‘Some student thought’.

(20) No examiner would notice every mistake.

According to the proposal in the previous section, the QNP ‘no examiner’ introduces a variadic function in the predicate that it takes as argument, i.e., ‘would notice every mistake’. The approach proposed in the previous section is such that the quantifier is able to introduce a variadic function on any node in the predicate that it combines with. In this case the variadic function introduced modifies the node of the CN ‘mistake’. The LF of the sentence, after QR-ing the quantifiers, is the following:

(21) $[_{S}$ $[_{NP}$ no examiner] $[\lambda_2$ $[_{S}$ $[_{NP}$ every mistake *on a paper which* x_2 *examines*] $[\lambda_1$ $[_{S}$ t_2 $[_{VP}$ $[_{V}$ would notice] $[_{NP}$ t_1]]]]]]]

Notice that the QNP ‘every mistake’ has also been QR’ed from its original position to solve the type mismatch. The binder that the QNP ‘no examiner’ introduces (i.e., λ_2) is co-indexed with its trace (i.e., t_2) as well as with the variable x_2, and so it binds the newly introduced variable.

The next step to get (10) is to embed (20) in the phrase ‘Some student thought’, which again introduces a variadic function on a node that is in its scope. In particular, the variadic function modifies the noun ‘mistake on a paper which x_2 examines’. The variable introduced by this variadic function (i.e., x_3) is co-indexed with the binder λ_3, which results from QR-ing ‘Some student’. The result is the following LF for (10):

(22) $[_{S}$ $[_{NP}$ Some student] $[\lambda_3$ $[_{S}$ t_3 $[_{VP}$ $[_{V}$ thought] $[_{S}$ $[_{NP}$ no examiner] $[\lambda_2$ $[_{S}$ $[_{NP}$ every mistake on a paper which x_2 examines *and* x_3 *turned in*] $[\lambda_1$ $[_{S}$ t_2 $[_{VP}$ $[_{V}$ would notice] $[_{NP}$ t_1]]]]]]]]]]]

The computation of the truth-conditions of (22) gives us the intuitively correct result, showing that the approach to QDR based on variadic functions successfully accounts for cases of multiple quantified contexts.[8]

This analysis of (10) shows why we need the variadic function that a quantifier introduces to be able to modify a particular node within the predicate the quantifier combines with. This is part of the proposal made in the previous section (see also footnote 7). In the case of (22) we need the variadic function that ‘some student’ introduces to operate on the QNP ‘every mistake on a paper which x_2 examines’, and not directly on the whole expression the quantifier ‘some student’ combines with. Otherwise, the variadic function introduced would not modify an expression *within* the scope of the propositional attitude verb, and so it could not affect the nominal phrase ‘every mistake’. The QNP the domain of which is restricted is embedded in an intensional context, so the restriction must be embedded as well, in order to get the correct truth-conditions.

Finally, consider simple cases of QDR such as (1):

(1) Every mistake was corrected.

In this case the quantifier is not embedded in another one responsible for introducing a variadic function that operates on ‘mistake’. However, the single quantifier in (1) does introduce a variadic function. If we QR the quantifier from its position at the superficial form, we obtain (23):

(23) $[_S [_{NP}$ Every mistake$] [\lambda_1 [_S [_{NP} t_1$ on x$] [_V$ was corrected$]]]]$

I suggested above that quantifiers, in virtue of their lexical meaning, might introduce variadic functions that operate on some element inside the predicate the quantifier takes as argument. In this case the variadic function operates on the trace, which can be conceived as a zero-place predicate. The value of x is given by a contextually determined assignment function, and in this case it will be *this exam*. As a result, we obtain the relevant reading of (1).

The present discussion indicates that a too rigid account of how variadic functions are introduced fails to account for the two phenomena discussed (i.e.,

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8 Alternatively, the variables x_2 and x_3 might be co-indexed with a different binder, or not co-indexed with any binder at all. These cases correspond to alternative interpretations of the sentence.

that of the limiting case of QDR and that of multiple quantified contexts). We need a flexible theory of variadic functions, both with respect to *whether* the quantifier introduces a variadic functions, and with respect to *where* it does so. For cases where no QDR is needed, we want to say that quantifiers do not introduce variadic function at all. In the case of (1)/(23) we want to say that the variadic function operates on the trace, and not on 'was corrected'. In the case of (10)/(22) the quantifier 'some student' restricts the quantifier 'every mistake', but not 'no examiner', so the variadic function it introduces must affect the interpretation of the former, but not of the latter.

The more general conclusion reached in this paper is that a theory that appeals to the apparatus of variadic functions, after several fine-grained adjustments, seems better prepared to deal with the two problems discussed (that of the limiting case of QDR and that of multiple quantified contexts) than an approach that postulates hidden variables in the LF of quantifier determiners or nouns.

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