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Communication in a Multi-Cultural World[1]

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ABSTRACT: The goal of this paper is to demonstrate that procedurally structured concepts are central to human communication in all cultures and throughout history. This thesis is supported by an analytical survey of three very different means of communication, namely Egyptian hieroglyphs, pictures, and Inca knot writing known as khipu. My thesis is that we learn, communicate and think by means of concepts; and regardless of the way in which the meaning of an expression is encoded, the meaning is a concept. Yet we do not define concepts within the classical set-theoretical framework. Instead, within the logical framework of Transparent Intensional Logic, we explicate concepts as logical *procedures* that can be assigned to expressions as their context-invariant meaning. In particular, complex meanings, which structurally match complex expressions, are complex procedures whose parts are sub-procedures. The moral suggested by the paper is this. Concepts are not flat sets; rather, they are algorithmically structured abstract procedures. Unlike sets, concepts have constituent sub-procedures that can be executed in order to arrive at the product of the procedure (if any). Not only particular parts matter, but also the way of combining these parts into one whole 'instruction' that can be followed, understood, executed, learnt, etc., matters.

KEYWORDS: Communication – concept – procedural isomorphism – structured meaning – Transparent Intensional Logic.

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In the beginning was the Word,
and the Word was with God,
and the Word was God.
(John 1:1)

0. Introduction

How do we communicate? How is it possible that we (more or less) understand each other despite different cultural and historical backgrounds, great cultural differences and language barriers? A seemingly simple answer might be this: because we are all human beings. Unlike machines, we *adapt* to new ideas, new environments, new cultures; we are able to *learn* from experience, we have the ability of *empathy*. Yet, how is it possible that we are able to learn a (new) language? On its standard conception, a language is a (potentially) infinite set of expressions. In order to reach such an infinity we need a 'clue instruction' that makes it possible to get to know *any* element of the infinite set in a finite number of steps.[2] In this paper I am going to present the idea that such a 'clue instruction' is a *procedurally structured concept*.[3]

I am not going to be involved in a particular theory of language; nor am I going to contribute to the endless discussions on where language comes from (see, for instance, Nordquist 2013). There are so many theories of language that it would be a futile contribution in this short study. I will assume that regardless of the way in which a meaningful expression is encoded, it is always a code endowed with a meaning.

In Transparent Intensional Logic (TIL), which is my background theory, we explicate structured meanings *procedurally*. An empirical expression *E* encodes an instruction of how, in any possible world at any time, to execute the procedure expressed by the expression as its meaning. Based on this explication I am going to present possible answers to the questions posed at the outset. My thesis is the following. We learn, communicate and

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[2] I accept Kant's finitist view in the sense that there is no upper bound on the number of steps we execute. No matter how many steps we may have executed, we can always move one step further. But at any point we will have acquired only a *finite* amount of experience and have taken only a finite number of steps.

[3] For the procedural theory of concepts, see Duží – Jespersen – Materna (2010, Chap. 2) and also Materna (1998).

think by means of *procedurally structured* concepts. Regardless of the way in which the meaning of an expression is encoded, its *meaning* is a *concept*. Yet I do *not* explicate concepts within the classical set-theoretical framework.[4] The moral I extract from this paper is this: Concepts are *not flat sets*; rather, concepts are *algorithmically structured, abstract procedures*. Unlike sets, concepts have *constituent sub-procedures* that can be executed in order to arrive at the product of the procedure (if any). Not only particular parts matter, but also the *way of combining these parts into one whole instruction* that can be followed, understood, executed, learnt, etc., matters. We explicate concepts as logical procedures that can be assigned to expressions as their context-invariant meaning; in particular, complex meanings, which structurally match complex expressions, are complex procedures whose parts are sub-procedures. For instance, the simple sentence "Tom is wise" encodes an instruction of how, in any possible world w at any time t, to evaluate its meaning in order to arrive at a truth-value. The respective procedure consists of these constituent sub-procedures: take the individual Tom; take the property of being wise; extensionalize the property with respect to the world w and time t of evaluation; produce a truth-value **T** or **F** according as Tom has the property of being wise in that world w and at that time t of evaluation.

Traditionally, concepts are often conceived of as mental objects. Yet already in 1837 Bolzano dealt a serious blow to the psychologistic tradition of concepts. In his *Wissenschaftslehre* Bolzano worked out a systematic realist theory of concepts, construing concepts as objective entities endowed with *structure*. Our theory embraces this conception.

The procedural character of structured mathematical concepts should be obvious. For instance, when one is seeking the solution of the equation $sin(x) = 0$ they are not related to the infinite set $\{..., -2\pi, -\pi, 0, \pi, 2\pi, ...\}$, because otherwise the seeker would immediately be a finder and there would be nothing to solve. On the other hand, relating the seeker to a particular syntactic term is not general enough. The Ancient Greek mathematicians, for instance, would solve such an equation using a different syntactic system. Any seeker, whether Greek or Babylonian, modern or extraterrestrial, is related to the *structured meaning* of "$sin(x) = 0$", which is the very *procedure* consisting of these constituents: applying the function sine to a real number x, checking whether the value of the function is zero, and if

[4] For instance, a Fregean concept is a characteristic function of objects (*Gegenstände*).

so abstracting over the value of the input number *x*. When solving the equation the seeker aims to execute this procedure in order to produce the infinite set of multiples of π.

This *procedural character* of mathematical concepts is *universal*. For instance, Ascher (2002) is an important contribution to a global view of mathematics. It humanizes our view of mathematics and expands our conception of what is mathematical. Ascher demonstrates that traditional cultures have mathematical ideas that are far more substantial and sophisticated than is generally acknowledged. Many ideas taken to be the exclusive province of professionally trained Western mathematicians are, in fact, shared by people in many societies. The ideas discussed come from geographically varied cultures, including the Borana and Malagasy of Africa, the Tongans and Marshall Islanders of Oceania, the Tamil of South India, the Basques of Western Europe, and the Balinese and Kodi of Indonesia. And Ascher reminds us of how *mathematical and logical procedures* are *universal* across any culture.

The same universal *procedural structures* govern our communication and reasoning not only in mathematics but also in ordinary life. As a semantic realist I am convinced that logic should assist in unearthing the objective structures underlying the expressions of a given language.

The rest of the paper is organized as follows. To demonstrate the thesis that procedural structures are central to human communication, in Section 1 I briefly examine three very different means of communication: Egyptian hieroglyphs, pictures, and Inca knot writing. I will argue that all these very different means of communication share a common procedural character; to wit, they are structures endowed with a procedural sense. Section 2 introduces the foundations of the logical background within which we define concepts as structured procedures, namely TIL, and the procedural definition of concepts is reproduced here. Concluding remarks are presented in Section 3.

1. Hieroglyphs, pictures, khipu

1.1. Hieroglyphic writing is not a pictorial script

The hieroglyphic writing system of the Ancient Egyptians is sometimes taken as the example *par excellence* of a *purely pictorial script*. Most prominently, Ludwig Wittgenstein draws an analogy between the pictorial semantics of the *Tractatus Logico-Philosophicus* and the alleged pictorial script of hieroglyphic writing:

> In order to understand the essence of the proposition [*Satz*], consider hieroglyphic writing, which pictures the facts it describes. And from it came the alphabet without the essence of the representation being lost. (Wittgenstein 1922, §4.016)

Yet there are many arguments against this conception. Here I will briefly reproduce the arguments of Jespersen – Reintges (2008). The authors criticize Wittgenstein's conception, claiming that Egyptian hieroglyphic writing is *not* a pictorial script. Their criticism can be summarized as follows. For more than three millennia, hieroglyphic writings were continuously used for the codification of a varied collection of Ancient Egyptian texts, ranging from the monumental religious corpora of the Pyramid Texts and the Book of the Dead and the elaborate historiographical inscriptions of Egyptian temples to all sorts of legal and administrative texts, bills, recipes, love letters, and so forth. The replacement of hieroglyphic writing by a Greek-based alphabet in late antiquity and early medieval Christian Egypt was *not* motivated by considerations of *efficiency*, but rather had an ideological basis, namely its association with the pagan Pharaonic culture. If the hieroglyphs were organized in a pictographic system, how come that we *cannot* read Ancient Egyptian texts right away, although we can readily identify hieroglyphic signs as graphic depictions of human beings, birds, fishes, reptiles, weapons, household equipment, and so on? The reason why we cannot read hieroglyphic texts is simply that the hieroglyphic writing system is *not pictographic in nature*. In the era of the Old Kingdom, the Middle Kingdom and the New Kingdom, about 800 hieroglyphs existed. By the Greco-Roman period, they numbered more than 5000. This means that the inventory of Egyptian hieroglyphs ran to maximally *5000 different characters*, yet the universe of discourse of such a highly complex culture as the one of Ancient Egypt certainly included far more than 5000 different *concrete objects* for depiction. Moreover, if we were to assume, counterfactually, that hieroglyphic writing were a pictographic system, the question naturally arises how it could possibly indicate *abstract objects*, such as truth, beauty, love, or justice.

Even the meaning of the word 'hieroglyph' throws doubts on the conception of a purely pictorial script. 'Hieroglyph' is a compound of 'hierós' (sacred) and 'glýpho' (engrave, carve), which is the translation of Egyptian *medu-netjer* ('God's words'). Hence Egyptian hieroglyphs are sacred engraved *words*. They consist of three kinds of glyphs: phonetic glyphs, including single-consonant characters that function like an alphabet; logographs, representing morphemes; and determinatives, which narrow down the meaning of logographic or phonetic words. Hence it should be clear that hieroglyphic writing has the capacity of rendering complex *structures*.

1.2. The structure of pictures

The second source I am going to exploit is Westerhoff (2005). The author analyses the structure of pictures, and argues (italics mine):

> Pictures differ from paintings as propositions differ from sentences. Paintings and sentences are tokens: spatio-temporally located physical objects. Different paintings can show the same picture, and different sentences can express the same proposition. (Westerhoff 2005, 605)

And he asks: What are *the parts of a picture*? The question is not as innocuous as it may sound. The mereology of ordinary physical objects is well-developed, but pictures are not ordinary physical objects. First of all, they are *not spatio-temporal*; rather, they are abstract objects. Secondly, they are *structured*: they are not like a heap of grain or a puddle of water the identity of which is preserved under various rearrangements of their parts. Pictures have parts which are *put together in a certain way*; if we destroy this way of putting the parts together the picture is gone.

Westerhoff criticizes the commonly accepted opinion that possible-world semantics is a proper tool for explaining the semantics of structures.

> Consider the sense in which states of affairs (possible worlds) could be taken to have parts. It is straightforward to argue that the state of affairs that John loves Becca has John as a part. But it is equally straightforward to argue that John's brain is part of the state of affairs that John loves Becca. But the mere parts (John's brain as opposed to John) are just any parts of that particular bit of the world we happen to be talking about, whether they take part in our *conceptualization* or not. (Westerhoff 2005, 609)[5]

[5] Similar arguments can be found also in Tichý (1995, 179-180).

Possible-world semantics receives a fare amount of criticism also in Blumson (2010) where the structure of pictures is investigated:

> Depictions, like thoughts and sentences, distinguish between different ways things might be; the Mona Lisa, for example, represents Lisa by distinguishing amongst the various possible ways which Lisa might have looked. It suggests that the content of the Mona Lisa, for example, should be analysed in terms of the possible worlds in which Lisa's appearance is as the picture portrays. (Blumson 2010, 135)

But how could this be so? First, one and the same painting can be seen from different perspectives in which Lisa's appearance is completely different. Second, and more importantly, the possibility of depicting *logical* or *a priori impossibilities* is directly problematic for the analysis of depictive content in terms of possible worlds. A straightforward argument against the possibility of capturing the structure of a picture by possible-world semantics is the Penrose triangle (see Penrose – Penrose 1958). It is a picture of an *a priori*, rather than merely *a posteriori*, impossibility, as illustrated by Fig. 1. The content of the picture cannot be analyzed as a subset of possible worlds, because the depicted triangle does not exist in any possible world.

Fig. 1. Penrose triangle

So much for Blumson's criticism of possible-world semantics; I would like to add the following note. True, there are theories that attempt to account for absurd objects (like round squares or Penrose triangle) by introducing a parallel logical space of logically impossible worlds (cf. Priest 1992). But just as little as the number five belongs to the domain of possible worlds and just as little as mathematical sentences are evaluated at possible worlds, so round squares or Penrose triangle should not be assigned to the domain of any impossible world. The very idiom of worlds, whether possible or impossible, is out of place, as soon as non-empirical objects like numbers and figures are involved. Yet I will show that terms like 'round square', 'the greatest prime' or 'Penrose triangle' are not meaningless expressions, though they lack a denotation, and we can handle them without the category of impossible worlds. Their meanings are empty concepts, which we explicate as procedures that do not produce any product.

1.3. Inca khipu

The last interesting means of communication that I am going to consider is the Inca knot writing known as *khipu* that has been used since 2000 BC. The Inca people of South America appeared to be the only civilization of all the major Bronze Age civilizations that apparently lacked a written language, an exception that embarrassed the anthropologists who habitually include writing as an attribute of a complex, highly developed culture deserving to be ranked as a *civilization*.

Khipu are textile artefacts composed of cords of cotton. There is a main primary cord from which many pendant cords hang, and there may be additional subsidiary cords attached to a pendant cord. Some khipu have up to 12 levels of subsidiaries. Each khipu cord may have one or more knots, see Fig. 2.

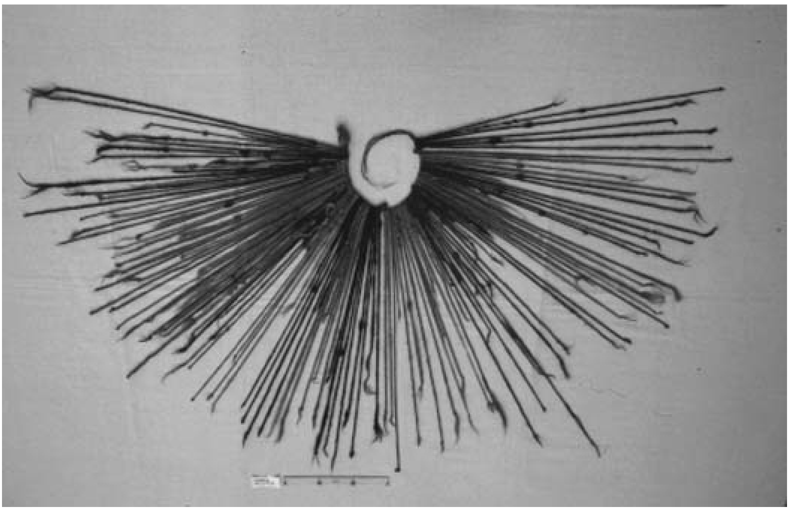

Fig. 2. Inca khipu

In the conventional view of scholars, most khipu represented decimal numbers for bookkeeping and census purposes. Hence the khipu were considered to serve as textile abacuses rather than written documents. Yet some well-informed colonial writers insisted that some khipu encoded stories and poems.

Recently, Urton recognized the depth of information contained in structured elements of khipu, and a growing number of researchers now think that khipu were mostly non-numerical and may have primarily been an early form of writing. A reading of the knotted string devices, if deciphered, could perhaps reveal narratives of the Inca Empire, the most extensive in America in its glory days before the Spanish conquest in 1532. How could cords encode a language? In an age when computers process immense amounts of information by the manipulation of sequences of 1's and 0's, it remains a frustrating mystery how prehistoric Inca record-keepers encoded a tremendous variety and quantity of information using only knotted and dyed strings. Yet the comparison between computers and khipu may hold an important clue to deciphering the Inca records.

In Urton (2003) a path-breaking theory is presented. The construction of khipu fibres constitutes *binary-coded* sequences which store units of information in a similar way as today's computers do. Urton begins his theory with the making of khipu, showing how at each step of the process binary either/or choices were made. He then investigates the symbolic components of the binary coding system, the amount of information that could have been encoded, procedures that may have been used for reading the khipu, the nature of the khipu signs, and, finally, the nature of the khipu recording system itself, emphasizing relations of semantic coupling. This research constitutes a major step forward in building a unified theory of the khipu system of information storage and communication based on the totality of construction features making up these extraordinary objects.

Needless to say, the meaning of the khipu coding system cannot be analysed within an intensional semantics such as possible-world semantics. Sets of possible worlds cannot be binary-recorded; rather, it is reasonable to assume that similarly as the binary code of a computer program is a record of the procedure to be executed, khipu writing is most probably a code of *structured* meanings best explicated as abstract *procedures*.

This completes our historical excursion into very different means of communication. Now I am going to introduce a recent theory of concepts defined as abstract procedures within the logical framework of Transparent Intensional Logic.

2. Transparent Intensional Logic (TIL)

2.1. Foundations of TIL

In possible-world semantics, which was the prevailing semantic theory in the last century, meanings are mappings defined on a domain of possible worlds, and meanings are co-intensional, i.e. identical, when they are necessarily co-extensional. As we have seen above, possible-world semantics is not a tool apt for analysis of structured meanings. Co-intensionality is nothing other than necessary co-extensionality. Moreover, possible-world intensions lack structure altogether. They are just flat set-theoretical mappings. The consequences for analysis of natural language are well-known; linguistic senses and attitude contents are too coarsely individuated; attitudes proliferate too rapidly, etc. Thus since the late 1960s many logicians have been striving for *hyperintensional semantics* and *structured meanings*.

Recent development can be characterised as an algorithmic or procedural turn. In (1994) Moschovakis put forward the idea of *meaning* as *algorithm*. Yet much earlier, in Tichý (1968; and 1969), Tichý had already formulated the idea of *procedural* (as opposed to set-theoretical denotational) *semantics*, according to which the sense of an expression is an algorithmically structured procedure detailing what operations to apply to what procedural constituents to arrive at the object (if any) denoted by the expression. Such procedures are rigorously defined as TIL *constructions*. Tichý developed a logical framework known today as *Transparent Intensional Logic* (TIL) (see Tichý 1988; and 2004).

Referring for details to numerous papers on TIL, and in particular to two recent books, to wit Duží – Jespersen – Materna (2010), and Duží – Materna (2012), in what follows I provide a brief summary of those features of TIL procedural semantics which we need for the definition of structured concepts. Formally, TIL is an extensional logic of hyperintensions based on the partial, typed λ-calculus enriched with a ramified type structure to accommodate hyperintensions.[6] Thus the syntax of TIL is Church's (higher-order) typed λ-calculus with the important difference that the syntax has been assigned a *procedural* (as opposed to denotational) semantics. TIL λ-terms do not denote functions-in-extensions, which are set-theoretical

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6 Parts of this section draw on material from Duží – Jespersen – Materna (2010, Chap. 1; Chap. 2).

mappings; rather they denote procedures (*constructions* in TIL's terminology) that produce functions or functional values as their products.[7] The sense of an expression is an abstract *procedure* detailing how to arrive at an object of a particular logical type denoted by the expression. TIL *constructions* are such procedures. Thus, abstraction transforms into the molecular procedure of forming a function, application into the molecular procedure of applying a function to an argument, and variables into atomic procedures for arriving at their values assigned by a valuation.

There are two kinds of constructions, atomic and compound (molecular). Atomic constructions (*Variables* and *Trivializations*) do not contain any other constituents but themselves; they supply objects (of any type) on which compound constructions operate. The *variables* x, y, p, q, ... construct objects dependently on a valuation; they are said to v-construct. The *Trivialisation* of an object X (of any type, even a construction), in symbols: ${}^0X$, constructs simply X without the mediation of any other construction; we say that ${}^0X$ is the *simple concept* of X. *Compound* constructions, which consist of other constituents than just themselves, are *Composition* and *Closure*. The *Composition* $[F\, A_1 \ldots A_n]$ is the operation of functional application. It v-constructs the value of the function f (v-constructed by the construction F) at the tuple-argument a (v-constructed by A_1, ..., A_n) if the function f is defined at a, otherwise the Composition is *v-improper*, i.e., it *fails* to v-construct anything.[8] The *Closure* $[\lambda x_1 \ldots x_n\, X]$ spells out the instruction to v-construct a function by abstracting over the values of the variables x_1, ..., x_n in the ordinary manner of the λ-calculi. Finally, higher-order constructions can be used twice over as constituents of composite constructions. This is achieved by a construction called *Double Execution*, ${}^2X$, that behaves as follows: If X v-constructs a construction X', and X' v-constructs an entity Y, then ${}^2X$ v-constructs Y; otherwise ${}^2X$ is *v-improper*, failing as it does to v-construct anything.

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7 I use the term 'function' as synonymous with the term 'set-theoretical mapping', that is Church's 'function-in-extension'. Church's functions-in-intension might correspond rather to our constructions of those mappings. Yet I hesitate to use Church's term 'function-in-intension', because Church did not define functions-in-intension, he only characterized them as *rules* specifying functions-in-extension.

8 We treat functions as properly *partial* mappings, i.e., mappings that may lack a value at some of their arguments.

TIL constructions, as well as the entities they construct, all receive a type. The formal ontology of TIL is bi-dimensional; one dimension is made up of constructions, the other dimension encompasses functions, i.e. mappings. On the ground level of the type hierarchy, there are non-constructional entities unstructured from the algorithmic point of view belonging to a *type of order 1*. Given a *base* of *atomic types* of order 1, the induction rule for forming functional types is applied: where α, β_1, ..., β_n are types of order 1, the set of partial mappings from $\beta_1 \times ... \times \beta_n$ to α, denoted '$(\alpha\, \beta_1 \ldots \beta_n)$', is a type of order 1 as well. Constructions that construct entities of order 1 are *constructions of order 1*. They themselves belong to a *type of order 2*, denoted '$*_1$'. The type $*_1$ together with the atomic types of order 1 serve as a base for the induction rule: any collection of partial mappings, of type $(\alpha\, \beta_1 \ldots \beta_n)$, *involving* $*_1$ in their domain or range is of a *type of order 2*. Constructions belonging to a type $*_2$ that construct entities of order 1 or 2, and partial mappings involving such constructions, belong to a *type of order 3*; and so on *ad infinitum*.[9]

For the purposes of natural-language analysis, the *atomic types* currently encompass these four:

ο = the set of truth-values {T, ⊥}
ι = the universe of discourse (a constant domain of individuals)
τ = the set of reals, doubling as times (time being a continuum)
ω = the logical space of logically possible worlds

(Possible-world) *intensions* are entities of type (βω): mappings from possible worlds to an arbitrary type β. The type β is frequently the type of a *chronology* of α-objects, i.e. a mapping of type (ατ). Thus α-intensions are frequently functions of type ((ατ)ω), abbreviated as '$\alpha_{\tau\omega}$'. We typically say that an index of evaluation is a world/time pair $\langle w, t\rangle$. *Extensional entities* are entities of some type α where $\alpha \neq (\beta\omega)$ for any type β.

In particular, a *property* of individuals is a function of type (((οι)τ)ω), abbreviated '$(o\iota)_{\tau\omega}$'. Relative to a world/time pair, there is a set (perhaps empty) of those individuals that have the relevant property at this dual index. A *proposition* is a function of type ((οτ)ω), abbreviated '$o_{\tau\omega}$'. That is, propositions are empirical truth-conditions modelled as temporally sensitive sets of possible worlds, as in possible-world semantics enriched with

[9] For details see, for instance, Duží – Jespersen – Materna (2010, Chap. 1.3; Chap. 1.4), or Duží – Materna (2012, Chap. 2).

temporal indices (in which a proposition is identified with its satisfaction class). The partiality of propositions allows them to fail to return a truth-value at the given world/time pair of evaluation. For instance, the proposition that the King of France is bold currently lacks a truth-value, because the *office* of King of France, of type $((\iota\tau)\omega)$, abbreviated '$\iota_{\tau\omega}$', is currently vacant. An office is a function from worlds to a partial function from times to individuals. There is currently no individual who is the King of France of whom it is either true or false that he is bold. Nonetheless, both the term 'the King of France' and the sentence "The King of France is bold" remain perfectly meaningful in TIL.

Empirical expressions denote *empirical conditions* that may or may not be satisfied at some empirical index of evaluation. We model these empirical conditions as *possible-world intensions.* (Non-empirical languages have no need for an additional category of expressions for empirical conditions.) Yet, a possible-world intension is *not* the meaning of an empirical expression *E*; rather it is merely the object denoted by *E*. The meaning of *E* is the construction encoded by *E*. Where w ranges over ω and t over τ, the following schematic Closure characterizes the logical syntax of an empirical language: $\lambda w \lambda t$ $[\ldots w \ldots . t \ldots]$. If the Composition $[\ldots w \ldots . t \ldots]$ v-constructs an α-object, the whole Closure constructs an object of type $\alpha_{\tau\omega}$, i.e. an α-intension.

Technically speaking, some constructions are modes of presentation of functions, including 0-place functions such as individuals and truth-values, and the rest are modes of presentation of other constructions. Thus, with constructions of constructions, constructions of functions, functions, and functional values in our stratified ontology, we need to keep track of the traffic between multiple logical strata. The ramified type hierarchy does just that. What is important about this traffic is, first of all, that constructions may themselves figure as functional arguments or values. Thus we consequently need constructions of one order higher in order to present those constructions being arguments or values of functions. As explained above, constructions that serve as arguments to operate on are supplied by atomic constructions, viz. Trivializations and variables. For instance, if a variable x v-constructs objects of type τ, then the variable belongs to $*_1$, the type of order 2, denoted '$x/*_1 \rightarrow \tau$'. The Closure $\lambda x\ [^0\!+\ x\ ^0 1]$ that constructs the successor function of type $(\tau\tau)$ belongs also to $*_1$; $\lambda x\ [^0\!+\ x\ ^0 1]/*_1 \rightarrow (\tau\tau)$. The Trivialization of this Closure, $^0[\lambda x\ [^0\!+\ x\ ^0 1]]$, is a construction belonging to $*_2$, the type of order 3, which constructs just the Closure $\lambda x\ [^0\!+\ x\ ^0 1]$.

We distinguish strictly between a *procedure* (construction) and its *product* (here, a constructed function), and between a *function* and its *value*. What makes TIL *anti-contextual* and *compositional* is the fact that the theory construes the semantic properties of the sense and denotation relations as remaining invariant across different sorts of linguistic contexts. We do not develop a special extensional logic for extensional contexts, an intensional logic especially for intensional contexts, and a hyperintensional logic especially for hyperintensional contexts. Logical operations are universal and context-invariant. What is context-dependent are the arguments on which these operations operate. In a *hyperintensional* context they are *constructions* themselves; in an *intensional* context the arguments of logical rules and operations are the *products* of constructions, that is set-theoretical *functions*; finally, in an *extensional* context we operate on functional *values*.[10]

At the outset of this paper I formulated the thesis that we learn, communicate and think by means of procedurally structured *concepts*. Hence now I am going to introduce the theory of concepts as formulated within the framework of TIL.

2.2. The TIL theory of concepts

TIL's procedural theory of concepts follows the principles formulated by Bolzano (1837, §49) and Church (1956). Church identifies concepts with the meanings of λ-terms; hence TIL concepts are constructions. However, in the new orthodoxy of structured meanings we encounter an outstanding issue, to wit, the granularity of *structures*. Since we explicate structured meanings procedurally, our basic idea is that any two terms or expressions are synonymous whenever their respective meanings are *procedurally isomorphic*. The notion of procedural isomorphism helps TIL to a principled account of hyperintensional individuation. This is a major issue, because only expressions with procedurally isomorphic meanings are synonymous and can be mutually substituted in hyperintensional contexts. Moreover, since we are building an *extensional* logic of hyperintensions, the extensional rules of substitution of *identicals* and existential generalization must be valid in all kinds of context, whether extensional, intensional, or hyperintensional.

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[10] For details on our extensional logic of hyperintensions, see Duží (2012) and (2013).

The degree to which meanings should be fine-grained was of the utmost importance for Church, and he proposed several so-called Alternatives (see Church 1993). Senses are identical if the respective λ-expressions that formalise the senses are (A0) *synonymously isomorphic* or (A1) mutually λ-convertible. (A0) is α-conversion and synonymies resting on meaning postulates; (A1) is α- and β-conversion; Church also considered Alternative (A1′), that is, α-, β- and η-conversion. There is also (A2), for completeness, which is logical equivalence. But logical equivalence is a too weak criterion of synonymy as already Carnap knew, and thus was not acceptable for Church.

TIL offers various principles of procedural isomorphism, all of which slot in between Church's Alternatives (A0) and (A1). One such would be Alternative (½), another Alternative (¾). The former includes α- and η-conversion, while the latter adds *restricted* β-conversion by name. In Duží – Jespersen – Materna (2010) we opt for Alternative (½), whereas in Duží – Jespersen (2013) we prefer Alternative (¾) to soak up those differences between β-transformations that concern only λ-bound variables and thus (at least appear to) lack natural-language counterparts. The *restricted* version of *equivalent* β-reduction by name consists in substituting free variables for λ-bound variables of the same type. It is just a formal manipulation with λ-bound variables that has much in common with η- and less with β-reduction. The latter is the operation of applying a function f to its argument value a in order to obtain the value of f at a (leaving it open whether a value emerges). No such features can be found in restricted β-reduction that substitutes variables for variables. It is just a formal simplification of the relevant construction.

The latest variant of procedural isomorphism encompasses α-conversion and β-conversion by value. Hence we are leaving out η-conversion, and β-conversion is restricted to conversion by value. There are two reasons for excluding η-conversion. First, it is rather peculiar to claim that two procedures are identical if they do not have the same number of constituents. Yet the η-expanded construction of the form $\lambda x\,[F\,x]$ has at least two more constituents than the corresponding η-reduced construction F, because it adds the steps of applying the function constructed by F to the value of the variable x followed by abstraction over the values of x. The second and more important reason is the fact that η-conversion is *not a strictly equivalent* transformation in the logic of partial functions such as TIL. To see why, consider this example. Let $F \rightarrow ((\alpha\beta)\gamma)$ v-construct a function f such that f is not defined at the argument v-constructed by $A \rightarrow \gamma$. Hence the Composition $[F A] \rightarrow (\alpha\beta)$ is v-improper, as it does not v-construct anything. However, the η-expanded construction $\lambda x\ [[F A]\ x] \rightarrow (\alpha\beta)$, where $x \rightarrow_v \beta$, constructs a degenerate function, which is a function undefined at all its arguments. True, due to the v-improperness of $[F A]$ the Composition $[[F A]\ x]$ is also v-improper, but λ-abstraction raises the context to the intensional level. Hence the Closure $\lambda x\ [[F A]\ x]$ v-constructs a degenerate function, which is an object, though a bizarre one. Hence the Compositions $[F A]$ and $\lambda x\ [[F A]\ x]$ are not strictly equivalent in the sense of v-constructing the same object for every valuation v.[11]

The reasons for excluding unrestricted β-conversion are these. Though β-conversion is the fundamental computational rule of the λ-calculi, it is underspecified by the rule (that we call 'by name') $[\lambda x\ C(x)\ A] \vdash C(A/x)$. The application procedure $[\lambda x\ C(x)\ A]$ can be executed in two different ways: 'by value' and 'by name'. If by name then according to the rule the *procedure A* is substituted for *x*. In this case there are two problems. First, conversion of this kind is not guaranteed to be an equivalent transformation as soon as partial functions are involved. This is due to the fact that *A* occurs extensionally as a constituent of the left-hand construction, whereas when dragged into *C* its occurrence may become intensional. Second, even in those cases when β-reduction is an equivalent transformation, it may yield loss of analytic information, because when executing β-reduction by name we do not keep track of the function that has been applied.[12] The idea of conversion by value is simple. Execute the procedure *A* first, and only if *A* does not fail to produce an argument value on which *C* should operate, substitute (the simple concept of) this *value* for *x*. This solution preserves equivalence, avoids the problem of loss of analytic information, and moreover, in practice it is more efficient.[13]

The granularity of the individuation of procedures is still an open problem. The variant I propose here is the strongest criterion we have at present. Yet we admit that slightly different definitions of procedural isomorphism are thinkable. These considerations are motivated by the

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[11] I am grateful to Jiří Raclavský for adducing the above example; see Raclavský (2010).

[12] For the notion of analytic information, see Duží (2010).

[13] For details, see Duží – Jespersen (2013).

fact that what appears to be synonymous in an ordinary vernacular might not be synonymous in a professional language like the language of, for instance, logic, mathematics or physics. Thus we are also considering whether it is philosophically wise to adopt several notions of procedural isomorphism. It is not improbable that several degrees of hyperintensional individuation are called for, depending on which sort of discourse happens to be analysed. But α-conversion and β-conversion by value should be on any list of Alternatives attempting to accommodate procedural isomorphism.

As we have seen above, TIL embraces the view that meanings must be structured by conceiving of meanings as procedurally structured constructions. An unambiguous expression has thus assigned a unique—up to procedural isomorphism—construction as its meaning. Hence constructions are good candidates to explicate concepts.

The full identification of concepts with constructions faces, however, two minor problems. First, up until now we did not take into account expressions with pragmatically incomplete meaning, that is, sentences and terms with indexical pronouns like '*her* father', "*He* is a philosopher", etc. These expressions are assigned *open* constructions with free 'indexical' variables as their meanings. For instance, the analysis of the above expressions amounts to these open constructions: $\lambda w \lambda t\ [{}^0Father\_of_{wt}\ her] \rightarrow_v \iota_{\tau\omega}$, $\lambda w \lambda t\ [{}^0Philosopher_{wt}\ he] \rightarrow o_{\tau\omega}$; types: $Father\_of/(\iota\iota)_{\tau\omega}$: attribute, that is, a function that, dependently on worlds w and times t, assigns to an individual another individual (their father); $Philosopher/(o\iota)_{\tau\omega}$; he, $her \rightarrow_v \iota$: pragmatic variables. We hesitate to claim that pragmatically incomplete expressions express concepts, because the evaluation of concepts should yield an object, provided the concept in question is not an empty one. Yet since concepts are procedures, their execution should always be, in principle, possible. It is not so with open constructions which await valuation of their free variables in order for them to be executed. Open constructions are procedures with formal parameters, and they cannot be executed until an actual parameter value is supplied. In case of expressions with pragmatically incomplete meanings the respective value of an argument (valuation of free indexical variables) is supplied by a situation of utterance. Only after the situation has done its job does one obtain a closed construction that can be executed.

Hence we might identify concepts with closed constructions. But here we must deal with the problem of procedurally isomorphic constructions.

Recall that any two unambiguous terms or expressions (even of different languages) are synonymous whenever their respective meanings are *procedurally isomorphic*. Thus synonymous expressions have the same meaning and express the same concept, yet they can be furnished with different—procedurally isomorphic—constructions. In other words, constructions are too fine-grained from the procedural point of view. For instance, the unambiguous sentence “Tom is wise” could have been assigned the following procedurally isomorphic constructions as its meaning: $\lambda w \lambda t\ [^0 Wise_{wt}\ ^0 Tom]$, $\lambda w_1 \lambda t_1\ [^0 Wise_{w_1 t_1}\ ^0 Tom]$, $\lambda w_2 \lambda t_2\ [^0 Wise_{w_2 t_2}\ ^0 Tom]$, ... Note that in natural language we do not render these distinctions, because in an ordinary vernacular we do not use bound variables.

Since procedural isomorphism is an equivalence relation, it factorizes the collection of constructions into equivalence classes. From the procedural point of view it is irrelevant which element of a particular class is singled out as its representative. Each equivalence class of constructions can be well-ordered. The representative element will be the first construction occurring in the given ordering. This construction is the unique *normal form* of all the elements of the equivalence class of constructions. The representative element is designated as a *concept*.

So, in general, the structured meaning of an expression is a construction. If an expression contains indexicals its meaning is an open construction; the meaning of a non-indexical unambiguous expression is the concept expressed by the expression. Having decided in favour of construing concepts as closed constructions, we can define some special categories of concepts like various kinds of empty concepts, analytical vs. empirical concepts, etc. Since this is out of the scope of this short study I refer to Duží – Jespersen – Materna (2010, Chap. 2). Yet I will briefly explain the very important category of *simple* concepts. Simple concepts are identified with Trivializations of *non-constructional* entities. These concepts are *simple* because they supply these entities without the mediation of any other concepts, by not having any other constituents but themselves. We assume that each competent language-user is acquainted with the simple concepts in use in order that communication may proceed smoothly; which, however, does not mean that some simple concepts cannot be refined. The refinement of a simple concept ^0X is an ontological definition of the entity X, which is a compound concept of X. For instance, in an ordinary vernacular we use simple concepts of zoological properties like ^{0}*Whale*, ^{0}*Cat*, ^{0}*Horse* without the need to define these properties. This is a matter of zoology.

Thus, for instance, the simple concept $^0 Whale$ is refined as the compound concept $\lambda w \lambda t\ [\lambda x\ [[[^0 Marine\ ^0 Mammal]_{wt}\ x] \wedge [^0 Cetacea\text{-}order_{wt}\ x]]]$, because zoology has explained to us that whales are marine mammals of the order Cetacea.

Concluding this section let me say a few words about the connection between *meaning* and *concept*. As illustrated above, we claim that meanings are concepts. Can we, however, claim the converse? This would be: concepts are meanings. A full identification of meanings with concepts would presuppose that every concept were the meaning of some expression. But then we could hardly explain the phenomenon of historical evolution of language, first and foremost the fact that new expressions are introduced into a language and other expressions vanish from it. Thus with the advent of a new ⟨expression, meaning⟩ pair a new concept would have come into being. Yet this is unacceptable for a realist: concepts *qua abstract* entities cannot come into being or vanish. Therefore, concepts outnumber expressions; some concepts are yet to be discovered and encoded in a particular language while others sink into oblivion and disappear from language, which is not to say that they would be going out of existence. For instance, before inventing computers and introducing the noun 'computer' into our language(s), the procedure that von Neumann made explicit was already around. The fact that in the 19th century we did not use (electronic) computers, and did not have a term for them in our language, does not mean that the concept (*qua* procedure) did not exist. In the dispute over whether concepts are discovered or invented we come down on the side of discovery.

Each of us may have their own conceptual system based on our own set of simple concepts. Yet since we are able to communicate and learn new languages, there is a common intersection that is shareable by all of us. Moreover, particular personal conceptual systems are gradually developed in the learning process; as we adapt ourselves to external stimuli and environment changes in general, we discover and learn new concepts, but also forget old ones. Concepts/procedures are the entities that we have in common across different languages, cultures, histories, different means of communication and different ways of encoding these procedures. They enable us to learn new languages and discover new means of communication.

3. Conclusion

In this paper I argued for the thesis that abstract, procedurally structured concepts are central for our communication, and that we learn, communicate, execute and discover concepts. In order to support this thesis, I adduced analytical survey of Egyptian hieroglyphs and Inca knot writing. These are very different ways of encoding meanings, the former embracing up to 5000 different signs, the latter just the khipu knots for 0's and 1's. Yet regardless of the nature of a particular recording system, large amount of information can be encoded. How could it be if these codes were records of potentially infinite sets of facts? And how could abstract objects be recorded? Yet both these ways (and many other writing systems) are capable of recording very complex texts including abstract objects. Thus, in my opinion, this is strong evidence in favour of the thesis that they encode procedurally structured concepts consisting of a finite number of constituents which can be executed in any possible world at any time. Hence, let me finish this paper by rephrasing John's prologue:

In the beginning was the Concept,
and the Concept was with God,
and the Concept was God.

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