Organon F 23 (1) 2016: 2-20

Logical Expressivist's Logical Constants

PAVEL ARAZIM

Department of Logic. Faculty of Arts. Charles University in Prague
Palachovo náměstí 2. 116 38 Praha 1. Czech Republic
pavel.arazim@centrum.cz

RECEIVED: 01-12-2014 ▪ ACCEPTED: 28-04-2015

ABSTRACT: I would like to show that the problem of logical constants can be helped by treating the problem of relationship between logic and human reasoning. Thus I will present some parallels between the respective dilemmas and show that choice of a proof-theoretic answer in one case induces an expressivist choice in the other and the other way round, as well. This does not mean that other options are closed, though the two selected ones are thus given a new plausibility. Furthermore, the proof-theoretical demarcations of logical constants can provide missing details into the expressivist story, as they say which constants and why can actually perform the expressivist job.

KEYWORDS: Demarcation – logical constants – logical expressivism – model theory – proof-theory.

1. The problem of logical constants

Logic, presumably as every other discipline, should have its own vocabulary, i.e. there should be some words which belong specifically to the purview of logic. One can distinguish between meta-vocabulary and object-vocabulary. Let us illustrate the distinction using the example of zoology. Members of its meta-vocabulary are, e.g. *species, kind, family* etc. Its object-vocabulary, then, includes such words as animal, dog or dolphin. Now logic obviously also has its meta-vocabulary, including, among others, *conse-*

quence, *premise* or *contradiction*. But we are interested here in the object-vocabulary, which plausibly includes the classical quantifiers, the signs for classical connectives and perhaps other signs as well. These are the ones which make the logical form of a meaningful sentence. Now how can we decide, which linguistic items to count as logical constants in this sense?

Obviously, no logician would think of the word *dog* as of something within the purview of logic. Whether the sentence *Every dog is a mammal* is true is something vindicated by a biologist (as far as it makes sense to verify such a sentence instead of accepting something like *Dog is a kind of mammal* as a partial definition). The same holds for the fact that the sentence *Some dogs can fly* is false. The truth value would change in both cases, if we substituted the word *dog* by *bird*, which would mean to break the laws of biological discourse. On the other hand the sentence *Every dog is or is not a mammal* can be judged true by a logician. And logicians would agree that the words *or* and *not* belong, unlike *dog* or *bird*, within the purview of logic. Thus also substituting other ones for them, such as *and*, would mean breaking the laws of logic.

It can thus be said that logic is, as every other discipline, distinguished by its specific vocabulary (by which I mean, once again, the object-vocabulary). This does not necessarily mean that logic is concerned specifically with just linguistic phenomena, just as zoology's having a specific object-vocabulary does not mean it is concerned only with linguistic matters. In case of logic, unlike in that of zoology, this is of course a much more attractive way to see its subject matter.

Anyway, the boundaries of the object-vocabulary are hardly clear. Does the modal operator on sentences *necessarily* belong to the logical vocabulary? It is disputable, whether a vocabulary of any discipline is quite definite. Intuitively, it seems (especially to someone acquainted with Quine 1951) that it is actually not. Yet clearly some specification must be at hand in every case, otherwise we would have no idea about the given discipline. Actually, there can be more non-equivalent specifications and none has to make a clear-cut distinction, but they still have to serve to elucidate the character of the given discipline.

Now, should there be something special about the case of logic, should, say, its boundaries be specified more definitely, getting more close to clear-cut boundary than in the case of other disciplines? The intuition probably is that we should be somewhat more demanding about the specificity of criteria of belonging to its vocabulary, i.e. criteria of logical constanthood.[1] Logic should, after all, be in one way or other, constitutive of our rationality and thus if anything is supposed to be definite, then it is logic. It appears that when one is somehow incompetent in logic, by which I do not mean that he lacks the academic skills and knowledge, but that (s)he is unable to master the behaviour of words such as *and* in every-day life, then (s)he cannot really understand anything else, indeed is incapable of being rational. Thus definiteness should be in general considered as a virtue of any suggested demarcation of logical constants.

2. The most important approaches

How can we try to demarcate the logical constants, then? In MacFarlane's (2009) helpful summary of the history of the present issue, we are offered quite a lot of possible approaches, yet only two of them not shortbreathed and actually alive (that is, besides the obvious possibility of scepticism, which, as I have tried to motivate in the introduction, should be at least tamed, if not refuted). One approach can be called a model-theoretical, the other one proof-theoretical. I will occasionally speak about the model-theoretic and proof-theoretic demarcation. Yet in both cases it would be more accurate to speak rather of a family of demarcations, as there are more possibilities, how to actually demarcate logical constants both model-theoretically and proof-theoretically. Yet no confusion should arise, as I will try to make clear in the subsequent sections. Let us see the two main approaches in some detail.

3. The model-theoretic demarcation

The problem of logical constants is rather significantly older than the two proposed solutions. Together with an ancestor of what I call here the model theoretical demarcation, it was identified by Bolzano. He was nevertheless generally rather sceptical about the possibility to give a non-arbitrary crite-

[1] This expression means that the members of logical object-vocabulary are treated by logic according to their specific meaning, unlike the members of the vocabularies of other discipline's, from the meaning of which logic abstracts, treats them rather as variables than as constants.

rion of logical constancy. For his contribution and further development of the issue till the time I cover in this article, see the trilogy of articles by Ladislav Koreň – Koreň (2014a; 2014b; 2014c).

The model-theoretical account was first proposed by the very founder of model theory, namely Alfred Tarski. In Tarski (1986), he proposes to demarcate the logical vocabulary, or rather the notions which can be denoted by logical vocabulary, in a way which generalizes the analogous demarcations of geometrical concepts. In what follows, a transformation is a bijection of some domain onto itself.

As the concepts of Euclidian geometry are invariant under similarity transformations of the universe (for example a triangle gets mapped on a similar triangle, though perhaps proportionally smaller or bigger), the affine geometry under affine transformations (so that a triangle gets mapped on a triangle, though possibly not a similar one) and topology under continuity transformation (thus the triangle will be mapped on something which might not be a triangle but still is a continuous figure, the transformation does not tear it apart), we can say that the respective geometries deal with concepts of increasing generality. Now, according to Tarski, there are also concepts which remain preserved under all the transformation of the universe onto itself and these are exactly the logical notions. For instance, take the existential quantifier. In model theory it can be seen as a second-order predicate, i.e. a predicate on sets. If we consider any bijection of the universe onto itself and with it the induced bijection of higher-order objects, such as the sets of primitive objects (i.e. members of the original domain), then the existential quantifier holds of any set if and only if it holds of its image under such a bijection.

This original idea had to be modified because otherwise some unwelcome concepts would also have to be counted as logical. In more recent model-theoretical demarcations, the logical notions are defined as the ones which are invariant not just under a bijection of a given universe onto itself but rather under bijections between different universes, i.e. domains of models (structures). This version of the conception, as well as its historical development, is captured by Gila Sher in Sher (1991).

In her book the reader may also find in more detail which elements of language are thus identified as logical constants. Let us just briefly mention that according to this approach logic is a rather broad discipline. For instance all the quantifiers which speak about cardinality of sets are logical. For example, we can think of a quantifier $\aleph_{131}$, which asserts that there are $\aleph_{131}$ things (satisfying the given formula). And there are many more quantifiers of quite different kinds which get counted as logical constants by these lights. Actually, even though the logic developed by Sher is actually first-order (as these quantifiers speak only of objects, not of sets of objects) we get a system, which is practically as strong as full second order logic (details about this relationship with the second-order logic can be found in Bonnay 2008; the salient properties of second-order logic are explained in Shapiro 1991). Thus logic incorporates most of the set-theory and when one accepts this demarcation, it even makes sense to say that logicism gets verified, i.e. mathematics is proven to be a part of logic.[2]

In closing this section, it is good to remark that in this orthodox form, the model-theoretic approach leads to revisionism, on the one hand, and leaves some questions about logical constants unanswered, on the other. As regards revisionism, it is a problematic charge, as it is doubtful whether we can speak of a list of logical constants the logicians generally agree on. Though, as I already pointed out, there is something like a mainstream. Perhaps the constants of classical first-order logic are the most standard list. Now, here we see that this demarcation suggest a substantial broadening of it. As regards the unanswered questions, it is clear, that modalities and in general the constants of intensional logics are not touched by this approach, as it is focused mainly on first-order quantifiers. A partial answer might, nevertheless, be still better than no answer.

4. Proof-theoretical demarcation(s)

Unlike the demarcations formulated using model-theory, the ones formulated using proof-theory form a very heterogeneous class. Actually, there are some potentially important differences in the model theoretical camp, as well, but they are less significant and less motivated (see Bonnay 2008).

But before getting into the differences among the proponents of the proof-theoretical approaches, let us first see the general idea which unites them.

---

2 This is because set theory becomes a part of logic and most substantial parts of mathematics can be reconstructed in it; but it would of course need more considerations.

This time it is Gerhard Gentzen who can be considered as the originator. While presenting his sequent calculus and his calculus of natural deduction (or, better, the corresponding forms of logical calculi) in Gentzen (1935), he said that the rules for the constants function as *sozusagen Definitionen*. They express what e.g. conjunction or existential quantifier are, or at least an important part thereof.

If we concentrate now on the sequent form of these rules for, say, constants of classical or intuitionistic logic, we see that they are fully schematic. The logical constants are the only concrete linguistic items, the accompanying formulas are just placeholders which can be filled in by any sentences. This fact led authors, such as Ian Hacking or Kosta Došen, to propose that exactly the items which can be characterized in such a calculus in this schematic manner should be regarded as logical constants. This corresponds to the fact that logic is plausibly supposed to be topic-neutral.

Let us see two examples of proof-theoretic characterization. I will show a characterization of conjunction and of existential quantifier in the form presented in Došen (1994). This approach is little bit unorthodox, as it countenances the double-line rules, the double-line expressing a mutual derivability (that is we can derive also upwards and not just downwards as usual).

$$\frac{\Gamma \Rightarrow \Delta, A \qquad \Gamma \Rightarrow \Delta, B}{\Gamma \Rightarrow \Delta, A \wedge B}$$

$$\frac{\Gamma, A \Rightarrow \Delta}{\Gamma, \exists x A \Rightarrow \Delta}$$

In the case of existential quantifier, there is a special requirement on the side formulae, namely that they do not contain any free occurrence of the variable x. Thus in the case of conjunction the rule shows that its inferential properties do not anyhow depend on a specific context, i.e. on what is being talked about. Just anything can occur among the formulae of Γ and Δ. This connective is thus independent of a specific makeup of the context. Imagine a similar rule for the word *dog*. I do not want to propose it in any specific form, but besides the word *dog*, it would certainly have to include other specific words, perhaps *mammal* and others. In this sense the rule would not be truly schematic. And this corresponds to the intuition that the logical object-vocabulary should be topic-neutral or universally applicable in rational discourse. Put otherwise, the rules governing the logical constants are special in the sense that they are independent of context – the rules apply no matter what a given discourse is about.

In the case of the rule of existential quantifier, we can say that it is not fully schematic, because of the proviso regarding the variable. This leads Ian Hacking to assert that the existential (as well as the general) quantifier is not completely schematic and thus topic-neutral, which makes it somewhat less logical than the connectives. While I am sympathetic to this stance, I would like to point out that the proviso is obviously very week and only negative, requiring something not to occur in the accompanying formulae.

Nevertheless, the case of existential (and in fact also the universal) quantifier shows that the notion of the inference rules being schematic might be in need of more specification. Perhaps being schematic is not a yes or no matter, but rather a matter of degree. Thus relaxing the requirements we get the classical quantifiers counted as logical constants and relaxing even more might lead to accept some modalities. Yet these further relaxations to allow them would be rather significant. The details might be found again in Hacking (1979).

Besides the possibilities to understand the schematic character of rules in different ways, there are more ways how the model-theoretical demarcators can legitimately differ. First of all, it is open which structural rules one accepts or whether one allows for multiple conclusions. And such choices do affect what one demarcates, as for example allowing multiple conclusions leads together with the other rules and relatively strict understanding of what makes a rule schematic to classical logic, while banning them, i.e. allowing only single conclusion, leads to intuitionistic logic.

A variation on this theme is the possibility to require the constants to preserve different structural rules. Thus Hacking (1979) demands that after an introduction of a new logical constant the structural rules be provable, i.e. they do not have to be stipulated for more complex formulae so that the new complexity does not conflict with the previously valid structural rules. Furthermore, according to Hacking, the rules also have to be conservative. On the other hand, Došen (1994) leaves open the possibility that introducing a new constant may, for instance, not be conservative (implication can make left weakening valid). These requirements on the rules are proposals to capture the notion of harmony of the rules for the logical constants, which was introduced in Dummett (1973) and further discussed in Dummett (1991). They prevent, among other things, such constants as Prior's *tonk* to become a part of logic.[3]

These technicalities might be substantial not just for a logician but also for a philosopher but let us put them aside, as we want to discuss rather the respective merits of the proof-theoretical approach in general, as compared with the model-theoretical one. For the present purposes another dispute internal to the proof-theoretical demarcators is more relevant. Some authors claim that the rules of the respective calculi are indeed definitions in the full-blown sense. Others are more modest, saying that these rules merely characterize or somehow analyse these expressions, possibly leaving some features of their meaning aside.

The attempts to fully define the constants by proof-theoretical means do indeed have their significant problems. The proof-theoretical definitions cannot actually distinguish the more desirable logical constants from some exotic and hardly acceptable ones. For example, Harold Hodes is forced to speak about obscure properties of some rules, such as their being primitively compelling (cf. Hodes 2004). Hacking and Došen adduce, on the other hand, plausible arguments for not considering the rules as definitions in the full sense. Hacking emphasizes that in order to understand for instance the rules for conjunction one has to understand the concept already (as we speak about the first conjunct appearing in its appropriate place AND the second conjunct as well). Došen shows that the rules lack some characteristics of definition. Most importantly they do not allow eliminating the constants in every context.

But as long as our concern is with the demarcation of logical constants it is not so important whether the rules fully define the logical concepts. It is enough that they pin down some of their features which distinguish them from extra-logical ones. Thus Gentzen's remarks about *sozusagen Definitionen* serve rather just as a motivation for this approach, not necessarily as a binding programme. The proof-theoretical demarcators thus do not have to embrace the thesis that the semantics of the constants has to be provided proof-theoretically.

---

[3] The tonk connective, introduced in Prior (1960) is defined by introduction and elimination rules in a natural-deduction calculus. Given a formula A, the rules say you can infer AtonkB for any other B and subsequently you can infer B.

To conclude this section, let us remark that the proof-theoretical approaches are generally not as revisionist as the model-theoretical ones. Furthermore, they also allow discussing the modalities and the language of intensional logics in general (though we did not focus on this issue here). Yet they do not give a fully definitive answer in these cases. But perhaps this is not only a bad thing, as it reflects some natural fuzziness of the notion of a logical constant. On the other hand, when we settle on a notion of schematicity of rule, the answer is definite.

5. Connections with human reasoning

Now I would like to discuss some broader implications of the presented demarcations, especially of the proof-theoretic ones. It is clear that both the proof-theoretic and the model-theoretic account can claim some degree of plausibility. But they give quite different results,[4] so we have to inquire into the differences of the disciplines which they describe and ask which of them is more adequately described as logic. The difference is even more striking if we consider that the two approaches point to different answers to the questions about the truth of logicism.

Logic obviously has to be connected to human reasoning in some way or another. The basic question is how much it can be independent of it. I would like to distinguish three basic approaches. My list is not supposed to be exhaustive, though I believe that the most relevant accounts are basically variations of them.

The first approach is psychologism. According to psychologism, logic is a discipline which studies how we human beings actually think or perhaps argue. (There are several ways of how to specify it.) Logic is thus a descriptive discipline and a given system is refuted when its disagreement with real praxis is demonstrated. It should be noted that although psychologism has been largely discredited by Frege's and Husserl's criticism, there are authors who try to revive it, for example Susan Haack in Haack (1978), Robert Hanna in Hanna (2006) or Johann van Benthem in van Benthem (2008).

---

4 Remember that the model-theoretic demarcation leads to the logic of generalized quantifiers, which is not much short of full second-order logic. The proof-theoretical ones, on the other hand, can end up demarcating just the classical first-order logic.

The second approach is Platonism, which makes logic practically independent of actual praxis. Logic has got a domain of specific entities, abstract objects, which exist independently of our discourse and which it studies. Discrepancies between its (correct) claims and actual human reasoning are to be ascribed to defects of our everyday use of reason. Frege often seems to be a Platonist. There has been, of course, a heated dispute about the adequacy of such a prima facie reading, but certainly many of his passages suggest Platonism, see e.g. Frege (1884).

The third variety of approaches is distinguished from the first two by taking logic to be a normative, not descriptive discipline. Neither the praxis of reasoning, nor the realm of abstract objects is described. Instead, logic determines which reasoning is and which is not correct, i.e. how one should reason. This vague idea can be concretized in different ways. It has to be explained why such norms are instituted in the first place and what their roles are. Furthermore, it should be clear whether the norms which are stated by logic are its original creation or whether they are rather codifications of norms which are already acknowledged in a reasoned argumentation. Do we have to decide for one of these two radically different forms of normativism, if we want to be normativists? Well, the difference can hardly be explained away, but I think a viable version of normativism has to have it both ways, though it might emphasize one of these aspects more. Yet a normativism which is based purely on codification of preexisting norms ceases to be a normativism just like the normativism which ignores the norms which actually live in our daily argumentation praxis. The first one would be end up being just a variety of psychologism, while the latter just a variety of Platonism (for whence would the norms stated by a logician derive their legitimacy?). Thus any normativist approach should somehow correspond to rules of reasoning which are actually adopted, yet it has to transcend them. Classical example of a normativist would be Kant (1954) and, at least according to some ways of reading some passages, Frege (1884).

But let us postpone these considerations about possible versions of normativism, because here I would like to consider the species which can be called expressivism and which was developed by Robert Brandom (a clear statement of it can be found in the first chapter of Brandom's 2000). But for the time being, let us leave normativism unspecified and briefly reflect on the compatibility of its general shape, as well as with that of psychologism and

Platonism, with the two dominant approaches to demarcating logical constants.

Of course the issue of logical constants and the issue of relationship between logic and reasoning are divided and many connections between them are possible, but still it will be readily acknowledged that psychologism hardly seems to be suggested either by the model theoretic or by the proof-theoretic approach. Their sheer abstractness seems to make them steer far away from actual practices, which are notoriously replete with fuzziness and heavily context-dependent. The case is perhaps more clear for the model-theoretic approach. It is simply given which expressions (of course, when they have their standard meaning) are invariant with respect to the aforementioned bijections. The source of this definiteness still may be our practices but once they establish that some words are invariant (again, with their meanings, i.e. the abstract objects), then they behave independently and it is up to us to discover their properties. And the development consists mainly in finding the logical concepts and attaching names to them. Psychologism may not be lost completely, but it gets in a very difficult position, when the model-theoretic approach is accepted. Truth of logic depends on us as little as the truths of mathematics, if not as truths of natural sciences.

How about the proof-theoretical approach and psychologism? First of all, it is highly disputable whether psychological observations about human reasoning can be formulated very well in the framework of a calculus of one of the described forms. And granted that, which structural rules should be admitted? Perhaps allowing or banning some structural rules can be said to provide for mapping different areas of human reasoning. But they can also be taken as different models of the same set of practices. Nevertheless, it does not seem very plausible that in our every-day reasoning we distinguish some rules as formal in the sense of sequent calculi. Thus psychologism, though not ruled out completely, does not interact very smoothly with the proof-theoretical account of logical constants.

The marriage of Platonism with model-theoretical demarcation has the best prospects to be a happy one. They share the strong sense of objectivity of logic. Indeed, it suggests itself that logical constants denote some quite specific objects which belong to a different realm than the more mundane ones (the members of a given domain of a model). One of the specifics of modern model theory is of course its relativization of ontology in the sense that there is not one universe of what there is, the domain of each model containing only some entities. But the entities which are supposed to be denoted by logical constants are nevertheless invariant over all the domains. Strictly speaking, though, the denotation of logical constants, i.e. the real shape of logical concepts, remains dependent on which models there are. By this I mean that Tarski's logical notions are identified with extensions induced over the models. And in a way this seems a little bit strange and undermines the notion of logic preceding all other knowledge and being independent of it. Though it might be legitimate to relax these foundationalist views, the Platonism just described seems to make logic dependent on the assumption that the Tarskian models represent all the possible discourses, all the possible ways our reasoning can be about something.

Keeping in mind the particularly good fit between Platonism and model-theoretic demarcation, we can expect that the relationship between the proof-theoretic demarcation and Platonism will be somewhat less harmonious. This time not some set-theoretical construct over a given domain but inferential steps (transitions from some propositions to other ones) or rather types of inferential steps are hypostatized. And these would be rather strange entities. Thus it seems to me that should one interpret the Gentzenian rules governing logical constants in a Platonic way, then they would have to be regarded just as different ways of introducing the logical notions described by model-theoretists. And since the Gentzenian demarcations demarcate weaker systems than the Tarskian ones, they are actually incomplete. Platonists can see them as different ways of illuminating the systems which should however be demarcated in the Tarskian manner.

Now what about the family of conceptions of logic which regard it as a normative discipline and though they differ from psychologism significantly, still take logic to be intimately (in a sense in which the connection is not intimate for Platonism) connected with human practices of reasoning? In this case, the proof-theoretical demarcations should square better but let us begin with the model-theoretical ones.

I do not claim that it is impossible to be a normativist and favour the model-theoretic approach at the same time. It has to be supposed that our practices are guided by rules which specify how to operate with certain logical notions. These notions should not be understood as existing independently of our practices (or more generally, of us), on pain of falling back to Platonism. The problem is that it is hardly intelligible how the extensionally, model-theoretically understood logical notions can be actually taken to be an object of human manipulation. I am not sure whether this would be only psychologically implausible or whether it would be downright impossible. But it seems that one has to be able to individuate a given logical notion with its infinite extension. And it should be clear that inferential practices should be finite, i.e. always consist of a finite number of acts. In case, say of conjunction, the situation does not have to be so grave, as we can envisage rules which specify the correct usage of it, that is according to the abstract object it denotes, i.e. its truth function. Yet if we consider the generalized quantifiers, such as $\aleph_{131}$, it is much more difficult to make sense of it as something which can be intelligibly referred to in a specification of a rule. First of all, it is clear that the concept of such infinite cardinality has no place outside a relatively narrow context of mathematics and thus cannot be regarded as universally applicable. Furthermore, the concept of $\aleph_{131}$ is extremely unsharp, we cannot say that we understand what we say when we use it in the way we know what we say when we use conjunction or the existential quantifier. Every concept contained in a rule is bound to be vague to some degree, but it seems that we move to a new level when we envisage rules, which rely on the concepts expressed by many of the generalized quantifiers.

Why should the union with proof-theoretic demarcations be more feasible? The first simple point is that, once again, these demarcations have weaker results, i.e. they pick fewer constants and thus make logic narrower. This means that they would be less demanding on rational creatures. Therefore the problems with psychological possibility decrease. More importantly, though, the form of Gentzenian rules suggests that they codify inferences, i.e. activities of certain kind. On the other hand, these activities do not necessarily correspond to the real practices. And, as we have seen, there are more possible proof-theoretic demarcations. Which one is then to be picked out as the correct one? More specifically, which structural rules should we accept for our logical calculus? These issues are nevertheless not my principal concern, as I am only trying to argue in favour of the proof-theoretic approaches in general. What matters now is that the potential discrepancies between such calculi and actual reasoning do not have to be as troubling as they were in the case of model-theoretical approaches. Even if the inferential steps codified in the calculi do not correspond directly to the actual inferences we make on daily basis, this does not mean that they could not serve to regulate actual inferential practices as norms. But of course not every norm should be regarded as relevant to a given practice. It is therefore important to understand what kinds of norms should be relevant to our reasoning and how they should relate to it. And, most importantly for us, whether the inferences codified by Gentzenian calculi can be regarded as such.

6. Logical expressivism

So far we have only sketched the normative account of logic. As a sketch, it can be elaborated in various ways. Here I would like to concentrate on one particular elaboration and see how it fits the proof-theoretic approach to logical constants.

The expresivism I would like to present here was presented by Brandom as a part of his inferentialism. According to inferentialism, meaning of a given word is constituted by the inferential properties of sentences containing it. Meanings of sentences, then, are determined by the sets of sentences they can be inferred from and by sentences that can be inferred from them (possibly with further premises). This has the consequence that not only formal but also material inferences are legitimate, i.e. the formal inferences are not seen as corrections of the material ones (understood as enthymemes). Nevertheless, they enable to correct our inferential practices, as they make explicit the implicit inferential commitments (by introducing conditionals) or incompatibility between statements (by introducing negation). Brandom thus explicitly provides a rationale for at least the conditional and the negation but it is not clear which one, whether e.g. the classical or the intuitionistic or still other ones.

MacFarlane (2009) takes Brandom's approach to logic to be pragmatic, contrasting it, among others, with the principled approaches, among which the proof-theoretical ones belong. It is certainly true that Brandom puts much more emphasis on the overall purpose of logic and its place in the overall epistemology than the authors who actually came up with the proof-theoretical demarcations, such as Hacking or Došen. Yet I would like to show that the proof-theoretic approach does not have to be seen as a different one but rather as compatible with logical expressivism. While Brandom gives a more broadly philosophical account of what logic is, the proof-theoretic demarcations give a more technical, clear-cut account. I suppose that Brandom's theory possesses its own appeal and may thus legitimize the proof theoretic account, while the proof-theoretic account has much more indirect attractions.

First of the attractions of proof-theoretical approach is the fact that the demarcation gets relatively unsurprising results. This is of course only a weak virtue, though it is not so unimportant that it corresponds to general intuitions and practice. Yet, of course, it is relative which logics one expects to be demarcated. Still, it is true that the generalized logics of proof-theoretic demarcators cannot be said to be standard in the way the classical first-order logic is, which is clear at least from the fact that students are first introduced to it and perhaps much later, if ever, to their generalized quantifiers and further developments.

More important is that this approach specifies the formality and therewith the generality of logic. The Gentzenian rules governing the use of a logical constant are formal in that they leave everything else unspecified, i.e. they do not depend on context.[5] Brandom requires of logical constants that they be conservative, for otherwise they could not really be used to perform the role of logic, namely to make our implicit inferential rules explicit. The constants demarcated by their formal Gentzenian rules are in general conservative (this depends also on the structural rules allowed in the calculus). Moreover, I am inclined to regard them as indispensable for our discourse, since it would not be rational without them, we could not do anything about the implicit rules and therefore would merely follow them similarly as physical objects follow the laws of physics. Here I am probably at odds with Brandom himself who would rather say that there might be discursive practices based just on implicit rules. To be a rational being, it is not enough to follow rules, but also to make them explicit and thus also to be able to question them.

In the same vein it holds that the constants specified by the proof-theoretic demarcation occur necessarily in any rational discourse. This is because of their formality. Once we engage in the enterprise of reasoning, the door for these constants simply gets open. The development of language presumably starts with some more concrete (thereby material) rules of inference and then by making the rules more and more general, we arrive at the completely formal ones, i.e. the ones governing the logical constants. For example, we can say that some expressions are more general than others, e.g. *dog* is more general than *Cocker Spaniel*, since its use is governed by more general rules. The rules for dog are presumably more schematic. For example its

[5] This is due to their being schematic, as was explained in the exposition of the proof-theoretical approach.

introduction rules demand less. Here I clearly do not mean only discursive rules, i.e. inferential rules in a narrow sense, but also the language-entry and language-exit transitions, envisaged in Sellars (1974) (a further discussion of the relationship of these transitions and inferential rules in the narrow sense in the framework of logical expressivism can be found in Peregrin 2014). Language then develops, among other things, by adding more general rules, which make more and more other rules explicit. Logical rules are somewhere at the end of this development. Ultimately it does not matter where, i.e. in which areas of reasoning our generalization gets started (if we forget about the possible controversies regarding the structural rules, as the differences on this issue can very well be motivated by applying logic to different areas). Logic is general and formal precisely because the logical constants can be arrived at from the standpoint of every specific discourse. This shows that logic is something which is bound to accompany rational discourse at its every step, i.e. it is a necessary, though of course not sufficient, condition of truth. Trying to assert something which contradicts logic is like playing a game which is lost at the very beginning. The development of language of course never stops and is not driven just by the increasing generalizations of the rules that are made explicit. And as logic can be seen as a final-point of the process of generalization, it is implicitly present in our discourse from the beginning, exactly because its rules can be arrived at no matter with which material rules we begin. Let us also not forget that there might be a legitimate discussion about which rules should be deemed as truly formal (recall the issue about the schematicity of the rules for existential quantifier).

But let us make the relationship between the formal and the material inferential rules more precise. I will try to tell the story beginning with the situation when we have only the material inferential rules. In such a language people can start to disagree about the inferences they make, some deem a given inference legitimate, others not. Such a situation can be solved only by stipulating the allowed inferential step, which amounts to stating the conditional (in case the rule is accepted), i.e. using a logical constant. This does not mean that the inference is now for ever taken to be legitimate only in its formalized shape (allowing to infer e.g. *Thunder will be heard soon* from *Lightning is seen now* only when we have the relevant conditional *If lightning is seen now, then thunder will be heard soon*), but it is possible that people might temporarily reason in a cautious mode, requiring the inference rules used to be stated explicitly.

The other point, however, is that such a process of explication and formalization of our inferential practices has to accompany our reasoning all the time because that is what makes it rational. And this process has to be guided by some norms which are given a precise form in the Gentzenian calculi. Yet to make the process of explication intelligible we have to study how the explicators, such as the conditional, behave. And to do this we consider them isolated from our actual material inferential practices. This isolation is effected by planting them into an artificial niche where no material inferences are valid, i.e. in the sequent calculus. Why do we need such a general ideal? I suppose it is so because the distance it thus acquires with respect to our actual inferential practices enables it to work as an impartial device, it constitutes its objectiveness (Wittgenstein's *Härte des logischen Muss*). And furthermore, by taking the rules governing the logical constants to be fully formal, i.e. that all the accompanying premises are only schematic, we guarantee that they do not actually change the inferential framework they are supposed to explicate, because the logical constants are the only actual linguistic items occurring in the rules. Thus by introducing them we do not make any other linguistic items enter into new inferential relationships and thus do not change their meaning. Or we do, as they enter into relationships with logically complex sentences, but equally so for all of them, thus not distinguishing any single one. The change of meaning which all the sentences and thus also all of their constituents undergo can be compared to the change of size, e.g. doubling, underwent by all the physical objects. As is well known, such a change would actually be unrecognizable and would in fact be as good as no change at all.

The change regarding the whole of language thus happens on a meta-level. Let us say that it is rather the way in which we treat language than the language itself. And it has to be said that language has its implicit logic even before the introduction of logical constants in the strictest sense. The material inferences can be said to constitute a sort of implicit logic, which nevertheless cannot ever be fully identified with its explication. Actually, it is very well thinkable that there are more explications in this sense. And therefore it does not make sense to speak of one true logic, though there are limits, and so not every system can form a logic.

And actually such an opinion is captured very well by the proof-theoretic approaches, because besides providing demarcations, they typically allow for pluralism. This pluralism, as was already mentioned, is engendered by the variety of possible structural rules that are or are not accepted. Logic is thus presented as something which is not independent of human activities and so it also is not definite as an independent entity, but it still has some boundaries, as by far not everything can be accepted as logic.

7. Conclusion

I hope to have given some plausibility to the thesis that the logical constants (and with them the bounds of logic as a discipline) can in fact be determined. This can be done by two different argumentation strategies using different theoretical backgrounds, which nevertheless support each other. One is the proof-theoretic approach with its technical clarity but in need of further philosophical motivation and the other one is Brandom's inferentialism and expressivism which is philosophically appealing but does not say very much about the problem of pluralism in logic; Brandom (2000) also does not give us any details on what the list of logical constants should be. The details of the proof-theoretical specification can be seen as formal regimentation (with the typical virtues and also vices, of course) of Brandom's philosophical insights, which can be seen, as he himself suggests, as developments of thoughts much older. When we ask, from the expressivist position, which logics can serve the expressive role, the proof-theoretical approach gives us a good, though not fully conclusive, answer. Or at least, it provides a framework for looking for such an answer.[6]

References

BONNAY, D. (2008): Logicality and Invariance. *Bulletin of Symbolic Logic* 14, No. 1, 29-65.

BRANDOM, R. (2000): *Articulating Reasons*. Cambridge: Harvard University Press.

DOŠEN, K. (1994): Logical Constants as Punctuation Marks. In: Gabbay, D. M. (ed.): *What Is a Logical System?* Oxford: Clarendon Press, 362-381.

DUMMETT, M. (1973): *Frege: Philosophy of Language*. London: Duckworth.

DUMMETT, M. (1991): *Logical Basis of Metaphysics*. London: Duckworth.

FREGE, G. (1884): *Grundlagen der Arithmetik*. Jena: hrsg. von Christian Thiel.

[6] The present article was supported by Program for Development of Sciences at the Charles University in Prague no. 13 Rationality in the Human Sciences, section *Methods and Applications of Modern Logic*.

GENTZEN, G. (1935): Untersuchungen über das logische Schliessen. *Mathematisches Zeitschrift* 39, 176-210.

HAACK, S. (1978): *Philosophy of Logics*. Cambridge: Cambridge University Press.

HACKING, I. (1979): What is Logic? *Journal of Philosophy* 76, No. 6, 285-319.

HANNA, R. (2006): *Rationality and Logic*. The MIT Press.

HODES, H. (2004): On the Sense and Reference of a Logical Constant. *The Philosophical Quarterly* 54, No. 214, 134-165.

KANT, I. (1954): *Kritik der reinen Vernunft*. Hamburg: Felix Meiner.

KOREŇ, L. (2014a): Quantificational Accounts of Logical Consequence I: From Aristotle to Bolzano. *Organon F* 21, No. 1, 22-44.

KOREŇ, L. (2014b): Quantificational Accounts of Logical Consequence II: In the Footsteps of Bolzano. *Organon F* 21, No. 3, 303-326.

KOREŇ, L. (2014c): Quantificational Accounts of Logical Consequence III. The Standard Model-theoretic Account: Quantificational Accounts Triumphant? *Organon F* 21, No. 4, 303-326.

MACFARLANE, J. (2009): Logical Constants. In. Zalta, E. N. (ed.): *The Stanford Encyclopedia of Philosophy*. Available at: http://plato.stanford.edu/archives/fall2015/entries/logical-constants/.

PEREGRIN, J. (2014): *Inferentialism: Why Rules Matter*. Palgrave Macmillan.

PRIOR, A. (1960): The Runabout Inference Ticket. *Analysis* 21, No. 2, 38-39.

QUINE, W. O. (1951): Two Dogmas of Empiricism. *The Philosophical Review* 60, No. 1, 20-43.

SELLARS, W. (1974): Meaning as Functional Classification. *Synthese* 27, No. 3, 417-437.

SHAPIRO, S. (1991): *Foundations without Foundationalism*. Oxford: Oxford University Press.

SHER, G. (1991): *The Bounds of Logic: A Generalized Viewpoint*. Cambridge: The MIT Press.

VAN BENTHEM, J. (2008): Logic and Reasoning: Do the Facts Matter? *Studia Logica* 88, No. 1, 67-84.