RELIABILISM, INTUITION, AND MATHEMATICAL KNOWLEDGE

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It is alleged that the causal inertness of abstract objects and the causal conditions of certain naturalized epistemologies precludes the possibility of mathematical knowledge. This paper rejects this alleged incompatibility, while also maintaining that the objects of mathematical beliefs are abstract objects, by incorporating a naturally acceptable account of ‘rational intuition.’ On this view, rational intuition consists in a non-inferential belief-forming process where the entertaining of propositions or certain contemplations results in true beliefs. This view is free of any conditions incompatible with abstract objects, for the reason that it is not necessary that S stand in some causal relation to the entities in virtue of which p is true. Mathematical intuition is simply one kind of reliable process type, whose inputs are not abstract numbers, but rather, contemplations of abstract numbers.

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Some have objected that on certain naturalized epistemologies, the possibility of mathematical knowledge is excluded. More specifically, if one maintains that the objects of our mathematical beliefs are abstract objects, and one imposes causal constraints on the conditions of knowledge, then, perhaps, knowledge of mathematics is not possible because of the incompatibility between the causal inertness of abstract objects and our causal conditions on knowledge. I think it is far from obvious that epistemological naturalism requires causal conditions on knowledge or justification, or, at least, requires causal conditions which are incompatible with knowledge of abstract entities. Moreover, I will suggest that the faculty of rational intuition is one promising source for our mathematical knowledge, and which is not inconsistent with a reliabilist naturalistic epistemology. Admittedly, this might strike some as odd. Epistemic naturalism is the view that epistemology ought to proceed scientifically, and rational intuition is alleged to be utterly mysterious. Yet, this is to forget that not all ‘naturalists’ need to reject universals or restrict their ontology to only what is scientifically acceptable. Epistemic naturalists may choose to remain neutral concerning these metaphysical issues.

If naturalism and knowledge of mathematics are, indeed, incompatible, one of two morals might be drawn: either one might take this to be a refutation of naturalism, since of course, we know mathematical truths; or, one might take this to be a refutation of the existence of abstract objects, for if we do have knowledge of mathematics, but we cannot have knowledge of abstract objects, then the subject-matter of our knowledge of mathematics cannot possibly be abstract objects. This forms the basis for the naturalistic attack...
on the *a priori*. Basically, the naturalistic argument first proceeds by articulating the causal conditions requisite for knowledge, and then claims that these causal conditions prevent any of our knowledge from being *a priori*. In other words, *a priori* knowledge is incompatible with plausible (naturalistic) constraints on an adequate theory of knowledge, for the reason that, since most purported *a priori* knowledge is of necessary truths, and the truth conditions for necessary truths refer to abstract objects, then it follows that the existence of *a priori* knowledge is impossible.

More often, however, concerns about mathematical knowledge are used in arguments against naturalized epistemologies. This is, essentially, a reverse of the above argument against the *a priori*. In order for *S* to have knowledge that *p* on a naturalistic analysis of epistemic concepts, it is argued, *S* must have some sort of causal relation to the fact that *p*; if this is not possible, *S* does not actually know *p*. Mathematical knowledge is a paradigm case where *S* does not stand in the appropriate causal relation. In effect, the argument alleges that the causal conditions on knowledge require a relation to particular, contingent features of the world, but one cannot be causally related to the truth-makers for necessary truths (i.e. abstract objects or features of other possible worlds). Thus, a naturalist has no satisfactory account of mathematical knowledge. Let’s call the above argument the *Incompatibility Thesis*.\(^1\) Essentially, what I am arguing is that this thesis is false—an epistemic naturalist\(^2\) can deny the incompatibility between naturalism and *a priori* knowledge of abstract entities by denying that naturalized epistemology requires a causal relation directly between a subject *S* and the entities referred to by the truth condition of *p*.\(^3\) Moreover, one can reject this alleged incompatibility between naturalism and mathematical knowledge by incorporating a naturalistically acceptable reliabilist account of ‘rational intuition’.

In fact, a reliabilist needs to allow for the possibility of rational intuition as a legitimate source of knowledge. Just as a reliabilist account of justification does not fundamen-

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\(^1\) Benacerraf argues that any adequate account of mathematical truth and knowledge needs to homogenously cover all cases, but that all accounts actually fail to mesh semantics with a reasonable epistemology. In one case, accounts of truth that treat mathematical and non-mathematical discourse in relevantly similar ways do so at the cost of leaving it unintelligible how we can have any mathematical knowledge whatsoever. Conversely, those accounts which attribute to mathematical propositions the kinds of truth conditions we can clearly know to obtain, do so at the expense of failing to connect these conditions with any analysis of the sentences which shows how the assigned conditions are conditions of their truth. Thus, epistemological naturalisms fail because they cannot satisfactorily bring together an account of mathematical truth and an account of mathematical knowledge. (Benacerraf [1973], pp. 661-662).

\(^2\) Epistemological naturalism does not require the denial of abstract objects (because epistemological naturalism does not entail *metaphysical naturalism*), though certainly many epistemological naturalists do make this claim (because many epistemological naturalists are also metaphysical naturalists). A metaphysical naturalist typically rejects the possibility of the existence of abstract objects because they are ‘non-natural’. There are some exceptions. Note in particular, Armstrong [1980], who appears to be able to combine a theory of universals with a version of metaphysical naturalism (only universals that are about what is scientifically respectable exist; other universals do not).

\(^3\) Indeed, not all naturalists endorse the Incompatibility Thesis. The Incompatibility Thesis actually has many variants in addition to the one under consideration, such as the claim that *a priori* knowledge is incompatible with the view that all knowledge is revisable. Some naturalists might endorse the above form of the Incompatibility Thesis while rejecting other variations. Perhaps, some naturalists would reject the Incompatibility Thesis in *any* of its guises.
tally depend on any particular way of forming beliefs, so too, it cannot rule out which particular ways of forming beliefs may, in fact, exist and be reliable. I think that this fact about reliabilism sometimes goes unnoticed, and that some philosophers fail to appreciate that one advantage of reliabilism is its ability to accommodate many diverse ways of forming beliefs, since it views this issue as a contingent matter. Thus, reliabilism has within it the means at least to accommodate rational intuition, were such a faculty to exist.

Perhaps, some of the conflict among the proponents and opponents of rational intuition results simply from a disagreement over what ‘rational intuition’ is. ‘Rational intuition’ is usually taken to be a faculty or process by which we come to know certain truths, and the knowledge given by this process is characterized as certain and immediate. Rational intuition is the means by which the truth-values, and in many cases, the necessity, of certain propositions are shown immediately to the mind.

As I see it, philosophers interested in rational intuition and its ability to help explain our mathematical knowledge offer one of two explanations. The first is a conception of rational intuition where the mind makes ‘contact’ with mathematical objects in a way which provides it information about these objects (Gödel is a main proponent of this view), while the second is a ‘no-contact’ theory, which claims that mathematical intuition does not make any contact with abstract entities, but it is nonetheless a source of mathematical knowledge whose subject-matter is these abstract entities. I am concerned with the second alternative. Proponents of the ‘no-contact’ theory agree that human beings exist entirely in space and time, and that abstract mathematical objects, if they exist, exist outside of space and time; but, they disagree that these two claims entail we cannot have knowledge of abstract objects. There is considerable variation in the explanation for how this type of ‘no-contact’ intuition works.

My own view is that rational intuition is a non-

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4 My concern is not to demonstrate the infallibility of intuition—I do not think that this is the case.  
5 Gödel is the most famous proponent of the ‘contact’ theory, and a variation on his account is offered by Maddy [1980]. Balaguer highlights the key features of the contact theory of mathematical intuition: 

(1) Mathematical intuition is analogous to sense perception; 
(2) Mathematical intuition involves a sort of information transfer between abstract mathematical objects and human beings (i.e. something is given to the mind in mathematical intuition—not the objects themselves, but “intuition data”); 
(3) Human beings do not exist entirely in space and time” ([1998], p. 27). Put broadly, Gödel thought that we acquire knowledge of mathematical objects in a similar way to how we acquire knowledge of physical objects—mathematical intuition is analogous to perception. Many find this sort of view on the face of it implausible, for two reasons: first, this requires ‘cross-realm’ contact (i.e. with a non-spatiotemporal realm); and second, abstract objects do not have the ability to “generate information-carrying signals” because they are inert. Regarding the first difficulty, one could try to deny cross-realm contact by claiming that minds themselves are non-spatiotemporal, or that abstract objects are spatiotemporal. The former claim is highly implausible, and we will wait to evaluate the latter until the discussion of Maddy’s view. Regarding the second difficulty, one might challenge the idea that mathematical intuition requires “information transfer.” This strategy seems more promising, though it is not clear what the ‘contact’ between S and the abstract object is really doing if it is not giving S information about the object—why not just switch to a no-contact view? Alternatively, one could claim that “information transfer” and “the passing of signals” seem to beg the question since it looks as if these are causal notions, and two non-spatiotemporal entities do not engage in causal relations (because, according to Balaguer, “information transfer makes sense only when the sender and receiver are both physical objects”) ([Ibid., p. 26].

6 This conception of intuition denies the “information-gathering” thesis. The views vary considerably. For instance, on Resnik’s [1997] view, intuition involves abstraction, where we abstract away from a collection of physical objects and arrive at a mathematical intuition about the set the objects exemplify.
inferential belief-forming process where the entertaining of propositions or certain contemplations result in true beliefs, as well as one being convinced of the truth of these propositions.

According to process reliabilism—one type of naturalized epistemology—a belief possesses justification if it is a result of a reliable belief-producing process. $S$ has a non-inferential justified belief that $p$ if and only if the belief-producing process used by $S$ to form the belief that $p$ is unconditionally reliable. A true belief is an instance of knowledge when the belief’s content connects in the appropriate way with the part of the world that determines its truth, whether the subject who has the belief is aware of this connection or not. The initial claim of incompatibility between mathematical knowledge and reliabilism seems to stem from reliabilism sourcing justification in an agent’s causal relation to the world. But, in fact, on process reliabilism, it is not necessary that $S$ stand in some causal relation to the entities in virtue of which $p$ is true, and so the view is free of any conditions incompatible with abstract objects. This is one key difference between the causal theory and reliabilism: On reliabilism, there need not be a causal connection between $S$ and the object $p$—the fact that $p$ is not required to participate in the generation of the justified belief that $p$; hence, reliabilism does not rule out the possibility of $S$’s having justified beliefs about abstract entities. Mathematical intuition is simply one kind of reliable process type, whose inputs are not abstract numbers, but rather, contemplations of abstract numbers.

Sosa [2003], however, is unsatisfied with this reliabilist solution, suggesting that a problem just reemerges at the next level. He claims that while the reliabilist may have avoided the problem of the causal inertness of abstract objects, she still has to explain how it is that we are able to understand numbers. Why should we think a contemplation of numbers is at all correlated to actual numbers? In other words, Goldman and other reliabilists have no explanation for how our processes of reasoning could put us reliably in touch with abstract objects. Sosa explains that the reason why this is a problem is because, since the process has no direct causal relation to abstracta, it needs an alternative explanation for its reliability—a process’s reliability generally depends on its being causally connected to a fact. “[I]t is still a mystery how these processes could be reliable about mind-transcendent facts without perception or some other causal mechanism to connect the two” (Sosa [2003], p. 179).

Does the causal inertness of abstract objects provide a basis for questioning the reliability of the process alleged to produce beliefs about them? Casullo argues that the

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7 See Goldman [1999] and Maddy [1984].

8 On a reliabilist foundational account of justification, there are two basic types of justification: the base is non-inferentially justified while the rest of our justification is inferential. $S$’s belief that $p$ at $t$ is justified if and only if: (1) $S$’s belief that $p$ at $t$ results from a process that is unconditionally reliable and belief-independent, or from a process that is unconditionally reliable but belief-dependent (non-inferential justification); or (2) $S$’s belief that $p$ at $t$ results from a process that is conditionally reliable, and the input beliefs into the conditionally reliable process are themselves justified (inferential justification).
causal inertness of abstract objects does represent a genuine obstacle to explaining the existence and reliability of rational intuition, which in turn, puts it at risk against being a genuine source of a priori justification:

The causal inertness of abstract entities ensures that they play no role in generating beliefs about them. Hence, if intuition is a reliable process, its reliability cannot be explained along the same lines as the reliability of our best understood cognitive processes. But, given that the underlying neurophysiological processes are unknown, we are not in a position to offer an alternative explanation. The belief that intuition is a reliable process introduces an explanatory gap, which reinforces the concerns about the reliability of the process (Casullo [2003], p. 138).

In other words, because the causal inertness of abstract entities precludes them from playing any sort of causal role, rational intuition, if it were a reliable cognitive process, would be unlike any other process we have, and its reliability would need an alternate explanation; but, we are not in any position to offer an alternative explanation.

Interestingly, Casullo thinks that an analogy with clairvoyance motivates this worry. Specifically, Casullo thinks that clairvoyance is a clear case of a process which is unable to produce justified beliefs. There is a general overall lack of scientific support for the existence and/or reliability of this alleged belief-forming process. Casullo argues that the events reported in clairvoyant beliefs do not appear to play a role in producing those beliefs. This indicates that clairvoyance produces beliefs in a manner different from those reliable cognitive processes that we do understand. Thus, the belief that clairvoyance is a reliable process introduces an explanatory gap, which reinforces the concern about the reliability of the process. Casullo concludes, “even if S’s belief that p is produced by a reliable process of clairvoyance, the presence of evidence that calls into question the possibility and reliability of clairvoyance suggests that…S’s belief is not justified (Casullo [2003], p. 137).” Like clairvoyance, since there is no adequate explanation for the workings of rational intuition, Casullo argues that we have reason to question whether the process exists, and if so, whether it is reliable. In both cases, there exists relevant evidence that calls into question the possibility and reliability of the process.

We see, then, that one major objection to the no-contact theory of intuition is that it does not explain how this faculty is reliable. But, the problem with this objection is that it

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9 Casullo also objects that there is too much controversy over rational intuition. Namely, while some people allege to possess this cognitive process, there is no sort of universal agreement. Casullo thinks it is unlikely that these people have a unique process not present in others; at best, the instances cited are anomalous. Moreover, there also exists disagreement over issues of cognitive access. Science has little to offer by way of supporting any of these claims.

10 Perhaps, Casullo’s reasoning that both intuition and clairvoyance cannot yield justified belief is also due in part to his strict requirements on reliabilism, where not only must process R be reliable, but S must have good reason to believe p is produced by process R and S must have no evidence which might cast doubt on the veracity of p or reliability of R, in addition to there being any such evidence available in the epistemic community. Casullo writes: “Even if we grant that intuition is a reliable belief forming process and S’s belief that p is produced by this process it does not follow that S’s belief that p is justified. In order for a reliable process R to justify a belief that p which it produces in S, S must have a justified belief to the effect that R produced the belief that p. The information available within S’s epistemic community regarding both the reliability of a belief forming process as well as the possibility of its existence is relevant to whether that process justifies the beliefs that it produces in S” (Casullo [1992], p. 579). But, I think that these requirements on justification are too demanding.
assumes that because intuition does not actually make contact with the abstract objects that it must then be “blind” and operate without “checking its work against the mathematical facts” (Balaguer [1998], p. 39). It also presupposes that the only way a belief can be about an object is if it is connected to the object in a certain way (usually causally). Katz explains that mathematical intuition does not need to contact its objects because the beliefs produced through this process are necessarily true. With perception, on the other hand, the objects of perception could have been different, so we need contact with these objects. The response to this line of defense is that even if mathematical truths are necessarily true, the view does not explain how we know that they are true. “There is still a problem explaining the actual correlation between our mathematical beliefs and the mathematical facts” (Field [1989], p. 238).

Yet, the fact that we cannot yet give a detailed account of rational intuition does not count against it. As BonJour explains, perhaps, we are not in a present position to know what sorts of cognitive acts human beings are capable of or what sorts of capacities we might possess. Moreover, why one would suppose that rational intuition does not fit into this picture? In other words, there are many aspects of our mental capacities and operations for which we do not yet have an adequate explanation, particularly on the physicalist-functionalist model: the nature of consciousness itself, the nature of qualia, the nature of conceptual thought, the nature of perceptual consciousness, the nature of introspective awareness, etc. So why is rational intuition singled-out? BonJour argues that, in order for this sort of argument to be compelling—that, because we have no explanation of rational intuition, it does not exist—it would also have to be used against consciousness itself. Thus, it is entirely reasonable to maintain that rational intuition exists, even in the absence of an account of its workings.

Sosa [1998] acknowledges that if intuition is to yield knowledge, it must be a reliable process. But, how do we go about demonstrating the reliability of intuition? Sosa admits that we do not have knowledge of the specific processes involved, and that there is still much we do not know about intuition; but, he argues the same is true for both perception and introspection—they are in an analogous situation. So, requiring a defense of one requires a defense of the others:

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11 BonJour also thinks that although he cannot yet provide a comprehensive defense of his view of mental content (i.e. where the purely intrinsic properties of a thought determine which property it is about), it is nonetheless worth pursuing because it can account for privileged access to the contents of our thoughts and to the abstract properties involved in those contents (i.e. by having them in mind rather than merely representing them in some indirect way). And, in any case, the rational insight theory does not require the truth of this “neo-Thomistic view.” The rational insight theory requires only that we do somehow have access to the contents of our thoughts, i.e. that we can actually think about and have in mind such things as properties and relations. Thus, BonJour says, his speculations over metaphysical commitments are a mere digression. “Perhaps this would not be so if a fairly complete explanation of how rational insight works were required for the moderate rationalist view to be tenable...[but] this does not seem to me to be so” (BonJour [2001], p. 675).

12 BonJour [2001], pp. 673-674. Now, BonJour does admit that the existence of rational intuition is not near as obvious as that of consciousness—although he does think apparent rational intuition to be just this obvious—and he admits that rational intuition can be called into question in ways consciousness cannot; this is why BonJour appeals to dialectical arguments and specific examples.

13 And especially in light of the fact that, according to BonJour, even the arguments against rational intuition appeal to it.
Why make such a demand? Why require, for the defense of intuition or introspection or perception, that one specify in general terms the conditions within which the respective faculty is sufficiently reliable? …Yes, it would give us a better understanding of how it is that we know the things we know through the exercise of the faculty. But what follows if we cannot? [E]ven if our minimalist conception of intuition is quite thin, the most defensible conceptions of introspection and perception seem comparably thin. Any indictment of intuition on grounds of thinness must be brought against introspection and perception as well, by parity of reasoning. If we doubt that we can know about the abstract through intuition, therefore, we must equally doubt that we can know about the inner through introspection, or about our surroundings through perception. For the moment, that seems defense enough of intuition (Sosa [1998], pp. 267-268).

Sosa emphasizes that even if the class of truths known through intuition is rather diverse, which makes determining the reliability of intuition difficult, this is also the case for perception and introspection. Intuition is much more on par with our putative reliable faculties than with the purportedly unreliable faculty of clairvoyance.14

According to Katz, the faculty of intuition offers the best explanation for our a priori knowledge of mathematics. Like Sosa [1998], Katz agrees that those who criticize intuition for failing to explain how a certain class of facts is acquired, need also to criticize introspection and perception for the same reason. Perception, introspection, and intuition are all mental processes resulting in acts of apprehension. Where they differ principally is in the objects which they enable us to apprehend (i.e. external worlds, our own mental states, and numbers and sets, respectively).15 “Intuition is like perception: internal representations are the source of knowledge but do not represent something psychological. What is represented in both cases is something objective…” (Katz [1981], pp. 195-196).

I recognize that more needs to be said here, for to the extent that one has no explanation for how rational intuition might work, this would certainly legitimize arguments

14 I would like to call attention to the fact that on one common explanation of sense perception, it is a process that begins with states internal to the subject (e.g. sense data). Hence, even if there is some sort of causal contact with an external object, the inputs to the perceptual belief-producing process are not the objects themselves, but the internal sensory-states (e.g. sense data) that were triggered by these objects. Of course, this is not to deny that there is a some sort of causal relation in the case of the perception, that is unexplained in the case of rational intuition—the defender of rational intuition has difficulty even construing an indirect connection between abstract objects and the contemplations. Certainly, it is possible that there is some sort of relation between the cognizer and the abstract object that could cause the contemplations, but what kind of relation this is exactly is not spelled out (perhaps, it might be something like an intentional relation, or a relation of acquaintance, or something similar).

15 However, both Hart and Casullo oppose this type of response by drawing attention to the fact that there cannot exist basic psychological processes that generate beliefs about objects that are causally inert. Hart writes: “You must not deny that when you learn something about an object, there is a change in you. Granted conservation of energy, such a change can be accounted for only by some sort of transmission of energy from, ultimately, your environment to, at least proximately, your brain. And I do not see how what you learned about that object can be about that object…unless at least a part of the energy that changed your state came from that object” (Hart [1977], p. 125). To be sure, when you learn new information, there is some sort a causal change, most likely in the brain, but it does not follow from this that the change in your brain state comes directly from the object in your environment that your state is about. See, for instance, Hart [1977]; Mcevoy [2004]; and Casullo [1992] for a more detailed discussion of this issue.
against its existence. Obviously, at least some sort of partial explanation is necessary, where positing rational intuition’s existence does some explanatory work in one’s overall framework. It may be that this is difficult in the case of rational intuition because of the potential circularity involved in attempting to establish a correlation between contemplations and true beliefs. But, a reliabilist often uses track record arguments in defending the reliability of her basic processes. There is no serious problem in using, for example, perceptual beliefs as a basis for justified belief that perception is reliable. Analogously, a reliabilist can use rationally intuited beliefs as a basis for justified belief that rational intuition is reliable.

While my view is not committed to the existence of abstract entities, if these entities do exist, knowledge of them can be accommodated within a reliabilist framework by adopting a no-contact theory of rational intuition. Moreover, the alleged difficulty of explaining rational intuition’s reliability does not speak against its existence. I acknowledge that the claim that a proposition’s truth conditions involve abstract objects, and yet, we have unproblematic knowledge of this proposition without any causal connection to these truth conditions, is controversial. But I think this account represents a promising direction for explaining our a priori mathematical knowledge via intuition. Obviously, there is much work to do in outlining what exactly the nature of this process is, but there is good reason to believe such a forthcoming account would be amenable to integration within a reliabilist framework. As explained before, my own view is that rational intuition is a non-inferential belief-forming process where the entertaining of propositions or certain contemplations result in true beliefs, as well as one being convinced of the truth of these propositions. Again, the numbers themselves do not literally have to be inputs to processes in order for me to have beliefs about them. It is the contemplation of mathematical entities, rather than the entities themselves, that may be the inputs to my mathematical belief-forming processes. In any case, our moral should be clear. The faculty of rational intuition is not inconsistent with a reliabilist naturalistic epistemology. What is more, this can aid the naturalist in reconnecting with the classical debate in that she leaves room for a “rationalist-like” capacity as a possible explanation for some of our a priori knowledge.

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