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Data, Phenomena and Mechanisms: The Logic of Construction of Bohr's 1913 Theory of the Hydrogen Atom[1]

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ABSTRACT: The paper aims at a unification of the two directions in contemporary philosophy of science: the direction which deals with the relation of data to phenomena with the direction which deals with the knowledge about mechanism and its employment in scientific thinking. It aims also at a reconstruction of the development of scientific knowledge which is characterized in contemporary philosophy of science as a movement from data, via phenomena, to mechanisms. An attempt will be made to show that this in fact amounts to an assignment of philosophical categories like data, phenomena, mechanism, etc. This unification and reconstruction draws also on the reconstruction of the main stages of the development of knowledge leading from A.-J. Ångström's measurement of the wave-lengths of spectral lines of hydrogen in 1868 to N. Bohr's theory of the hydrogen atom proposed in 1913.

KEYWORDS: Bohr's hydrogen atom – data – laws of phenomena – mechanisms – phenomena.

1. Introduction

This paper has two aims. The first is to unify two directions in contemporary philosophy of science: a) the direction which draws on the

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seminal works of J. Bogen and J. Woodward to deal with the relation of data to phenomena and b) the direction which utilizes here the article Machamer – Darden – Craver (2000) to deal with the knowledge about mechanism and its employment in scientific thinking. This unification aims to delineate the main stages of the development and growth of scientific knowledge via thinking and shall draw on the results achieved in the framework of philosophy of science in the last 20 years or so, as well as on the reconstruction of the main stages of development of knowledge leading from A.-J. Ångström's measurement of the wavelengths of the spectral lines of hydrogen in 1868 to N. Bohr's theory of the hydrogen atom in proposed 1913.

The second aim, located at a "meta" level, is to show that the reconstruction of the development of scientific knowledge (which is characterized in contemporary philosophy of science as a movement from data, via phenomena, to mechanisms) in fact amounts to an assignment of philosophical categories (e.g., *data*, *phenomena*, *mechanism*, etc.) to the respective stages of this development. From this "meta" point of view it will become obvious that our choice of the above-given episode in the history of physics can contribute to the explication of these categories, and at the same time, by bringing in additional categories, serve to broaden the network of categories by means of which philosophy of science approaches and reflects the natural sciences. The categories that we will introduce in addition to those used in the more recent philosophy of science literature (e.g., data, phenomena, and mechanism) are as follows: *phenomena as derived from the law of phenomena*, the latter being derived from the knowledge of and by thinking about a mechanism, and where these phenomena and laws of phenomena have a different epistemological status as compared to those *phenomena derived from data*, and from *laws of phenomena derived* from these latter phenomena. With respect to the category mechanism we will also introduce the additional category pairs *ground-grounded* and *reason-reasoned*.

2. J. Bogen, J. Woodward and P. Machamer on data, phenomena, and mechanisms

J. Bogen and J. Woodward in their seminal works criticize "those who hold that scientific theories explain what we observe and who then go on to tie the relevant notion of observation rather closely to sensory perception" (Bogen – Woodward 1988, 306). In opposition to those who approach scientific theories and explanations in this way, they propose a differentiation between the category *data* and the category *phenomena* as follows (1988, 305-306):

> Data … play the role of evidence for the existence of phenomena, for the most part [they] can be straightforwardly observed. However, data typically cannot be predicted or systematically explained by theory. By contrast, well developed scientific theories do predict and explain facts about phenomena. Phenomena are detected through the use of data, but in most cases are not observable in any interesting sense of the term[2] … Facts about phenomena … are evidence for the high-level general theories by which they are explained.

So, in their view, while claims about data serve as evidence for claims (facts) about phenomena, claims about phenomena serve as evidence only for claims which are already part of general theories explaining/predicting claims about phenomena but not about data. Stated in a more general way: "we need to distinguish what theories explain (phenomena or facts about phenomena) from what is uncontroversially observable (data)" (Bogen – Woodward 1988, 314). Data can also be subjected to explanations, but because they are produced by irregular coincidences of myriad particular causes, such explanations:

> when they can be given at all, will be highly complex and closely tied to the details of particular experimental arrangements … Thus, explanations of data will often lack generality … Moreover, the factors involved in the production of any given bit of data may be so disparate and so numerous, and their co-occurrence so rare, that the details of their interaction may be both epistemically inaccessible and difficult to model theoretically. Exhibitions of dependency-relations of the sort that would be achieved by explicit derivations or the tracing of specific causal mechanisms may prove impossible because of computational intractabilities. (Bogen – Woodward 1988, 326)

[2] This claim was recently restated as follows: "phenomena need not be observable … asking whether phenomena are observable is often not the right question to understand how … reasoning works. This is because the reliability of … reasoning often has little to do with how human perception works" (Woodward 2011, 171).

For the relation of data to phenomena it holds that the latter are derived from the former:

> The problem of detecting a phenomenon is the problem of identifying a signal in a sea of noise, of identifying a relatively stable and invariant pattern of some simplicity and generality with the recurrent features – a pattern which is not just an artifact of the particular detection technique we employ or the local environment in which we operate. (Woodward 1989, 396-397)

Bogen and Woodward also explain, as already stated above, the difference between the category *phenomena* and the category *data* in terms of the category *observation*. While the former need not be observable, the latter can usually be directly observed in the following sense:

> Data are *records* or *reports* – accessible to the human perceptual system and available for public inspection. Some data (e.g., reports of measurement results written in laboratory notebooks, or the drawings of the field geologists) are produced by human perceivers. But ... many data are produced by nonhuman measurement and recording devices ... Data constitute observational evidence to investigate phenomena. (Bogen – Woodward 1992, 593)

So, for example, they claim that when measuring the temperature at which a sample of lead melts, the value – the datum – of each thermometer reading is observable (cf. Bogen – Woodward 1988, 319). The following diagrammatic summary of the views of Bogen and Woodward can be given (cf. Schindler 2007, 167):

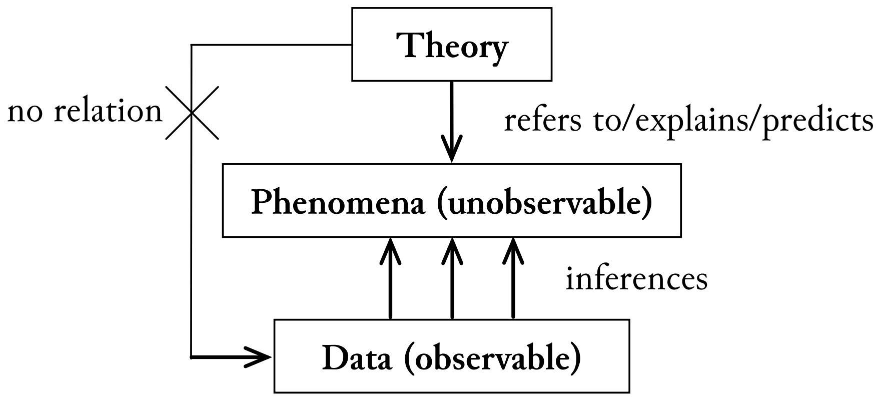

Diagram 1: S. Schindler on Bogen's and Woodward's views on data, phenomena and theory

What "Theory" in this diagram stands for can be understood if one takes into account the fact that Bogen and Woodward characterize the process of systematic explanation to which claims about phenomena are subjected as "explanation of an outcome due to the assertions of the details of a mechanism" (Bogen – Woodward 1988, 323). The category of mechanism was delineated by means of additional categories explicated in Machamer – Darden – Craver (2000) and Darden (2008). In Machamer – Darden – Craver (2000) one can identify the following triple of categories falling under the category *working of the mechanism*: *start-up/set-up conditions*, *intermediate activities*, and *terminating activities*. The first category involves the following subordinated categories: *entities*, *relevant properties of entities given at the beginning of the activities*, *the mutual interactions of these entities*, *entities and activities as mutually correlative*, *the spatial distribution, orientation, and relations of the entities and activities*, *the temporal characteristics of the activities*. To the category *intermediate activities* are subordinated categories *intervening entities and activities*, understood as *producers of new states in entities and/or of new entities and activities*. The third category involves categories *endpoint of activities*, *parameters and state of the mechanism*.

This triple of categories with their subordinated categories, once unified with the set of categories given by Darden (2008, 965), can be summarized in Table 1.[3]

	Main categories			
	Components	*Spatial arrangement of components*	*Temporal aspects of components*	*Contextual locations*
Subordinated Categories	set-up/start-up & terminating conditions, entities, initial & intermediate & terminating activities, modules	localization structure orientation connectivity compartmentalization	order rate duration frequency	location within a hierarchy location within a series

Table 1: Categories involved in the category *mechanism*

[3] The category *module* stands for reasoning about groups of components of the mechanism.

3. From Ångström to Bohr and “back”

3.1. From data to phenomena: A.-J. Ångström

We now start to deal with a level of scientific knowledge to which A. Sommerfeld assigned, in a backward glance, the category – when expressed in German as *empirisches* (Sommerfeld 1921, 222), understood in English as *empirical data* (Sommerfeld 1923, 68).

Ångström’s aim was to provide a detailed atlas of the wavelengths of spectral lines given in solar light.[4] In his experiments he used two ready-made gratings, numbered as (I) and (II), the former with 4501 lines, the latter with 2701 lines engraved on a length, declared by the producer of the gratings as being equal to 9 Paris lines. The basic equation used by Ångström for the computation of the wavelength λ of the particular lines was $\lambda = e \cdot \sin\varphi$ for the spectrum of the first order, for its N-th order holds $\lambda = N \cdot e \cdot \sin\varphi$, where e stands for the width of the grating space and φ for the angle of diffraction of the light waves, the latter being measured by a theodolite. In addition, he modified these equations in order to take into account the variation of temperature and orientation of the gratings, and then computed the data for the spectra – in Frauenhofer notation – C, F of hydrogen. The resulting values were as follows (Ångström 1868, 18):

Line	1. spectrum in 10^{-7} mm	Number of observations	2. spectrum in 10^{-7} mm	Number of observations	3. spectrum in 10^{-7} mm	Number of observations
C	6562.27	2	6561.97	3	6562.16	1
F	4860.67	2	4860.62	2	4860.80	3

Table 2: Data for the spectral lines C and F of hydrogen computed in Ångström (1868)

Based on all the data for grating (I), as well as all the data for grating (II), he then computed the *weighted arithmetic mean* values of the wavelengths of the spectral lines of solar light. For the four lines of hydrogen C, F, near G, and h – in the more modern notation H_α, H_β, H_γ, and H_δ to

4 For a detailed analysis of his experiments and the conceptual basis of his computations see Landauer (1898, 6-33) and Baly (1912, 12-29).

which we will hold from now on – he obtained the following mean values (Ångström 1868, 31 – 32):

Line	Wavelength in 10^{-7} mm
H_α	6562.10
H_β	4860.74
H_γ	4340.10
H_δ	4101.20

Table 3: Average wavelengths of the spectral lines of hydrogen computed in Ångström (1868)

These values were used by J. J. Balmer in his 1884 formulation of the first truly unifying formula for the wavelengths of the spectral lines of hydrogen.

From the *epistemological* point of view it holds that in his *thought* operations Ångström employed the statistical analysis based on the weighted statistical mean, which enabled him to pass in mind from what we label by the philosophical category *data* to what can be labeled by the philosophical category *phenomena*. Thus, *data* as a philosophical category stand in our approach for an *entity in mind* and simultaneously for *a certain level of attained knowledge about the world*. Both are information about occurrences given to us in our practical operations (manipulations) with things in the world; *occurrences in the world of things are thought in mind as data*.

In the case of the above given Table 2, the data for spectra stand for the meaning of a sentence like "The wavelength of the line C (H_α) in the 1st spectrum on grating (I) is $6562 \cdot 10^{-7}$ mm". In our view, contrary to that of Bogen and Woodward, the meaning of sentences of this type does not refer to something observable in the world, or, stated in more general categories, to something accessible to human sensory perception. *Occurrences as states of affairs in the world in the range of 10^{-7} mm are not perceptible* and, of course, the *very meaning, in the sense of information, stated in a sentence like the one given above, is not perceptible either*. So, in our reconstruction here *data do not refer to perceptible state of affairs*, and *so neither do phenomena derived from them*. In fact, the meaning of a statement is never perceptible, since meanings as information can only be thought of because they are entities in the mind on which one can perform thought operations which yield other thought entities. In Ångström's approach such operations were, for example, the computations of the weighted arithmetic mean for the wavelength and his abstraction from the impact of changes of the atmospheric pressure.

3.2. From phenomena to laws of phenomena: J. J. Balmer, J. R. Rydberg, and W. Ritz

The results given in Ångström (1868) were the point of departure for the development of spectral analysis leading to what W. Ritz labeled by the epistemological category *empirische Gesetze* (Ritz 1903, 266) and N. Bohr by the category-concept hybrid *empirical spectral laws* (Bohr 1922, 1).

J. J. Balmer was able to derive one formula, which he viewed as an "expression of a law" (Balmer 1885a, 552), and which unified the knowledge of the wavelengths of the four spectral lines H_α, H_β, H_γ, and H_δ.[5] He drew on the wavelengths of the spectral lines of hydrogen as computed by Ångström: $\lambda_\alpha = 6562.10 \cdot 10^{-7}$ mm, $\lambda_\beta = 4860.74 \cdot 10^{-7}$ mm, $\lambda_\gamma = 4340.10 \cdot 10^{-7}$ mm, and $\lambda_\delta = 4101.20 \cdot 10^{-7}$ mm. He started by unifying the above given four wavelengths by means of a basic wavelength $\lambda_0 = 3645.6 \cdot 10^{-7}$ mm, so that they are given as multiples of the following coefficients $\frac{9}{5}, \frac{4}{3}, \frac{25}{21}$, and $\frac{9}{8}$, respectively. By multiplying the second and third coefficients by $\frac{4}{4}$ he obtained four coefficients which can be written as $\frac{9}{9-4}, \frac{16}{16-4}, \frac{25}{25-4}$, and $\frac{36}{36-4}$, and thus unified into the formula $\frac{m^2}{m^2-2^2}$, where m stands for the positive whole numbers 3, 4, 5, and 6. The four wavelengths λ_α, λ_β, λ_γ, and λ_δ can be then unified, by means of a basic wavelength $\lambda_0 = 3645.6 \cdot 10^{-7}$ mm, into the formula

$$(1) \qquad \lambda = \lambda_0 \frac{m^2}{m^2 - 2^2} \quad (m = 3, 4, 5, 6)$$

Formula (1) can be viewed as embedded into a phenomenal spectral law, so that holds:

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5 For an analysis of this derivation see Banet (1966) and (1970), respectively; the latter article draws on Balmer's manuscripts.

$$(2) \qquad \forall x \; [H(x) \;\&\; I(x) \rightarrow \lambda(x) = \lambda_0 \, (\frac{m^2}{m^2 - 2^2})]$$

Here "$H(x)$" stands "x is hydrogen," "$I(x)$" for "x is incandesced," and "$\lambda(x)$" for "the wavelength of radiation emitted by x." This law has the following two features. It can be used already for the purpose of explanation of the already known four different wavelengths of the spectral lines of hydrogen. It thus fulfills the requirement of the so-called *functional interdependence* imposed on scientific explanation by J. Woodward:

> The law occurring in the explanans of a scientific explanation of some explanandum *E* must be stated in terms of variables or parameters variations in the value of which will permit the derivation of other explananda which are appropriately different from *E*. (Woodward 1979, 46)

We can provide here the first epistemological delineation of the category *scientific law of phenomena: it unifies in mind a number of different, already known phenomena pertaining to entities of the same kind and provides a unified account of these phenomena.*

The importance of such a phenomenal law is that it stood in Balmer's reasoning not only for a unification of antecedently known phenomena, but was also used by him for the derivation of phenomena whose referents were not known to him when he stated formula (1). In fact, on the basis of formula (1) and the hypothesized value $m = 7$, he predicted the existence of another spectral line of hydrogen, with the predicted wavelength $\lambda = 3969.65 \cdot 10^{-7}$ mm. Only later did he learn that such a spectral line had already been discovered together with other spectral lines symbolized as H_ε, H_ζ, H_η, H_θ, and H_ι. Thus, the scope of applicability of formula (1) was extended to the values 7 through 11 of the parameter m. In addition, in the second communication in Balmer (1885b), he again broadened, based on the additional knowledge he obtained about spectral lines, the scope of application of the formula (1), so that the values of the parameter m ranged now for the positive whole numbers 3 through 16.

In the ensuing development of spectral analysis, phenomenal spectral laws were generalized in order to cover also the spectral lines of substances other than hydrogen. As we shall see, the works of J. R. Rydberg and W. Ritz appear as crucial to this investigation.

Rydberg in (1890) employed the term "wave number," abbreviated as n in the sense of "*the number of wavelengths in 1 centimeter*" (1890a, 13), that is, $n = 10^8 \cdot \lambda^{-1}$, where λ is expressed in units used by Ångström. He also drew on the already acquired knowledge that the spectra of particular substances are ordered in series. Based on this he found that the wave numbers assigned to the spectral lines of different substances can be described by the following unified formula (1890, 40):

$$(3) \qquad n = n_0 - \frac{N_0}{(m+\mu)^2}$$

Here "*m*" stands for a positive whole number labeled "number of the term;" N_0 is a universal constant, while constants n_0 and μ stand for specific constants characterizing particular spectral series of a substance, and where n_0 is the limit which n approaches when $m = \infty$. By transforming Blamer's formula (2) so that it now holds for wave numbers ($\lambda = \frac{10^8}{n}$ and $\lambda_0 = \frac{10^8}{n_0}$) he obtained the formula $n = n_0 \cdot \frac{m^2 - 2^2}{m^2}$ and thus $n = n_0 - \frac{4n_0}{m^2}$, thus $n = 4n_0 \left(\frac{1}{2^2} - \frac{1}{m^2}\right)$. By comparing the former formula with formula (3) he found out that for its universal constant holds $N_0 = 4n_0$ and so as $\lambda_0 = \frac{10^8}{n_0}$, he computed in (1890) that $N_0 = 109721.60$.

Finally, in early 20th century, W. Ritz formulated "a new law of the series spectra" (Ritz 1908, 521), making it possible to combine the already known formulas of spectral series of a substance, so that one could obtain new formulas of other series of this substance that can be used for the prediction of the existence of these other series. This law he stated in an autoseminar paper added to his article (1908) as follows:

> While the hitherto known laws of the series spectra connect with one another the lines of *one* series ... it is shown here, that there exists a simple relation also between different series of one element: by means of additive or subtractive combinations, either of the series formula or constants given in them, new formulas are being formed, which allow to compute completely the new lines discovered in recent years from those known earlier. (Ritz 1911, 162)

So, for example, as shown above, for the lines H_α, H_β, and H_γ, Balmer's formula (1) in Rydberg's reformulation yields:

$$n_\alpha = N_0\,(\frac{1}{2^2} - \frac{1}{3^2}),\; n_\beta = N_0\,(\frac{1}{2^2} - \frac{1}{4^2}),\; n_\gamma = N_0\,(\frac{1}{2^2} - \frac{1}{5^2})$$

By applying Ritz's combination principle, one obtains from these three formulas the following two additional formulas:

$$n_\beta - n_\alpha = N_0\,(\frac{1}{3^2} - \frac{1}{4^2}),\; n_\gamma - n_\alpha = N_0\,(\frac{1}{3^2} - \frac{1}{5^2})$$

Thus, one is able to predict the existence of a spectral line for which should hold the formula

$$n = N_0\,(\frac{1}{3^2} - \frac{1}{m^2})\;(m = 4, 5)$$

This prediction was confirmed by Paschen (1908), who – based on a series of experiments – proved the existence of a spectral series corresponding to the last formula.

Based on the last formula and Balmer's formula (1), Ritz stated the universal formula:

$$(4)\qquad n = N_0\,(\frac{1}{n^2} - \frac{1}{m^2})$$

The phenomenal spectral law (2) then turns into the phenomenal spectral law

$$(5)\qquad \forall x\,[H(x)\ \&\ I(x) \rightarrow n(x) = N_0\,(\frac{1}{n^2} - \frac{1}{m^2})]$$

Here "$n(x)$" stands for "the wave number of the spectral line emitted by x."

From the *epistemological* point of view it holds that Ritz's law (5) can be used not only (as can like Balmer's formula (1)) to explain and predict particular wavelengths, but also to explain the laws for Balmer's and Paschen's series. We can thus provide a reinterpretation of the category *law of phenomena* given above. *A scientific law of phenomena unifies in mind a number of different phenomena and laws of phenomena which pertain to entities of the same* *kind and provides a unified account of them.* One can give also a generalization of the above quoted requirement of *functional interdependence* imposed by Woodward on scientific explanations:

> The law occurring in the explanans of a scientific explanation of some explanandum phenomenon E and explanandum law of phenomena L must be stated in terms of variables or parameters variations in the value of which will permit the derivation of other explananda phenomena and explananda laws of phenomena which are appropriately different from E and L.

This generalization is based on Woodward's view that "the scientific explanations typically have as their explananda generalisations rather than singular sentences ... the scientific explanation of particular facts is an activity which is derivative or parasitic on explanation of generalisations" (Woodward 1979, 63).

4. Bohr's mechanism of the hydrogen atom

In Part I of his article (1913), Bohr sets as his aim to deal with "the mechanism of the binding of electrons by a positive nucleus" (Bohr 1913, 2). He conceptually approaches this mechanism by means of Planck's view of radiation which he employs drawing on the "general acknowledgment of the inadequacy of classical electrodynamics in describing the behavior of system of atomic size" (Bohr 1913, 2). At the same time, Bohr clearly differentiates in his approach two distinct but still interrelated phases, *one*, in which he will provide "a basis for a theory of the constitution of atoms", and another one, based on the first one, in which he will give a "an account ... for the law of the line spectra of hydrogen" (cf. Bohr 1913, 2).

As the object of his thought operations Bohr chooses "a simple system consisting of a positively charged nucleus of very small dimensions and an electron describing closed orbits around it" (Bohr 1913, 3), and where this object is subjected to two additional suppositions: the mass of the electron is negligible in size as compared to that of the nucleus, and the velocity v of the electron is much smaller than the velocity of light c. That choice enables him to bypass the issue of mechanical instability, and, when unified with the first supposition, provides the conceptual framework for the treatment of the hydrogen atom. The latter supposition, characterized by means of the epistemological category *idealization*, states that $\frac{v}{c} = 0$ holds. By the introduction of this idealization Bohr can abstract in mind from the impact of relativistic effects.

Bohr initially presupposes that no energy is radiated; thus he can apply classical mechanics to the case of the movement of the electron, on an elliptical orbit, around the nucleus located in the focus of this orbit. If e and m stand for the charge and mass of the orbiting electron, E for the charge of the nucleus, and $2a$ for the major axis of the orbit, then it holds (Bohr 1913, 3):

$$(6) \qquad \omega = \frac{\sqrt{2}}{\pi} \cdot \frac{W^{\frac{3}{2}}}{eE\sqrt{m}}, \; 2a = \frac{eE}{W}$$

Here ω stands for the orbital frequency of the electron, while W stands for the energy to remove it from the vicinity of the nucleus to infinity.

Then, Bohr gives up the supposition that energy is not radiated; this requires stabilizing the atom radioactively. To achieve this stabilization, he brings in Planck's idea of a discrete energy being radiated by a vibrator with the frequency v, and where the energy radiated in one emission is τhv, where τ is positive whole number and h Planck's universal constant. He applies this idea of Planck in such a way that he now supposes the electron, being initially at a great distance from the nucleus, as bound by the nucleus and settled in an orbit around it. To this binding Bohr applies Planck's idea in such a way that the energy being emitted in this process is given by the relation $E = \tau h \frac{\omega}{2}$, where ω stands for the orbital frequency of the electron on its final orbit in which it settles.[6] This relation, when combined with relations given in (6), yields

$$(7) \qquad W = \frac{2\pi^2 me^2 E^2}{\tau^2 h^2}, \; \omega = \frac{4\pi^2 me^2 E^2}{\tau^3 h^3}, \; 2a = \frac{\tau^2 h^2}{2\pi^2 meE}$$

Bohr interprets these relations in such a way that for the respective values of τ one obtains the values for W, ω, and $2a$ which characterize the

[6] Bohr uses in Part I two additional applications of Planck's view on quanta of energy; we do not deal with them in this paper. On this see Heilbron – Kuhn (1969).

configurations of the atom when no energy is radiated by the atom. These configurations he views as characterizing the states of the atom; so he assigns to them the term "stationary states." By bringing in the values for e, m, and h, and by supposing that $e = E$, he computes for $\tau = 1$ the linear dimension $2a$ of the atom of hydrogen, its optical frequency ω, and its ionization potential $\frac{W}{e}$; all these computed values correspond to those which are computed on the basis of experiments. At the end of his treatment of the atom itself, Bohr gives the following brief summary of his main assumptions:

> (1) The dynamical equilibrium of the systems in the stationary states can be discussed by help of the ordinary mechanics, while the passing of the systems between different stationary states cannot be treated on that basis.
> (2) That the latter is followed by the emission of a homogeneous radiation, for which the relation between the frequency and the amount of energy emitted is the one given by Planck's theory. (Bohr 1913, 7)

In our view to Bohr's treatment of the constitution of the hydrogen atom one can assign the category *ground* understood as *working of the ground's mechanism*. This category involves as subordinated those categories which are given in Machamer – Darden – Craver (2000) and Darden (2008) and which were stated already above.

5. The spectra explained and predicted

After Bohr dealt with the working of the mechanism of the hydrogen atom, he moved on to the derivation of the formula for the spectral lines of hydrogen. As shown above, Rydberg's formula (5) holds for the spectral lines of hydrogen. It can be transformed as follows. The relation $\lambda = c \cdot T$, where c is the speed of light and T the period of its movement, is transformed into $n = \frac{1}{c \cdot T}$, and by introducing the magnitude frequency υ as $\upsilon = \frac{1}{T}$ one obtains $\upsilon = c \cdot n$. So, by multiplying the formula (5) by the magnitude c, one obtains

$$(8) \qquad v = c \cdot N_0 \left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

For $c = 3 \cdot 10^{10}$ cms^{-1} and $N_0 = 109675.0$ cm^{-1} one obtains $c \cdot N_0 = 3.29\ 10^{15}$ s^{-1}. Bohr compared this value, obtained by computation from the historically prior form of the formula for hydrogen, with the value he computed on the basis of the formula he derived from his theory about the hydrogen atom. He proceeded in the following way. He interprets the experimental process of discharge in vacuum tube when the spectrum is experimentally produced as a process of moving the orbital electron to a great distance from the nucleus. This latter process he views as corresponding to the creation of a stationary state when the electron is bound by the nucleus. For the energy W radiated in this binding holds the relation (7), and so as the object of his thought derivations is still hydrogen, he puts the absolute values of the charge of the orbiting electron and that of the nucleus as identical. He thus obtains from (7) the relation

$$(9) \qquad W_\tau = \frac{2\pi^2 m e^4}{\tau^2 h^2}$$

Then, the energy produced when the atom passes from the state characterized by τ_1 to that characterized by τ_2 is as follows:

$$(10) \qquad W_{\tau_2} - W_{\tau_1} = \frac{2\pi^2 m e^4}{h^2}\left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2}\right)$$

This energy he puts equal to hv, where v is the frequency of radiation, so he obtains

$$(11) \qquad v = \frac{2\pi^2 m e^4}{h^3}\left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2}\right)$$

From here Bohr's thought movement goes into two directions. One deals with the expression $\frac{2\pi^2 m e^4}{h^3}$ in (11) and the second with the expression in the brackets of (11). The former expression is used by Bohr for the recomputation of the constant given above as $c \cdot N_0$; for the values of e, m, and h it yields the value $3.10\ 10^{15}$ s^{-1}. The expression in the brackets is used by

Bohr in the *explanation* of already known spectral series and in the *prediction* of the existence of spectral lines as yet undetected. By putting $\tau_2 = 2$ and for a varying τ_1 he explains Balmer's series and for $\tau_2 = 3$ he *explains* Paschen's series. He *predicts*, by putting $\tau_2 = 1$, or 4, or 5, the existence of spectral series both in the extreme ultra-violet and the extreme ultra-red, which "are not observed, but the existence of which may be expected" (Bohr 1913, 8). Already in 1914 the series corresponding to $\tau_2 = 1$ was detected; few years later the series for τ_2 equal to 4 and 5 were also experimentally detected.[7]

Finally, still in Part I, Bohr leaves the framework of reflections on hydrogen atom and its spectra, and gives an interpretation of Ritz combination principle as holding for all substances producing line spectra:

> The circumstance that the frequency can be written as a difference between two functions of entire numbers suggests an origin of the lines in the spectra in question similar to the one we have suggested for hydrogen, i.e. that the lines correspond to a radiation emitted during the passing of the system between two stationary states. This may account for the different sets of series in the line spectra emitted from substances in question. (Bohr 1913, 11)

What *epistemological lessons* can be drawn here? Once Bohr succeeded in the above reconstructed two directions of though movement, his *knowledge has grown*. What we mean by this can be understood when we put side by side the initial formulation (8) of the spectral formula for hydrogen and the spectral formula (11) derived by Bohr:

$$(12) \quad \upsilon = c \cdot N_0 \left(\frac{1}{n^2} - \frac{1}{m^2}\right), \ \upsilon = \frac{2\pi^2 me^4}{h^3} \left(\frac{1}{\tau_2{}^2} - \frac{1}{\tau_1{}^2}\right)$$

By their comparison it becomes readily seen that the expression "$c \cdot N_0$" is restated as $\frac{2\pi^2 me^4}{h^3}$, and so the formula for the phenomenon of spectral emission is completely tied to its explanans, the latter being Bohr's unification of Rutherford's atom with Planck's views on the quanta of radiation. The symbols "e", "h" and "m" together with their respective meanings ap-

7 On this see Lyman (1914), Brackett (1922) and Pfund (1924).

pear in the explananda and predicanda because there are shifted here in the process of explanation and prediction from Bohr's explanans – the understanding of the *ground*'s mechanism producing the respective phenomena.[8] By this comparison one finds out also that the interpretation of the symbols "*n*" and "*m*" is different from that of "τ_2" and "τ_1." While the former pair stands just for an ordered pair of numbers, the latter stands for the sequence of orbits of the electron.

The growth of knowledge becomes readily seen also when one reconstructs the respective scientific laws into which the formulas (12) are embedded. We reconstruct them as follows:

$$\forall x\ [H(x)\ \&\ I(x) \rightarrow \upsilon(x) = c \cdot N_0\ (\frac{1}{n^2} - \frac{1}{m^2})],$$

$$\forall x \forall y [El(x)\ \&\ N(y,x)\ \&\ H^*(z,x,y)\ \&\ D(z)\ \&\ \frac{v(y)}{c} = 0 \rightarrow \upsilon(x) =$$

$$\frac{2\pi^2\ m(x)^{(1)} E(y)^2 e(x)^2}{h^3}\ (\frac{1}{\tau_2(x)^2} - \frac{1}{\tau_1(x)^2})]$$

In the antecedent of the second law "*El*(*x*)" stands for "*x* is an electron," "*N*(*y*,*x*)" for "*y* is the nucleus orbited by *x*", "*H*(*z*,*x*,*y*)" for „*z* is hydrogen composed of *x* and *y*", „*D*(*z*)" for "*z* is disturbed," and "$\frac{v(y)}{c}$" for "the ratio of the speed of *y* and speed of light." In the consequent "$m^{(1)}(x)$" stands for "mass of *x* subjected to one idealization".

By comparing these two laws, one finds out that what has changed is the very meaning of the term "hydrogen," that is, the universe of discourse to which the law can be applied at all. While in the former law, symbolized as "*H*," it is understood as, say, a substance with a certain atomic weight, reacting in certain proportion with other substances and with a certain ionization potential, in the latter, symbolized as "*H**," it is understood as being set up by an electron orbiting the nucleus. What has changed also is the understanding when the hydrogen is at all radiating: in the former law it has to be incandesced, in the latter law it has to be disturbed from the outside by an input of energy, so that the electron is shifted from one orbit

[8] In order not to overburden the text we use here only the category *explanans* as the basis for the thought derivation of both the explananda and predicanda; thus refraining from the use of the category *predicans*.

to another. Finally, Bohr's equation given in the law holds only when the idealization holds that the orbiting speed of the electron inside the hydrogen must be much smaller than that of light.[9]

Based on these analyses, we come to the conclusion that while it is possible to assign both to the knowledge which is the point of departure of the thought movement *to* the ground's mechanism as well as to the knowledge which is obtained by explanation and prediction *from* the knowledge about the ground's mechanism the same category pair *phenomena* and *law of phenomena*, this pair of philosophical categories does not capture adequately the above reconstructed cases of growth of knowledge. The phenomena and laws of phenomena as the epistemic points of departure for the movement to ground's mechanism and as the latter's epistemic consequences differ mutually in some fundamental aspects. This difference, as can be seen from Diagram 1, was not taken into account in the article Bogen – Woodward (1988). As a consequence of the absence of this account, the whole cycle *data* → *phenomena* → ... → *phenomena*, reconstructed for the first time in Bogen – Woodward (1988) fails to express the extension of knowledge. This failure could be turned into an argument arguing in favor of a modified Hempelian view from the fifties, namely, that *even if the phenomena are not observable, still it does not make any sense to make a detour through theory and the knowledge of mechanism given in it*; one starts and ends with the same knowledge expressed by the category phenomena. A remedy to this deficit will be given now.

6. The epistemological lessons

Based on the above given reconstructions of the growth of knowledge, we propose the following, more subtle differentiation between philosophical categories. Knowledge which was till now labeled by the category *phenomena* is now labeled either by the category *appearance* or by the category *manifestation*, and knowledge which was till now labeled by the category *law of phenomena* is now labeled either by the category *law of appearance* or by the category *law of manifestation*. The philosophical category *appearance* is assigned to that level of knowledge and thinking where

[9] On the reconstruction of the structure of scientific laws containing idealizations see Nowak (1972).

the phenomena are thought as yet independently and prior to their unification in scientific laws, where the latter have the status of *laws of appearance*; it holds for them the characterizations of the category *law of phenomena* given above. The category *law of appearance* characterizes knowledge and thinking before the knowledge about the ground's mechanism is derived from them. Once the very mechanism of the ground is understood, the laws of phenomena and the phenomena derived (explained and/or predicted) have already the status of *laws of manifestation* and of *manifestations*. The category *law of manifestation* can be characterized as follow: *it unifies in mind a number of different phenomena pertaining to entities of the same kind, and where this unification stands for the derivation based on the understanding how the phenomena are produced by the working of the mechanism of the ground of these phenomena*.

How can the difference between categories *appearance* and *law of appearance*, on the one hand, and *manifestation* and *law of manifestation*, on the other hand, be epistemologically explicated? As shown above, the laws of spectra and thus also the frequencies computed on their basis, once derived from Bohr's understanding of the working of the mechanism of the atom, contain already the symbols e, m, h, τ_1, and τ_2. These symbols, as well as their respective meanings, are part of the explananda/predicanda and have their origin in their common explanans – Bohr's understanding of the working of the mechanism, which is based on the meaning of such terms as "electron moving on in orbit above the nucleus," "quantum of radiation emitted," "stationary orbits," etc. This can be stated in more general, epistemological terms as follows. Concepts introduced at the level of knowledge and thinking characterized by the category *working of the mechanism of the ground* – as the explanans – are shifted in the process of explanation/prediction to the explananda/predicanda. So, while at the level of knowledge and thinking characterized by the categories *appearances* and *laws of appearances*, concepts standing for the knowledge about the working of the mechanism are as yet not given, they are already given at the level knowledge characterized by the categories *laws of manifestations* and *manifestations*. This is the difference between knowledge characterized by the categories *appearance and laws of appearance* and knowledge characterized by the categories *law of manifestation* and *manifestation*.

Based on this differentiation of categories, we propose – as an alternative to S. Schindler's Diagram 1 – the following diagrammatical representation of the sequence of categories reconstructed till now.

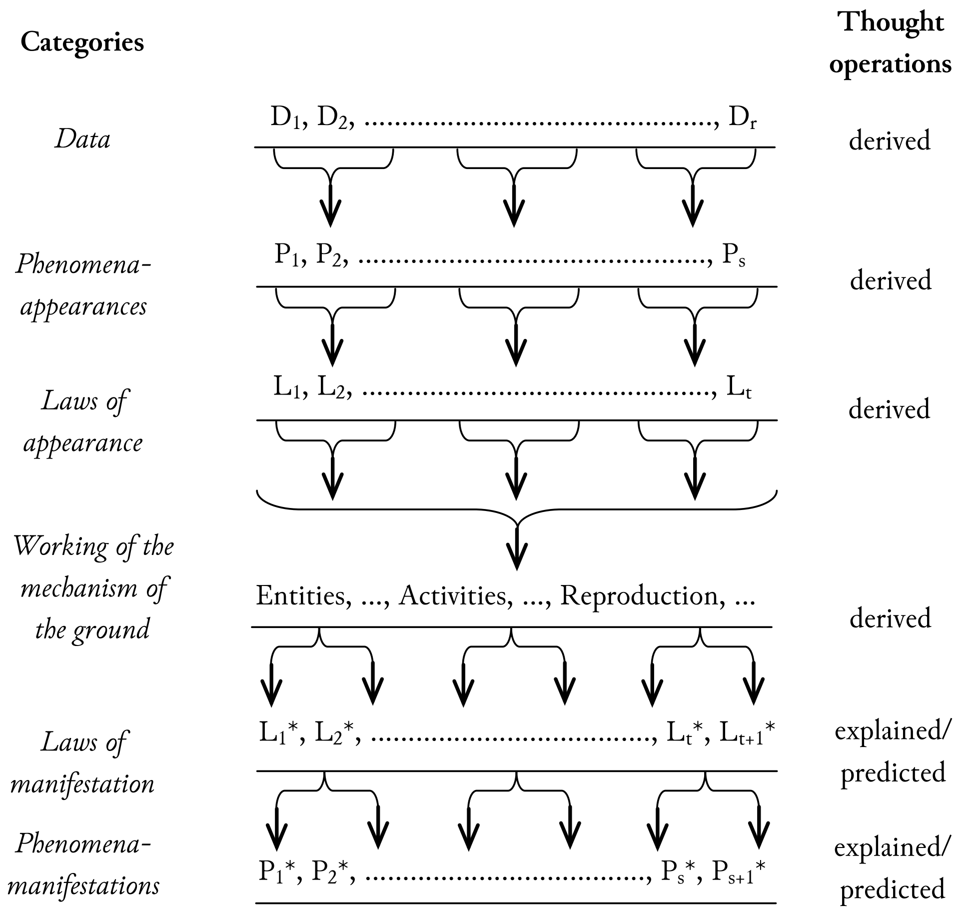

Diagram 2: Sequence of categories reconstructing
the growth of scientific knowledge

Here the scientific laws L_1*, L_2*, ..., L_t* stand for a reinterpretation by means of explanation of the scientific laws L_1, L_2, ..., L_t, while the laws L_{t+1}*, ..., stand for laws of manifestation which do not have their counterpart in the laws of appearance; they are *predicanda* laws and not explananda laws. The same holds for manifestations expressed as P_1*, P_2*, ..., P_s*; they are the reinterpretations, based on explanation from the laws of manifestation, of the appearances P_1, P_2, ..., P_s, while the manifestations P_{s+1}*, ..., are *predicanda phenomena* derived from the laws of manifestations.

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