

Tichý's Possible Worlds

JIŘÍ RACLAVSKÝ

Department of Philosophy, Faculty of Arts, Masaryk University
Arne Nováka 1, 602 00 Brno, Czech Republic
raclavsky@phil.muni.cz

RECEIVED: 30-04-2014 • ACCEPTED: 11-08-2014

ABSTRACT: Pavel Tichý originally published his interesting conception of possible worlds in 1968. Even though he modified it over the following twenty five years, its core remained unchanged. None of his thirty journal papers or books containing the notion of possible worlds was a study in metaphysics. Tichý (and most of his followers) always introduced the notion in the context of other investigations where he applied his Transparent intensional logic either to the semantic analysis of natural language or to the explications of other notions. Tichý presented his conceptions using rather short descriptions occurring on a number of places; his proposal appears not only fragmentary but also somehow incoherent. The main contribution of this paper is thus not only a complete survey of Tichý's development of his conception but also a certain completion of the very proposal.

KEYWORDS: Actualism – combinatorialism – possible worlds – possible world semantics – Transparent intensional logic.

1. Introduction

It is well known that the modern story of possible worlds began with Wittgenstein's states-of-affairs (see Wittgenstein 1921/1922). Carnap (1947) introduced a linguistic mirror of this conception (viz. state-descriptions) to formal semantics. Through a development best summarized elsewhere (e.g. Copeland 2006), Kripke (1963) and others supplemented formulas of modal logic with model-theoretic semantics based on the no-

tion of possible worlds. During the 1970s, Lewis proposed basics of his theory (1973); Plantinga (e.g. 1970), Adams (1974), Stalnaker (1976) and others significantly contributed to the debate. As the reader will be aware, the contemporary debate revolves around Lewis' landmark monograph (1986); further notable books include Armstrong (1989), Divers (2002), Nolan (2002) and Yagisawa (2010).

The present paper deals with a remarkable conception of possible worlds developed by the post-Prague Spring refugee Pavel Tichý (1936 Brno – 1994 Dunedin) who moved to New Zealand where he became a professor of philosophy. Tichý never wrote a systematic paper on possible worlds, nor a book, which is one of the reasons why his conception is not well known. Another reason might be simply that he exposed the notion outside metaphysical debate, in the context of different investigations – where he applied his system of intensional logic.¹

My main objective is to present Tichý's proposal, while focusing on its recent version. Though his conception of possible worlds is shared and sometimes discussed by his sympathisers (cf. below), the present paper provides its first complex survey.

Tichý developed his views in four stages:

- 1) around 1969 (the papers published in 1968, 1969 and 1971)
- 2) 1971–79 (especially the unpublished book 1976 and a number of related papers mainly in the late 1970s)
- 3) between 1980 and 1988 (partly in cooperation with G. Oddie)
- 4) 1988 and after (his supreme proposal published mainly in 1988).

However, such division is somehow artificial because the stages are overlapping and a number of ideas persisted throughout. One of my main goals then is to show a unity of Tichý's conception – even if there is an apparent incompatibility between its historical variants.

None of Tichý's expositions of the notion is extensive: it is usually less than half of one page. Tichý's most comprehensive treatise can be found in his excellent monograph *The Foundations of Frege's Logic* (see Tichý 1988), but the core part of its section '36. Possible Worlds' covers only three pages. If we compare Tichý's presentation with rivalling proposals of 1970s and early 1980s, we may say that it is likewise brief. After Lewis' monograph

¹ As remarked by one of the reviewers, there is also a reason that Tichý took possible worlds as primitive; however, this is not so simple (cf. below).

(1986) and subsequent philosophical debates, however, such proposals seem sketchy. Though I fill some gaps in presentation of Tichý's position, limits of the paper format force me to omit an extensive discussion of his conception. Recall also that Tichý as well as his followers are more interested in logical matters than in philosophy, which is why there has been no philosophical elaboration of Tichý's conception so far.

I will, of course, add explanatory remarks that should help the reader to compare Tichý's conception with other proposals. But it should be kept in mind that the contemporary understanding of possible worlds and the classification of their conceptions was proposed later (cf., e.g., Haack 1978, Menzel 2014) than Tichý developed his views. Since Tichý is a distinctive thinker, his conception is sometimes difficult to subsume into one particular category.

In the following Section (2), I explain Tichý's first conception followed by an immediate assessment (Section 3). In Section 4, I introduce its second and third stage. Then, in Section 5, I describe Tichý's supreme conception. In Section 6, I briefly explain his intensional logic and semantics and provide a conclusion (Section 7).

2. The first stage: procedures and intensional basis

During the late 1960s, Tichý faced limits of analysis of our conceptual scheme by means of classical extensional logic. Being well acknowledged with algorithms and related problems, he proposed in his textbook introducing logic as a framework of science (1968a), to enrich predicate logic by means of higher-order logic developed by Church (e.g., Church 1940). From a debate between Tichý (e.g., Tichý 1966) and Czech extensional logicians it follows that he knew Tractarian/Carnapian ideas concerning the content of empirical/non-empirical sentences, whereas the latter ones are valid in all circumstances.

Tichý's paper "Intensions in Terms of Turing Machines" published in *Studia Logica* (see Tichý 1969) benefited from his previous thoughts; a shorter version already appeared in a Czech philosophy journal (see Tichý 1968; for its English translation, see Tichý 2004). Tichý presented there his first compact conception of possible worlds.

As Tichý (1969, 7–9) explained, the aim of empirical investigation is to provide results of tests or *procedures* examining which *attributes* (i.e. proper-

ties and relations) are possessed by investigated objects. The sentence such as “The object *A* is heavier than *B*” is a *record* of this: it amounts to saying that *A* was positively tested on having that attribute.

To verify the sentence, one executes the procedure consisting of testing *A* on being such and such. The meaning of the sentence is identified with a complex concept – the procedure. Procedure is an older term for *algorithm*. Procedures split into two kinds, empirical and non-empirical. For an execution of any empirical procedure, one has to introduce state of the external world.

According to Tichý (1969, 9), language is based on an *empirical system* which consists of a particular fixed finite set of individuals and a fixed register of elementary tests, such as BE HEAVIER THAN *B*, called *intensional basis*.

Tichý always repeated this idea, only deleting “finite” and replacing “empirical system” with “*epistemic framework*” and “set of individuals” with “*universe of discourse*”. He admitted *partial attributes* to be in intensional basis (cf., e.g., Oddie – Tichý 1982, 234, 3.1.a). Tichý also dismissed the idea of *intensional independence* of tests employed in Tichý (1969) because he soon realized (though he did not mention) the force of Kemeny’s (1951) objection to Carnap (and Wittgenstein) according to which some intuitive attributes depend on others; in other words, that atomic facts are not necessarily independent.

Possible worlds are systems of possible outcomes of applications of procedures collected in intensional basis:

each combinatorial possibility as to the outcome of applications of all the tests in the intensional basis to all individuals (or to couples of individuals etc.) must be regarded as a conceivable state of the external world. Let us call these possibilities briefly *possible worlds with respect* to the empirical system. (Tichý 1969, 9)

Tichý also counted the number of possible worlds, which reminds us of Wittgenstein’s counting in Wittgenstein (1921/1922, 4.42). Tichý added that pure semantics cannot decide or assume which world is the actual one.

The rest of Tichý’s paper focuses on defining *intensions* as classes of equivalent procedures (procedures are equivalent if they yield the same outputs for the same systems of inputs and states of external world). Inten-

sions were thus not functions from possible worlds (W s) as we know it from possible world semantics and Tichý's later writings.²

In the subsequent paper "An Approach to Intensional Analysis" published in *Noûs* as Tichý (1971), he reminded the reader of his definition of intensions in Tichý (1969) and proposed his novel method of using Church (1940)'s typed λ -calculus for analysis of natural language (cf. Section 6).

3. Brief characterization of Tichý's conception

Since Tichý never made a substantial change to the above picture, we may already briefly compare it with rivalling proposals.

Despite being inspired by Tractarian/Carnapian *combinatorialism*,³ Tichý does not subscribe to its contemporary Armstrong's (1989) version (the differences between the two will be more evident from the next two sections). Since Tichý's worlds are external to language (though they are linked to epistemic base of a language), his conception of possible worlds is not *linguistic 'ersatzism'*⁴ as, e.g., Hintikka's (1969) conception.

Though Tichý's views seem to contain some ideas of *abstractionism*,⁵ Tichý maintained that the mathematical nature of the combinations leads to the conclusion that *logical space*, i.e., the set of possible worlds, has to be *homogenous* (see Tichý 1988, 179). He thus rejected Stalnaker's idea (1976) to set apart our actual world from the other possible worlds, which are only admitted as (useful) fictions. Modal *fictionalism* was dismissed by Tichý before it came into existence:

² In intensional semantics and logic, *intensions* are functions from possible worlds (W s). Intensions include *propositions* whose values are truth-values, *properties* whose values are classes of objects, *relations* whose values are classes of n -tuples of objects, etc. That an object *instantiates* (exemplifies, possesses) a property in (or: at) a given possible world W means that the object occurs in the *extension* of that property in (at) W .

³ The view that (alternative) possible worlds are results of recombination of meta-physical elements.

⁴ The view that (alternative) possible worlds are mere maximal consistent sets of sentences.

⁵ The view that alternative possible worlds are mere abstractions from the actual (real) world.

If unrealized determination systems [i.e. ‘possible worlds’] are mere fictions, then so is the realized one. (Tichý 1988, 179)

Obviously, Tichý is an *actualist* about possible worlds: all possible worlds exist (yet only one of them is actualized).

Tichý’s worlds are not concrete entities, they are classes of tests and each test is distinguished from its empirical execution (compare it with a computer program as such and its concrete execution). Tichý’s conception is thus not (Lewisian) *concretism*. Tichý sharply rejected Lewis’ conception as an absurdity (cf. Tichý 1988, 177–180).⁶

Lewis proposal is the only rival conception of possible worlds Tichý discussed and explicitly referred to.⁷ In Tichý (1975, 91–92), he objected also to Lewis’ construal of actuality. According to Tichý, the word “actual” stands for the identity function on worlds; this explains why an addition of the word to a descriptive phrase is redundant. He maintained that we, ignorant of many facts, are not omniscient, thus we are hardly capable to identify the *actual possible world*. Tichý repeated this idea in a number of places.

In Tichý (1971, 274–277) and also in Tichý (1988, 180–183), he passionately argued against *varying domains* and possible individuals. Without a demarcation of a domain, quantification over x cannot be logically satisfactory. The alleged examples of *possible individuals* (Kripke’s Pegasus, 1963) are only examples of this or that individual concept – Tichý instead used his terms *individual role* or *individual office* (an entity an individual can occupy). Tichý explicated individual offices as intensions having individuals as values. He elaborated and generalized the theory of offices to interesting heights.⁸

Tichý was a strong *antiessentialist* and ‘*haecceitist*’ (cf. Raclavský 2008; 2011). Though individuals do not have a genuine essence (but see Cmorej

⁶ According to Tichý (1988), Lewis’ worlds are not in space, since they are causally unrelated to our world. Thus, they are simply nowhere because space is by definition all-embracing. The ‘newspeak’ Lewis proposed does not help explain modality as appearing in everyday sentences.

⁷ The other metaphysical conception Tichý provably knew was Plantinga’s early theory (1970), but he criticized Plantinga (1972)’s essentialism.

⁸ See Tichý (1987), which is an adaptation of his introduction to Tichý (1976). For its application see Tichý (1978d); for his analysis of Anselm’s ontological proof see Tichý (1979).

1996; 2001), individual offices do have *essences* (an essence is a unique sum of defining properties called “requisites”). Tichý quantified over all individuals there are and, with the help of another variable, over all individual offices. Tichý is thus an *actualist*, though not a paradigmatic one: alleged possible individuals, excluded by a paradigmatic actualist from quantification, are treated by Tichý as individual offices and these are in the domain of quantification.

According to Tichý, the kind of *existence* which is applicable to individuals is *trivially* applicable to them. But he introduced (e.g., in Tichý 1979; or Tichý 1988; 1976) a kind of existence *nontrivially* applicable to offices; to say that the Pope exists in a given world W amounts to saying that the Pope-office has a holder in W . Nontrivial existence ascribable to properties (relations) says that the property has an instance in W (cf. e.g. Raclavský 2010 for more).

4. The second and third stage: intensions, primary / derived attributes, nexuses

The second stage of Tichý's considerations on possible worlds begins after the 1971-paper. At that time, Tichý (1974) disproved Popper's definition of *verisimilitude* (likeness of theories of truth) and worked on a positive proposal within the framework of his intensional logic, which he published in Tichý (1976a). This was followed by its slightly modified version in Tichý (1978d).⁹ Recall that truthlikeness measured within the framework of intensional logic (Tichý, Oddie, Niiniluoto and others) is closely related to the topic because it deploys possible worlds underlying scientific theories.

Especially in Tichý (1978d), he focused more on entities in intensional basis. He emphasized the difference between *primary attributes* which are collected in the intensional basis and *derived attributes*. The idea is explained further in his unpublished book Tichý (1976) where he remarked that to find out whether Xantippa is a widow one has to ascertain some more basic facts, e.g. that Socrates is dead. The idea goes back to the

⁹ Tichý's former pupil Graham Oddie wrote a whole book (see Oddie 1986) elaborating on and defending Tichý's approach to truthlikeness. In Oddie (1987), he indirectly supports Tractarian inspiration for Tichýan worlds.

aforementioned Kemeny's criticism of Carnap and it seems to be a predecessor of Kim's idea of *supervenient properties*.¹⁰

The second feature distinguishing the second period from the first is that *intensions* are not equivalent classes of procedures, but total or partial functions (as mappings) from possible worlds (cf. footnote 3).

A significant Tichý's work of this period is an extensive unpublished monograph "Introduction to Intensional Logic" (see Tichý 1976) written between 1973 and 1976. The book contains the first main, atemporal, version of his logical system (see Section 6) and its applications to natural language. Its important part is a large chapter on *subjunctive conditionals* and related phenomena, which makes a third distinctive feature of this stage.¹¹

Most of Tichý's papers published between 1975 and 1979 are nothing but selections from this book. Needless to say that his Czech and Slovak namesakes Pavel Materna (known for his later popularization of Tichý's logic) and Pavel Cmurej (who also utilizes his system) persuaded Tichý to adopt *temporal parameter* and most of his papers published at that time already contained its adoption in a rudimentary form. A typical example of a brief exposition of possible worlds in such a paper can be found in "New Theory of Subjunctive Conditionals" (see Tichý 1978). Note the similarity with his first stage proposal:

The aim of the investigation delimited by an epistemic framework is to determine exactly how the attributes from the intensional base are distributed through the universe of discourse at various moments of time. Before the investigation gets off the ground, the investigator faces a range of possibilities on that score. These possibilities are usually called, somewhat dramatically, *possible worlds*, and the totality of possible worlds is known as the *logical space* of the framework. (Tichý 1978, 435)

A feature more characterizing the third stage is Tichý's emphasis on *nexus* (*connections*), i.e. higher-order relations such as cause-effect relation

¹⁰ Later, Tichý wrote a paper (co-authored by Oddie, 1990) on resplicing properties over a supervenience base.

¹¹ Tichý published three papers on subjunctive conditionals. The first one (Tichý 1976b) disproves Lewis-Stalnaker's theory. The last one (Tichý 1984) disproves, among others, Tichý's own theory from (1978). In Tichý (1984), the truth of subjunctive conditionals depends, *inter alia*, on a certain unarticulated, tacitly assumed parameter (such as usual laws of nature).

(or even attitudes), as discriminating between possible worlds. He puts them in an intensional basis:

We thus see that a possible world is not fully described in terms of observable events such like X's being dry, X's being stuck, and X's burning, taking place at definite times. What makes a world the world it is are also *connections* which hold between such events. Worlds differ from one another not just in what observable events take place in them, but also which events have the power bring others about. (Tichý – Oddie 1983, 136)

However, we may find that Tichý already considered nexuses to be among the main characteristics of worlds in his (1978, 435).

Thus, the only decisive feature differentiating the third stage from the second one is Tichý's stress on *temporality*. In the third stage, he significantly employed verb tenses and also events/episodes for characterizing possible worlds.¹² Here is an illustration from the introductory parts of the papers which utilize temporality in explication of ability and freedom:

worlds must be allowed to *branch*: there must be worlds which are populated by the same individuals and whose histories are the same up to a certain time, and different after that time. (Tichý – Oddie 1983, 135)

As the possession of an attribute by an object is a time-dependent affair, the possibilities are, more particularly, *possible histories* of the distribution. (Oddie – Tichý 1982, 228)

5. Tichý's late conception: determination systems

Tichý's late conception is clearly recognizable from its stress on the idea of determination systems. These are discussed only in his masterpiece *The Foundations of Frege's Logic* (see Tichý 1988) and in Tichý (1994), which is a posthumously published introduction to his just written, but unfinished, book *Meaning Driven Grammar* which aimed to provide a highly ambitious semantic analysis of natural language. In his explanations, Tichý presup-

¹² See mainly his excellent analyses of verb tenses and temporal adverbials in (1980) and analyses of episodic verbs and verb aspects, as well as *events* and *episodes*, in (1980a). See also his philosophical defence of temporal dependence of truth (1980b).

posed a reader who has only a cursory knowledge of possible worlds; he even cited no his earlier relevant paper.

To introduce determination systems, Tichý deploys the notion of fact:

What are facts? The notion of fact is correlative with that of determiner. To each fact there corresponds a determiner in such a way that the fact consist either in determiner's singling out a definite object or in its failing to single out anything at all. The fact that Scott is the author of *Waverley*, for instance, consists in determiner A's [= THE AUTHOR OF WAVERLEY's] singling out Scott. The fact that the author of *Waverley* is a poet consists in the proposition P's [= THAT THE AUTHOR OF WAVERLEY IS A POET's] singling out [the truth-value] T. (Tichý 1988 178)

Obviously, this is nothing but Tichý's 'tests-on-individuals' story. Instead of "attribute" or "office", Tichý uses the term "*determiner*"; determiners are explicated (in Tichý 1988, 198) as possible world intensions. Instead of "test with a positive/negative outcome", Tichý says *fact*; facts are actual (i.e. obtaining) or possible. Tichý explicated the notion of fact as proposition because he considered *A IS GREATER THAN B* and *B IS SMALLER THAN A* being one and the same intuitive fact. Tichý's most elaborated view on the notion of fact appears in his re-examination of Wittgenstein's *Tractatus* (cf. Tichý 1994a).

Intuitive possible worlds are *total* (i.e. maximal) collections of facts:

Thus the determiner's [= THE AUTHOR OF WAVERLEY] picking out George IV is a *possible* fact and so is its picking out nothing at all. Hence a possible world, if conceived as a totality of possible facts, will be fully characterized by an assignment of objects (of appropriate kinds) to some determiners. Let us call such assignment a *determination system*. A determination system is thus any many-to-one correspondence associating (some) individual determiners with individuals, (some) truth-value determiners with truth-values, etc. (Tichý 1988, 178)

Briefly, a determination system specifies one combinatorial possibility as to what objects are determined (or singled out) by what intensions at what times. (Tichý 1988, 199)

The determiners involved in intensional basis can be of various types:

[determiners] may come classified into categories, like colours, heights, propositional attitudes, and the like. (Tichý 1988, 199)

Realize that every combinatorially possible association of determiners with determinees is a determination system only *relative to a given epistemic framework*.

Let us assume an illustrative epistemic framework with a universe only containing $A(\text{lan})$ and $B(\text{arbra})$ and intensional basis only containing attributes (determiners) MAN and WOMAN. When ignoring partiality, there are exactly 16 mappings associating the determiners with their possible values (cf. Tichý 1988, 178), i.e. 16 determination systems:

	<i>MAN</i>	<i>WOMAN</i>		<i>MAN</i>	<i>WOMAN</i>
1.	$\{A, B\}$	$\{A, B\}$	9.	$\{B\}$	$\{A, B\}$
2.	$\{A, B\}$	$\{A\}$	10.	$\{B\}$	$\{A\}$
3.	$\{A, B\}$	$\{B\}$	11.	$\{B\}$	$\{B\}$
4.	$\{A, B\}$	\emptyset	12.	$\{B\}$	\emptyset
5.	$\{A\}$	$\{A, B\}$	13.	\emptyset	$\{A, B\}$
6.	$\{A\}$	$\{A\}$	14.	\emptyset	$\{A\}$
7.	$\{A\}$	$\{B\}$	15.	\emptyset	$\{B\}$
8.	$\{A\}$	\emptyset	16.	\emptyset	\emptyset

As Tichý explains in (1988, 178-179), since determiners are not always mutually independent, not every combinatorially possible determination system is *realizable*. The two determiners MAN and WOMAN cannot pick out overlapping classes of individuals. Thus, not every determination system is a possible world. On the other hand, every possible world is a determination system. Given our intensional basis and universe of discourse, only the determination systems whose numerals are written in bold (4, 7-8, 10, 12-16) count as (intuitive) possible worlds; we therefore have 9 possible worlds altogether.¹³

¹³ In Tichý (1988, 179), Tichý speaks about 8 worlds but, when discussing an entirely analogous example in his (1994, 60), he mentioned the number 9, which is, I maintain, the correct one (my reason occurs in the next paragraph).

Before proceeding further, I should state a few remarks explaining Tichý's position. The possible world no. 16 seems to be a suspicious *empty world*. However, this idea is mistaken. Firstly realize that Tichý's worlds are not individuated by a certain population of individuals.¹⁴ The possible world no. 16 is simply a world in which *A* and *B* are tested on instantiation of the properties MAN and WOMAN with a negative result. The individuals have different properties than the two primary ones contained in our miniature intensional basis, e.g. the property BE NOT A MAN.¹⁵

Unrealizable determination systems in some ways resemble *impossible possible worlds* which are discussed so much today (cf., e.g., Berto 2013; Vacek 2013). Impossible worlds are proposed for a variety of reasons, none of them are important for Tichý. For instance, he prevented the problem of logical omniscience by employing hyperintensional entities (cf. Section 6). Recall also that Tichý proposed possible worlds in connection with the idea of subject investigating the external, empirical reality. Both empty and impossible worlds are beyond such considerations. In other words, they are excluded on the basis of Tichý's pre-theoretic motivation.

Surprisingly, Tichý (1988) did not employ and develop an idea which suggests itself, viz. that a determination system is only a single '*slice*' (or 'time point', Kuchyňka – Račlavský 2014) of a possible world, which is thus a sequence of such slices. However, an anticipation of this idea can be found in (Oddie – Tichý 1982, 228, cf. the quotation above), its full expression is this:

we must think of it [i.e. possible world] as a world *history*, a whole course of events unfolding in time. In other words, a world must be conceived of not as a single distribution of the traits from the intensional base through the universe of discourse, but as a series of such distributions, one for each moment of time. (Tichý 1994, 62)

¹⁴ Vladimír Svoboda (2001) imagined that individuals may escape 'our world', moving thus to the world no. 16, by losing any (remarkable) property and reaching a kind of 'limbo'. A closer examination of this idea (e.g. Račlavský 2008b) leads to the conclusion that such proposal contradicts Tichý.

¹⁵ Some metaphysicians reject 'negative' properties but Tichý belongs to the opposite camp: BE A NON-FERROUS, for instance, is a good property the possession of which can be sensibly empirically tested. When properties are explicated as possible world intensions (cf. Račlavský 2011, I.2) any trace of negativity evaporates, because no intension displays any mark of negation.

So far, we have an intuitive notion of possible world as an entity which contains facts whereas a fact consists in that a determiner picks out a determinee. What then are the logically simple entities W_1, W_2, \dots, W_n which are the arguments of Tichý's intensions?

The question is very important because Tichý's construal appears to be incoherent: Tichý often speaks about possible worlds in the spirit of combinatorialism but then he says that in his type-theoretic logical framework possible worlds are members of the type ω , which is an atomic type thus its members are primitive – this seems to mean that they do not possess combinatorialistic features.¹⁶

Oddie (1986, 125) first reacted to the problem and suggested to call these W s *proto-worlds* and assumed their correlation with the 'thick' possible worlds. Tichý in (1988, 194–200) enlightened us of the link between the two kinds of worlds as follows. (The following explanation appeared firstly in Raclavský 2009, 9–11.)

Firstly recall Carnap's notion of explication: it consists in the replacing of an intuitive notion – here the notion of possible world as a collection of facts – by its rigorous mate, its explicatum. Now there is a small complication. Tichý explicated facts as possible world propositions, which are classes of world/time couples. As noted already by Stalnaker (1976) and even Adams (1974), neither mentioned by Tichý, there is a question how can possible worlds be classes of propositions when propositions are classes of worlds (or world/time couples).¹⁷

Tichý realized this *circularity problem* (cf. Tichý 1988, 194) and solved it by carefully distinguishing between logically primitive and complex entities and their role in explication.¹⁸ For Tichý, propositions are the primary goal of his investigation, thus possible worlds must be taken as *logically primitive*, while propositions will then be defined in terms of worlds.

¹⁶ As Pavel Cmorej reminded me in personal communication, these possible worlds are not quite pure entities because a system of functions based on such worlds encodes the properties of intuitive determination systems. Thus, they are rather surrogates of 'full blooded' worlds as we will show in our argument.

¹⁷ Oddie (1986, 125) considered a slightly different puzzle: possible worlds have to be specified even by some functions defined on worlds (e.g. nexuses between propositions); but how then can one specify a single world? Oddie's solution anticipated Tichý's.

¹⁸ A logically primitive entity differs from another entity of the same sort only by its different numeric identity.

But to explain anything, propositions must be tied into the system of *explication* to the intuitive notions because it does matter whether a particular class of world/time couples explicates the intuitive proposition ALAN IS A MAN or rather ALAN IS NOT A MAN. Such a link of a possible world proposition to its intuitive correlate is guaranteed by an *interpretation* of the members of ω and the two truth-values. Interpretation is obviously a reverse of explication:

Now to interpret the basic category ω is to assign to each of its members a unique determination system. ... The determination systems which are assigned, within the [given epistemic] framework, to ω -objects are called the *possible worlds* of the framework. (Tichý 1988, 199–200)

Let us briefly reflect on Tichý's position. Tichý's contribution to metaphysics consists in explaining possible worlds as determination systems. This is a refinement of the idea that possible worlds are total classes of compatible facts.¹⁹ For logical investigations, however, Tichý decided to use only *surrogates* of these worlds, namely the logically primitive entities W_1, W_2, \dots, W_n , which are interpreted by the possible worlds as determination systems (or rather their chronologies).

A metaphysician can be perhaps a bit uncomfortable with the fact that Tichý's framework, used by him and others for a number of useful explications of various notions, does not provide a 'full blooded' explication of the notion of possible world. Yet there is a way out of this suggested by Raclavský (2009a) who utilized ideas and apparatus from Raclavský (2008a) and Raclavský – Kuchyňka (2011).²⁰ We can retain Tichý's framework as it is, with all those explications, but we must rethink the role of W s. I suggest understanding W s as a mere *modal factor* (I suppress some questions related to interaction with temporality). A proposition, for instance, is clearly a function whose values depend on logical modality (modal factor). Possible worlds are then explicated as some other, complex entities which reflect the intuitive features of classes of facts, whereas facts are some structured entities,

¹⁹ Which goes back to C. I. Lewis (see Lewis 1923).

²⁰ The original motivation is Raclavský's solution (2007; and 2014 with Kuchyňka) to the puzzle studied already by Petr Kolář in his (e.g. 2002): Tichý rejected the idea of facts as structured entities by evoking the famous aRb fact; but then, there is the problem of how to preserve correspondence theory of truth which is based on some sort of isomorphism between sentences and their significance (facts).

not possible world propositions. It is interesting that such possible worlds would be very close to the worlds of Tichý's first stage conception (recall that those worlds were classes of structured procedures expressed by sentences).

6. Possible worlds in Tichý's Transparent intensional logic

In this section, I am going to briefly explain the role of possible worlds (as Ws) in Tichý's semantics and logic. It is important to stress that Tichý adopted the notion of possible world for utilization in philosophical logic, not as an object of metaphysical considerations; this is somehow obscured by the fact that in Tichý's work logic and philosophy go hand in hand.

An admissible introduction to Tichý's semantics likens it to Montague's semantics which was published mainly around 1970, cf. the posthumous collection (see Montague 1974). Tichý referred to Montague's work in (1971) and it seems that he became acknowledged with Montague's results after developing his own system. Unlike Montague, Tichý made a straightforward adaptation of Church's typed λ -calculus because he enriched Church's type basis containing o (two truth-values) and ι (individuals) by the type ω . Over such basis, there are numerous functions of composite types; for instance, the type of propositions is $(o\omega)$ (in another notation: $\omega \rightarrow o$). After his (1971), Tichý adopted partial functions, thus some intensions can be without a value (gappy). In the late 1970s, he admitted the temporal parameter τ ; *intensions* (e.g. propositions, properties, ...) are then functions from world/time couples (equivalently speaking, they are functions from worlds to 'chronologies' of objects; for the sake of brevity, I will omit temporal parameter).

Tichý's technical treatment of intensions is distinct from Montague's (cf. e.g. Tichý 1978b). Tichý's terms standing for intensions are λ -abstractions over possible worlds; but such λ -abstraction $\lambda w[\dots w\dots]$ can be combined with variable w ranging over possible worlds in order to express a recourse to a value of that intension in the value of w . This way Tichý could sensibly assign intensions to expressions having modally conditioned reference (common sentences, descriptions, predicates, etc.) in every context, including the transparent one,²¹ which is the main difference from Montague's approach.

²¹ This is why he called his system *Transparent intensional logic*, *TIL*.

λ is a variable binding operator and λ -formalism is capable of expressing subtle differences in scope. This enables a logically lucid treatment of *de re/de dicto* propositional attitudes as well as scopes of modal operators. Let us add that *modal operators* are classes of propositions, being of type $(o(o\omega))$, which means that they are ‘quantifiers’ of possible worlds. Famous puzzles concerning modality are quickly solvable in TIL (cf. Tichý 2004; Raclavský 2009).

Already in (1970), Lewis published a criticism of intensional (or possible world) semantics. His main argument roughly says that possible world intensions are too coarse-grained to be meanings of expressions. In other words, intuitive meanings have a more fine-grained structure. We thus need *hyperintensional entities* which would stand in semantic scheme between expressions and extensions/intensions signified by them. For another argument, when X believes that $1+1=2$, we can hardly entail that X believes Fermat’s Last theorem, despite that “ $1+1=2$ ” and the famous theorem stand for one and the same proposition which is true in all possible worlds.

After Tichý (1971), he found a solution to the problem. He realized that his λ -terms can be read in an extensional way as representing usual set-theoretic objects, e.g. possible world intensions, or they can be read in ‘intensional’ way as representing entities which determine the set-theoretic objects. Tichý called those ‘intensional’ entities “*constructions*” and he later explained (in his 1986) that he borrowed the term from geometry where a point or circle can be constructed one way or another. Constructions are structured, abstract, extra-linguistic entities which are akin to algorithmic computations (Tichý 1986, 526). For a defence of constructions and their careful description see esp. Tichý (1988).

Constructions can be used for capturing features of intuitive entities that cannot be explicated by means of mere possible world intensions. Tichý stressed (already in his 1976) mainly their usefulness as the explicata of *meanings*. His semantic scheme is thus entirely Churchian (‘Fregean’): an expression E expresses (in L) a construction C (= the meaning of E) whereas C constructs an intension or extension (= the significance of E). For instance, the meaning of the sentence “Fido is a dog” is the propositional construction $\lambda w[\mathbf{Dog}_w \mathbf{Fido}]$, which constructs the proposition FIDO IS A DOG. Propositional attitudes are explicated in Tichý (1988) as attitudes towards propositional constructions, not towards mere propositions; this proposal blocks undesirable consequences embraced by intensional semantics and solves the problem of *logical omniscience*.

The type of constructions is split. Tichý's late system is thus a special *ramified theory of types*. Consequently, it is easily capable of avoiding a number of *paradoxes concerning 'propositions'* (propositions not in the strict sense of possible world semantics). One of the best known paradoxes concerning possible worlds and 'propositions' is Kaplan's (1994). In Tichý (1988, sec. 42, 218 and *passim*), he reminded the reader of the fact proven by Cantor that there are more mappings from (nonempty) set S to (nonempty) set S' than there are members of S . Thus, there are more propositions than worlds: no mapping can associate, in 1-1 fashion, every proposition with a certain world (or world/time couple). Consequently, no matter how one explicates relations such as belief, assertion,... between individuals and propositions, "there will always be a proposition such that at no world/time couple is it the only proposition believed, or asserted, by George IV." (Tichý 1988, 219). Tichý demonstrated the situation and immediately proved Theorem 42.1, the corollaries of which provide his solution to the *Liar paradox* (which bears resemblance to his solution in his 1976).

7. Conclusions

To repeat the most essential features of Tichý's conception of possible worlds, Tichý proposed it in close connection with his sophisticated and extensive logical system. Similarly as other intensional logicians/semanticists, he used possible worlds for semantic analysis of natural language and also further explication of various notions of our conceptual scheme such as fact, causality, event, ability, freedom, propositional attitudes, subjunctive conditionals and modalities. Though it is problematic to subsume his conception under the known ones, "combinatorialism" and "actualism" seem to be the most appropriate labels.

Tichý's first proposal was published in 1968. His conception underwent various modifications, but its main character remained unchanged. In the first stage, possible worlds were classes of certain procedures (algorithms) consisting mainly of tests on individuals. In the second and subsequent stages, they were intuitively total classes of facts but Tichý technically treated them as arguments for his intensions, i.e. as logically primitive entities. The combinatorial character was underlined by his late conception in which the intuitive possible worlds are explicitly specified as some determination systems; determiners are then explicated as intensions.

A tension in textual evidence is caused by the fact that Tichý always described possible worlds in the spirit of combinatorialism but, at the same time, he treated them as logically primitive entities of his logical framework. Above, I have reconciled the two views utilizing Tichý's late suggestion that the former worlds are only intuitive, pre-theoretic possible worlds, while the latter worlds are their rigorous explicata.

Tichý usually stressed intensional dependence of attributes/determiners and also the idea that worlds are individuated even by the nexuses and attitudes which are realized in them. In the late 1970s, he added temporality. Tichý always preferred fixed domain; he modelled 'possibilia' as individual offices, i.e. some intensions. Since his recent type-theoretic framework is explicitly ramified, his approach can avoid various paradoxes concerning possible worlds and 'propositions'.

Some questions not answered by Tichý, but posed and answered by some his followers, concern the structured nature of facts, the nature of correspondence, and the possibility to construct possible worlds from structured facts. Another important task is to elaborate consequences of temporality involved in Tichý's conception of possible worlds.²²

References

- ADAMS, R.M. (1974): Theories of Actuality. *Noûs* 8, No. 3, 211-231.
- ARMSTRONG, D.M. (1989): *A Combinatorial Theory of Possibility*. Cambridge University Press.
- BERTO, F. (2013): Impossible Worlds. In: Zalta, E. N. (ed.): *The Stanford Encyclopedia of Philosophy*. (Winter 2013 Edition). Available at: <http://plato.stanford.edu/archives/win2013/entries/impossible-worlds/>.
- CARNAP, R. (1947): *Meaning and Necessity*. The University of Chicago Press.
- CHURCH, A. (1940): A Formulation of the Simple Theory of Types. *The Journal of Symbolic Logic* 5, No. 2, 56-68.
- CMOREJ, P. (1996): Empirické esenciálne vlastnosti. *Organon F* 3, No. 3, 239-261.
- CMOREJ, P. (2001): Esencializmus verzus antiesencializmus. In: *Na pomedzí logiky a filozofie*. Bratislava: Veda, 91-113.
- COPELAND, B.J. (2006): Meredith, Prior, and the History of Possible Worlds Semantics. *Synthese* 150, No. 3, 373-397.
- DIVERS, J. (2002): *Possible Worlds*. Routledge.

²² Thanks to Pavel Cmorej and both anonymous reviewers for their valuable suggestions.

- HAACK, S. (1978): *Philosophy of Logics*. Cambridge University Press.
- HINTIKKA, J. (1969): *Models for Modalities*. Dordrecht: Reidel.
- KAPLAN, D. (1994): A Problem in Possible-World Semantics. In: Sinnott-Armstrong, W. – Raffman, D. – Asher, N. (eds.): *Modality, Morality and Belief: Essays in Honor of Ruth Barcan Marcus*. Cambridge University Press, 41-52.
- KOLÁŘ, P. (2003): *Pravda a fakt*. Praha: Filosofia.
- KEMENY, J.G. (1951): Review of R. Carnap, Logical Foundations of Probability. *The Journal of Symbolic Logic* 16, 205-207.
- KRIPKE, S.A. (1963): Semantic Considerations on Modal Logic. *Acta Philosophica Fennica* 16, 83-94.
- KUCHYŇKA, P. – RAČLAVSKÝ, J. (2014): *Pojmy a vědecké teorie* [Concepts and Scientific Theories]. Brno: Masarykova univerzita.
- LEWIS, C.I. (1923): Facts, Systems, and the Unity of the World. *The Journal of Philosophy* 20, No. 6, 141-151.
- LEWIS, D. (1970): General Semantics. *Synthese* 22, 18-67.
- LEWIS, D. (1973): *Counterfactuals*. Cambridge (Mass.): Harvard University Press.
- LEWIS, D. (1986): *On the Plurality of Worlds*. Blackwell.
- MENZEL, C. (2014): Possible Worlds. In: Zalta, E. N. (ed.): *The Stanford Encyclopedia of Philosophy*. (Summer 2014 Edition). Available at: <http://plato.stanford.edu/archives/sum2014/entries/possible-worlds/>.
- MONTAGUE, R. (1974): *Formal Philosophy*. New Haven – London: Yale University Press.
- NOLAN, D. (2002): *Topics in the Philosophy of Possible Worlds*. Routledge.
- ODDIE, G. (1986): *Likeness to Truth*. Dordrecht: Reidel.
- ODDIE, G. (1987): The Picture Theory of Truthlikeness. In: Kuipers, T.A.F. (ed.): *What is Closer-to-the-Truth?* Amsterdam: Rodopi, 25-45.
- ODDIE, G. – TICHÝ, P. (1982): The Logic of Ability, Freedom, and Responsibility. *Studia Logica* 41, 2-3, 227-248.
- ODDIE, G. – TICHÝ, P. (1990): Resplicing Properties in the Supervenience Base. *Philosophical Studies* 58, No. 3, 259-269.
- PLANTINGA, A. (1970): World and Essence. *Philosophical Review* 79, 461-492.
- RAČLAVSKÝ, J. (2008): Reformulating Tichý's Conception of Bare Individuals. *Organon F* 15, No. 2, 143-167.
- RAČLAVSKÝ, J. (2008a): Conceptual Dependence of Verisimilitude Vindicated. A Farewell to Miller's Argument. *Organon F* 15, No. 3, 369-382.
- RAČLAVSKÝ, J. (2008b): Holá individua nejsou bez vlastností [Bare Individuals are not without Properties]. In: Zouhar, M. (ed.): *Jednotliviny, všeobecniny, významy*. Bratislava: Filozofický ústav SAV, 158-172.
- RAČLAVSKÝ, J. (2009): *Jména a deskripce: logicko-sémantická zkoumání* [Names and Descriptions: Logico-Semantical Investigation]. Olomouc: Nakladatelství Olomouc.
- RAČLAVSKÝ, J. (2009a): Structured Language Meanings and Structured Possible Worlds. In: Munz, V. A. – Puhl, K. – Wang, J. (eds.): *Language and the World*. Wien: Austrian Ludwig Wittgenstein Society, 349-350.

- RACLAVSKÝ, J. (2010): Co obnáší kontingentní existence individuí [What is Contingent Existence of Individuals?]. *Organon F* 17, No. 3, 374-387.
- RACLAVSKÝ, J. (2011): *Individua a jejich vlastnosti: studie z intenzionální metafyziky [Individuals and Their Properties: Studies in Intensional Metaphysics]*. Olomouc: Nakladatelství Olomouc.
- RACLAVSKÝ, J. – KUCHYŇKA, P. (2011): Conceptual and Derivation Systems. *Logic and Logical Philosophy* 20, 1-2, 159-174.
- STALNAKER, R.C. (1976): Possible Worlds. *Noûs* 10, No. 1, 65-75.
- TICHÝ, P. (1966): K explikaci pojmu obsah věty [On Explication of the Notion of Content of a Sentence]. *Filosofický časopis* 14, 364-372.²³
- TICHÝ, P. (1968): Smysl a procedura [Sense and Procedure]. *Filosofický časopis* 16, 222-232.
- TICHÝ, P. (1968a): *Logická stavba vědeckého jazyka [The Logical Structure of Scientific Language]*. Praha: Státní pedagogické nakladatelství.
- TICHÝ, P. (1969): Intensions in Terms of Turing Machines. *Studia Logica* 26, 7-52.
- TICHÝ, P. (1971): An Approach to Intensional Analysis. *Noûs* 5, No. 3, 273-297.
- TICHÝ, P. (1972): Plantinga on Essence: A Few Questions. *The Philosophical Review* 81, No. 1, 82-93.
- TICHÝ, P. (1974): On Popper's Definitions of Verisimilitude. *British Journal for the Philosophy of Science* 25, No. 2, 155-188.
- TICHÝ, P. (1975): What Do We Talk About? *Philosophy of Science* 42, No. 1, 80-93.
- TICHÝ, P. (1976): *Introduction to Intensional Logic*. Unpublished book manuscript.
- TICHÝ, P. (1976a): Verisimilitude Redefined. *The British Journal for the Philosophy of Science* 27, No. 1, 25-42.
- TICHÝ, P. (1976b): A Counterexample to the Stalnaker-Lewis Analysis of Counterfactuals. *Philosophical Studies* 29, No. 4, 271-273.
- TICHÝ, P. (1978): A New Theory of Subjunctive Conditionals. *Synthese* 37, No. 3, 433-457.
- TICHÝ, P. (1978a): Verisimilitude Revisited. *Synthese* 38, No. 2, 175-196.
- TICHÝ, P. (1978b): Two Kinds of Intensional Logic. *Epistemologia* 1, 143-164.
- TICHÝ, P. (1978c): *De Dicto* and *De Re*. *Philosophia* 8, No. 1, 1-16.
- TICHÝ, P. (1978d): Verisimilitude Revisited. *Synthese* 38, No. 2, 175-196.
- TICHÝ, P. (1979): Existence and God. *The Journal of Philosophy* 76, No. 8, 403-420.
- TICHÝ, P. (1980): The Logic of Temporal Discourse. *Linguistics and Philosophy* 3, No. 3, 343-369.
- TICHÝ, P. (1980a): The Semantics of Episodic Verbs. *Theoretical Linguistics* 7, No. 3, 264-296.
- TICHÝ, P. (1980b): The Transiency of Truth. *Theoria* 46, 2-3, 165-182.
- TICHÝ, P. (1984): Subjunctive Conditionals: Two Parameters vs. Three. *Philosophical Studies* 45, No. 2, 1-13.
- TICHÝ, P. (1986): Constructions. *Philosophy of Science* 53, No. 4, 514-534.

²³ Tichý's papers are reprinted in Tichý (2004).

- TICHÝ, P. (1994): The Analysis of Natural Language. *From the Logical Point of View* 3, No. 2, 42-80.
- TICHÝ, P. (1994a): The *Tractatus* in the Light of Intensional Logic. *From the Logical Point of View* 3, No. 2, 32-41.
- TICHÝ, P. (1987): Einzeldinge als Amtsinhaber. *Zeitschrift für Semiotik* 9, 13-50. Transl. to Slovak as Tichý, P. (1994): Jednotliviny a ich roly [Particulars and their Roles]. *Organon F* 1, No. 1, 29-42; No. 2, 123-132; No. 3, 208-224; No. 4, 328-333.
- TICHÝ, P. (2004): *Pavel Tichý's Collected Papers in Logic and Philosophy*. Svoboda, V. – Jespersen, B. – Cheyne, C. (eds.). Dunedin: University of Otago Press; Praha: Filozofia.
- TICHÝ, P. (1988): *The Foundations of Frege's Logic*. Walter de Gruyter.
- TICHÝ, P. – ODDIE, G. (1983): Ability and Freedom. *American Philosophical Quarterly* 20, No. 2, 135-147.
- SVOBODA, V. (2000): Where Do All the Individuals Go? In: Childers, T. – Palomäki, S. (eds.): *Between Words and Worlds (A Festschrift for Pavel Materna)*. Praha: Filozofia, 21-33.
- VACEK, M. (2013): Impossibilists' Paradise on the Cheap? *Organon F* 20, No. 3, 283-301.
- WITTGENSTEIN, L. (1921/1922): *Tractatus logico-philosophicus*. Routledge & Kegan Paul.
- YAGISAWA T. (2010): *Worlds and Individuals, Possible and Otherwise*. Oxford University Press.